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The Microscopic Calculation of the Collective Gyromagnetic Ratio of the Deformed Nuclei around Barium

Obliczenia mikroskopowe kolektywnych czynników giromagnetycznych jąder zdeformowanych z okolic baru

Микроскопические вычисления колективных гиромагнитных факторов ядер из области бария

INTRODUCTION

The theoretical description of the collective rotational states of deformed nuclei involves determining not only the rotational energies and connected with them moments of inertia but also the intrinsic quadruple moments Q_2 and the gyromagnetic ratios g_R describing the magnetic moments μ of these states.

The present paper deals with g_R factor for even-even neutron-deficient nuclei with proton number $50 < Z < 82$ and neutron number $50 < N < 82$.

The equilibrium deformations and quadruple moments of these nuclei are calculated in ref. [1, 2]. The theoretical results for moments of inertia are presented in ref. [3, 4].

Those theoretical investigations show that the nuclei around Ba have two minima in the potential energy surface versus quadruple deformation ε. One minimum corresponds to prolate shape of nucleus /$Q_2 > 0$/ and the second one to oblate shape /$Q_2 < 0$/ Both the minima have similar depths. The nuclei around Ba show slight hexadecapole deformation ε_4 in equilibrium points.

It is difficult to decide which of the two minima corresponds to the real situation in ground states of nuclei. The theoretical values of J and $|Q_2|$ for both the shapes lie near the experimental data. Measurements of magnetic moments of rotational states would give additional information about the shapes of nuclei in equilibrium points. The theoretical predictions of factors are very important for this purpose.

METHOD OF THE CALCULATION

The collective rotation of a deformed nucleus takes place around the axis perpendicular to the nuclear symmetry axis and is associated with the collective angular momentum $\vec{R}$. In an odd nucleus $\vec{R}$ couples with the third angular momentum component K of the odd particle to the total angular momentum $\vec{I}$. In the ground state of an even-even nucleus we have $\vec{R} = \vec{I}$.

The collective motion of protons and neutrons connected with the $\vec{R}$ gives rise to a magnetic moment which is associated with the operator [5]:

$$\vec{\mu}_{cou} = \sum_i \vec{\mu}_i = \sum_i \left(g_s^i \vec{s}_i + g_i \vec{l}_i \right) \tag{1}$$

where the sum runs over the paired nucleons.

Gyromagnetic ratio is introduced by the relation

$$\vec{\mu}_{cou} \equiv g_R \vec{R} \tag{2}$$

The cranking approximation gives the following formula for the gyromagnetic ratio g_R

$$g_R = \frac{\hbar^2}{\mathcal{J}} \sum_{\alpha\beta} \frac{\langle \Phi_\alpha | \hat{\mu}_x | \Phi_\beta \rangle \langle \Phi_\beta | \hat{I}_x | \Phi_\alpha \rangle}{\mathcal{E}_\beta - \mathcal{E}_\alpha} \tag{3}$$

$\hat{I}_x = \sum_i \hat{j}_x^i$ generate the rotation around x axis and $\mathcal{J}$ is the moment of inertia of the nucleus. $\mathcal{E}_{\alpha(\beta)}$ denotes the energy of the state $|\Phi_{\alpha(\beta)}\rangle$

When we include the pairing interaction by using the BCS approximation we come to the following expression for g_R [6]

$$g_R = \left\{ \mathcal{J}_p + (g_s^p - 1) W_p + g_s^n W_n \right\} / \mathcal{J} \tag{4}$$

where

$$W = 2\hbar^2 \left\{ \sum_{\nu\nu'} \frac{\langle \nu' | \hat{j}_x | \nu \rangle \langle \nu | \hat{s}_x | \nu' \rangle}{E_\nu + E_{\nu'}} \left(u_\nu v_{\nu'} - u_{\nu'} v_\nu \right)^2 + \sum_{\mu\mu'} \frac{\langle \mu | \hat{s}_x | -\mu' \rangle \langle -\mu' | \hat{j}_x | \mu \rangle}{E_\mu + E_{\mu'}} \left(u_\mu v_{\mu'} - v_\mu u_{\mu'} \right)^2 \right\} \tag{5}$$

The moment of inertia is calculated, using the cranking model formula

$$\mathcal{J} = 2\hbar^2 \left\{ \sum_{\nu\nu'} \frac{|\langle \nu | \hat{j}_x | \nu' \rangle|^2}{E_\nu + E_{\nu'}} \left(u_\mu v_{\mu'} - u_{\mu'} v_\mu \right)^2 + \sum_{\mu\mu'} \frac{|\langle \mu | j_x | -\mu' \rangle|^2}{E_\mu + E_{\mu'}} \left(u_\mu v_{\mu'} - u_{\mu'} v_\mu \right)^2 \right\} \tag{6}$$

In formulae [5, 6] E_ν denotes the quasi-particle energy; u_ν, v_ν are the BCS occupation factors. The states μ, μ' correspond to the third angular momentum components $K_\mu = -K_{\mu'} = \frac{1}{2}$

PARAMETERS OF THE CALCULATION

The numerical calculations were performed for even-even nuclei in the new region of deformation $50 < Z, N < 82$ /the neutron deficient nuclei/.

We assume the Nilsson potential /with $\langle \vec{l}^2 \rangle$ term/ as a single-particle potential. Quadruple ε and hexadecapole ε_4 deformations are taken into account [7].

We adopt the following values for the parameters and of the Nilsson potential [1]:

a/ the same as for the rare earth i.e.

$\varkappa_p = 0.0637$, $\mu_p = 0.60$ $\varkappa_p = 0.0637$, $\mu_p = 0.42$

b/ the extrapolated set; the same values for $\varkappa_p$ and μ_p as in the case /a/ and $\varkappa_p = 0.0637$, $\mu_n = 0.491$

The numerical values g_R obtained in the cases /a/ and /b/ are close, the differences are smaller than 5%, so we give later only the values for the case /a/.

We take into account 24 levels nearest to the Fermi level and the following pairing strength $G_n \cdot A = 25$ MeV for neutrons, and $G_p \cdot A = 28.5$ MeV for protons solving the pairing equation [1]. The equilibrium deformation of nuclei were calculated using the Strutinsky prescription [7].

RESULTS

The results are presented in Fig. 1. The values of g_R are drawn versus nucleon number A for the well deformed nuclei

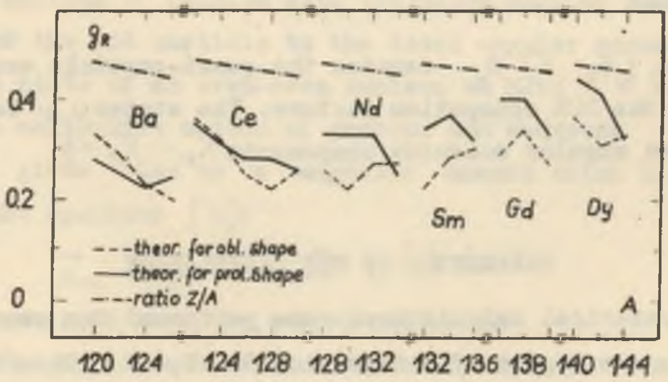

Fig. 1. The theoretical values of g_R versus mass number A for even nuclei around Barium. The numbers corresponding to oblate /solid lines/ and prolate /dashed lines/ of nuclei. The dashed-dotted lines show the Z/A ratio

Edef$\geqslant$2 MeV/. Two values of g_R are given for each nuclei, they correspond to the two possible /in the theoretical way; see Fig. 2/ shapes of nucleus, the oblate and the prolate ones .

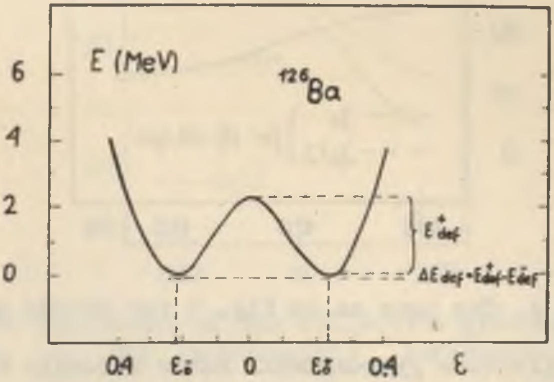

Fig. 2. The dependence of potential energy of ^{126}Ba on quadruple deformation ε .

The dashed dotted lines correspond to the ratio of proton number over mass number A.

The efect of the two last terms in the equation /4/ / W_p and W_n / is shown in Figs 3 and 4. It is seen that their contribution to the gyromagnetic ratio of Xe isotopes in 20% and

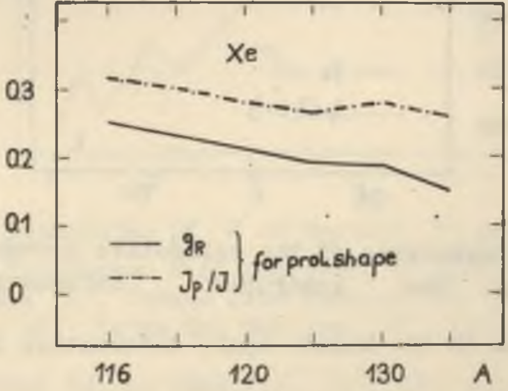

Fig. 3. The comparison of the values g_R calculated for Xe isotopes with the leading term J_p/J in formula /4/ for g_R .

more, for the heavier nuclei the contribution is smaller /e.g Ce isotopes/.

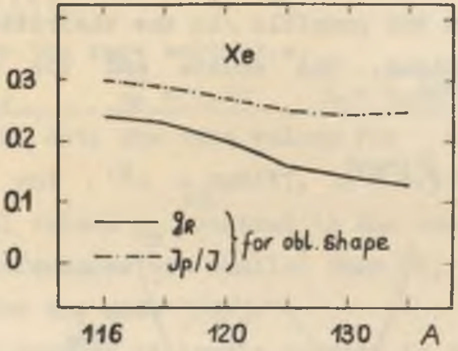

Fig. 4. The same as on fig. 3 for oblate minimum

The collective gyromagnetic ratio strongly depends on the deformations ε /fig. 5/ and ε_4 /fig. 6/. The dependence of g_R of ^{126}Ba on the hexadecapole deformation is especially strong.

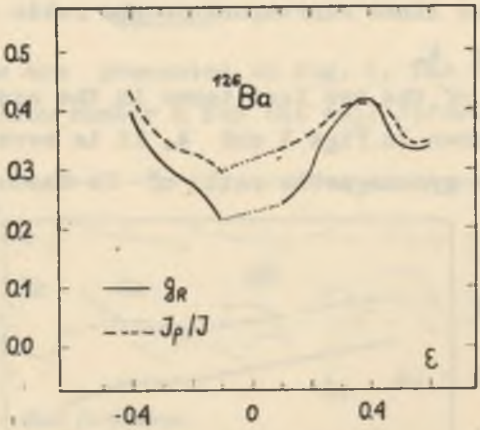

Fig. 5. The dependence of the collective gyromagnetic ratio of ^{126}Ba on the quadruple deformation ε

The gyromagnetic ratio does not depend strongly on the pairing forces strength, with the change of G by 10% the change g_R less than 10% follows. The single particle

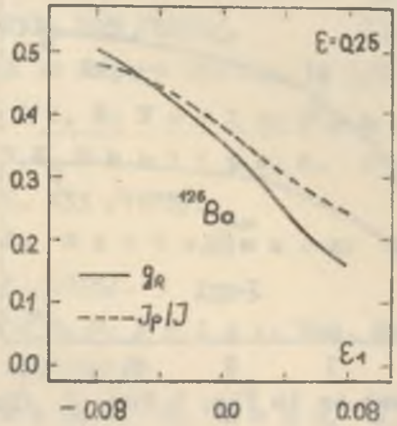

Fig. 6. The dependence of the collective gyromagnetic ratio of ^{126}Ba on the hexadecapole deformation ε_4.

structure of the gyromagnetic ratio is presented in figs. 7, 8. W and J of protons and neutrons are plotted versus the

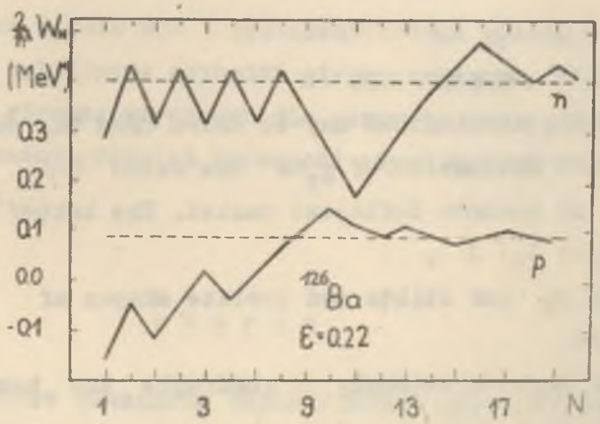

Fig. 7. The dependence of W of protons W_p and neutrons W_n of ^{126}Ba on the number N of the pairs of states taken in the sum in formula /5/. Dashed line denotes the values of W with all terms in the sum /5/ included.

pairs of states in the sums /5/, /6/. The pairs are ordered in magnitude of the contribution to the total sum /in the descending order/. It is seen that the largest element in the sum W_p /or W_n / is the same order of magnitude as the whole

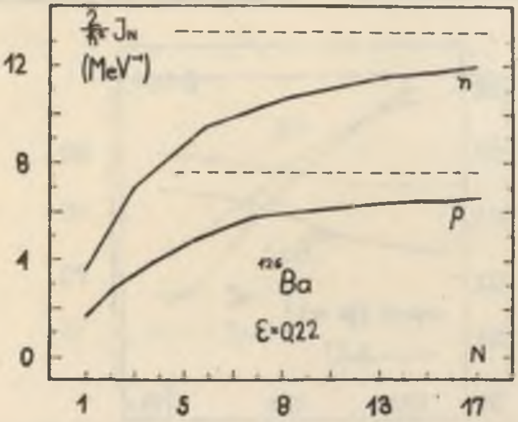

Fig. 8. The same as in Fig. 7 for J /formula 6/

sum, so we can state that factor W is not a good collective parameter of a nucleus. The moment of inertia presents a more collective nature, the largest element in the sum is equal to about 12 % of the total.

CONCLUSIONS

The following conclusions may be drawn from the calculation:

1/ Frequently used estimation of g_R as the ratio Z/A is wrong in the case of neutron deficient nuclei. The better estimation gives the ratio I_p/I .

2/ The values of g_R for oblate end prolate shapes of nucleus are comparable.

3/ The g_R factor depends strongly on quadruple and hexadecapole deformation.

4/ The g_R factor is a good collective parameter.

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STRESZCZENIE

W ramach modelu BCS i jednocząstkowego potencjału Nilssona wyliczono kolektywne czynniki giromagnetyczne. Podano wartości czynników giromagnetycznych dla parzysto-parzystych jąder z okolic Ba. Zbadano również zależność czynników giromagnetycznych od deformacji.

РЕЗЮМЕ

В работе вычислены гиромагнитные факторы парных ядер из области бария. Исследовано тоже зависимость гиромагнитных факторов от деформации. Все расчёты проведены в модели BCS с одночастичным потенциалом Нилссона.

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