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# On a Perfect Fluid with Nonlocal Interactions

O cieczy doskonałej z oddziaływaniami nielokalnymi

Об идеальной жидкости с нелокальными взаимодействиями

## I. INTRODUCTION

The nonlocal theory of fluids has been introduced by Eringen in [1] where both localized equations of motion and a constitutive theory for nonlocal non heat conducting stokesian fluid are formulated. Nonlocality is introduced into the theory in two ways. First, classical field quantities - free energy density $\psi$, entropy density $\eta$, and stress tensor $\bar{t}$ .- - become functionals defined over the motion of the body $\bar{x}$ and the temperature distribution $\theta$, and over their time and space derivatives (e.g. $\bar{v}$, grad $\bar{v}$, $\dot{\theta}$ etc.). They are also functions of the latter quantities taken at a given point $\bar{x}$. Second, some new expressions, so called localization residuals, appear in the equations of motion. The residuals are functionals and functions of the same character as those of the fields. They describe a direct long-range transfer of momentum, moment of momentum, energy and entropy between various parts of the body. It is remarkable in Eringen's theory that if residuals are independent of $\bar{v}$ (and $\dot{\bar{v}}$, $\dot{\theta}$, and $(\mathrm{grad}\,\bar{v})^{T}$ ), then the body force residual $\hat{\bar{f}}$ vanishes identically. This result, however, arouses two doubts. One, the assumption that residuals do not depend on velocity (nor on other rates) is inconsistent with the principle of equipresence, cf. e.g. [2]. Two, the vanishing of the body force residual seems to be a strong restriction to the theory since one can easily imagine a medium with non zero nonlocal force that could generate energy and entropy effects.

This paper aims at presenting a nonlocal fluid where the long-range force does not generally vanish. It is assumed that the fluid is locally ideal (incompressible, unviscous, nonpolar) and non heat conducting. The nonlocal effects consist in long-range transfer and production of momentum, internal energy and entropy, and in general they depend on the distance between interacting particles, on their relative velocity and their temperatures. It is shown that even such simple assumptions lead to significant modifications of velocity profiles for a flow of such a nonlocal fluid.

## II. EQUATIONS OF MOTION

The basic equations of motion together with the Clausius-Duhem inequality for a nonlocal non heat conducting ideal stokesian fluid take the form (cf. [1] ):

$$\mathrm{div}\,\bar{v} = 0,$$

$$\varrho\dot{\bar{v}} - \varrho\bar{f} + \mathrm{grad}\,p = \varrho\hat{\bar{f}}, \tag{1}$$

$$0 = \hat{\bar{l}} - \bar{x}\times\hat{\bar{f}},$$

$$\varrho\dot{\varepsilon} - \varrho h = \varrho\hat{h} - \varrho\hat{\bar{f}}\cdot\bar{v},$$
$$-\frac{1}{\theta}\varrho(\dot{\psi} + \eta\cdot\dot{\theta}) \geqslant \varrho(\hat{b} - \frac{1}{\theta}\hat{h} + \frac{1}{\theta}\hat{\bar{f}}\cdot\bar{v}). \qquad (1)$$

The notation is adopted from [1] . So we have: $\varrho$ = mass density, $\bar{f}$ = external body force density, p = hydrostatic pressure, $\varepsilon$ = internal energy density, h = external energy source, and $\hat{\bar{f}}$ = nonlocal body force density or momentum residual, $\hat{\bar{1}}$ = = nonlocal body couple density or the residual of moment of momentum, $\hat{h}$ = nonlocal energy production or energy residual, $\hat{b}$ = nonlocal entropy production or entropy residual. The body $\mathcal{B}$ occupies the region $\mathcal{V}$ of 3-dimensional Euclidean point space $E_3$. The fields $\bar{v}(\bar{x})$ and $\theta(\bar{x})$, $\bar{x}\in\mathcal{V}$, designate the velocity and the temperature of a particle occupying the position $\bar{x}$, respectively. We assume that all the fields and residuals are sufficiently regular to possess time and space derivatives of desired order. A superposed dot stands for the material time derivative. Two bars above a letter signify a tensor quantity.

The localization residuals obey the following identity, [1]:

$$\int_{\mathcal{V}} \varrho\left\{\hat{\bar{f}}, \hat{\bar{1}}, \hat{h}, \hat{b}\right\} \cdot dV(\bar{x}) = 0, \qquad (2)$$

which ascertains that the global nonlocal production of momentum, moment of momentum, energy and entropy in the whole region $\mathcal{V}$ is naught. In relations (1), on the right-hand side, we have expressions describing total effects of nonlocal transfer and production of the quantities mentioned above:

$$\hat{\bar{f}},$$
$$\hat{\bar{1}}_0 = \hat{\bar{1}} - \bar{x}\times\hat{\bar{f}},$$
$$\hat{h}_0 = \hat{h} - \hat{\bar{f}}\cdot\bar{v}, \qquad (3)$$
$$\hat{b}_0 = \hat{b} - \frac{1}{\theta}\hat{h} + \frac{1}{\theta}\hat{\bar{f}}\cdot\bar{v}.$$

Of course, upon (1.3) we have $\hat{\bar{1}}_0 = 0$ so that $\hat{\bar{1}}$ is fully determined by $\hat{\bar{f}}$.

The body is acted upon by an external body force $\bar{f}$, external pressure $p$ acting on the boundary $\partial\mathcal{V}$, and external rate of supply of heat $h$. The body is forced to change its spatial configuration and temperature distribution. At the same time the processes of transfer and production of energy and entropy take place. They are all described by the quantities (3). In order to determine the response of the body to a given external action one must solve the equations (1) together with a set of constitutive equations which express the quantities (3) as functionals of the independent variables $\bar{x}$ and $\theta$. This is the scope of the following section.

## III. CONSTITUTIVE EQUATIONS

As was mentioned in the Introduction we now come to consider a medium that is locally a perfect non heat conducting fluid. For such a fluid we have:

$$\bar{\bar{t}} = -p\bar{\bar{I}}, \qquad \bar{q} = 0, \qquad \psi = \psi(\theta), \qquad \eta = \eta(\theta), \qquad (4)$$

where $\bar{\bar{I}}$ is the identity tensor and $\bar{q}$ is a heat flux vector. For the nonlocal residuals (3) we adopt the constitutive assumption that they are functionals of $\bar{x}'$, $\bar{v}(\bar{x}')$ and $\theta(\bar{x}')$ over the whole region $\mathcal{V}$ and functions of $\bar{x}$, $\bar{v}(\bar{x})$ and $\theta(\bar{x})$ at a certain point $\bar{x}$. Having introduced the abbreviation

$$\Lambda(\bar{x}) = \left\{ \bar{x}, \ \bar{v}(\bar{x}), \ \theta(\bar{x}) \right\} \qquad (5)$$

one can write the following general expressions for residuals:

$$\begin{aligned} \hat{\bar{f}}(\bar{x}) &= \bar{\mathcal{F}}\left[\Lambda(\bar{x}'); \Lambda(\bar{x})\right], \\ \hat{h}_0(\bar{x}) &= \mathcal{H}_0\left[\Lambda(\bar{x}'); \Lambda(\bar{x})\right], \qquad (6) \\ \hat{b}_0(\bar{x}) &= \mathcal{B}_0\left[\Lambda(\bar{x}'); \Lambda(\bar{x})\right], \end{aligned}$$

where the bracket stands for the functional dependence on $\Lambda(\bar{x}')$, $\bar{x}' \in \mathcal{V}$, and for the function dependence on $\Lambda(\bar{x})$.

The constitutive equations (6) must be subject to several conditions of general character, cf. [2] and [3]. They guarantee proper behaviour of the functionals $\mathcal{F}$, $\mathcal{H}_0$ and $\mathcal{B}_0$ so as not to violate the fundamental principles and laws of physics. Below only those conditions are cited which will be referred to later on. The constitutive postulates state that the form of the constitutive functionals must

(a) not violate the Clausius-Duhem inequality (1.5) (the entropy principle);
(b) be invariant with respect to arbitrary rigid motions (the principle of objectivity);
(c) provide the vanishing of the global effect (2);
(d) tend to the form of a local theory in the classical continuum limit (the principle of correspondence).

Here we add one more postulate which states that

(e) nonlocal interactions are additive with respect to subbodies.

The last condition is not of such general and common character. It follows from the assumption that the nonlocal interactions are two-body interactions. One can equally well derive a nonlocal theory without taking that condition into account. Now let us consider the consequences of the above axioms.

The condition (e) together with the natural hypothesis of continuity and boundedness of functionals $\mathcal{F}$, $\mathcal{H}_0$ and $\mathcal{B}_0$, upon the Friedman-Katz theorem [4], leads to the following integral representation of (6):

$$
\begin{aligned}
\hat{f}(\bar{x}) &= \int_{\mathcal{V}} \rho\, \tilde{f}(\Lambda(\bar{x}'), \Lambda(\bar{x}))\cdot dV(\bar{x}'),\\
\hat{h}_0(\bar{x}) &= \int_{\mathcal{V}} \rho\, \tilde{h}_0(\Lambda(\bar{x}'), \Lambda(\bar{x}))\cdot dV(\bar{x}'),\\
\hat{b}_0(\bar{x}) &= \int_{\mathcal{V}} \rho\, \tilde{b}_0(\Lambda(\bar{x}'), \Lambda(\bar{x}))\cdot dV(\bar{x}'),
\end{aligned}
\tag{7}
$$

where the generating functions or influence functions $\bar{\bar{f}}$, $\tilde{h}_0$ and $\tilde{b}_0$ characterize the nonlocal properties of the material of the body while the integral represents the dependence on the shape and volume of the body.

Condition (c) after substituting (7) into (2) leads to the following equations:

$$\begin{aligned}
&\iint_{\Upsilon\Upsilon} \bar{\bar{f}}(\Lambda',\Lambda)\cdot dV'\, dV = 0,\\
&\iint_{\Upsilon\Upsilon} \bar{x}\times\bar{\bar{f}}(\Lambda',\Lambda)\cdot dV'\, dV = 0,\\
&\iint_{\Upsilon\Upsilon} (\tilde{h}_0(\Lambda',\Lambda) + \bar{v}(\bar{x})\bullet\bar{\bar{f}}(\Lambda',\Lambda))\cdot dV'\, dV = 0,\\
&\iint_{\Upsilon\Upsilon} (\tilde{b}_0(\Lambda',\Lambda) + \frac{1}{\theta(x)}\tilde{h}_0(\Lambda',\Lambda))\cdot dV'\, dV = 0,
\end{aligned} \tag{8}$$

where $\Lambda' = \Lambda(\bar{x}')$ and $dV' = dV(\bar{x}')$. The above identities do not restrict a class of motions of the body $\mathcal{B}$, viz. the class of variables $\Lambda$. They must be satisfied for all thermomechanical processes, i.e. for all motions and all temperature distributions. What is more, from the same nonlocal material defined by the functions $\bar{\bar{f}}$, $\tilde{h}_0$ and $\tilde{b}_0$ one can make up bodies of arbitrary volumes and arbitrary shapes. Then, one can force them to move in an arbitrary way. Therefore it follows that the relations (8) must be satisfied identically for both all functions $\Lambda(\bar{x})$ and all regions $\Upsilon$. If the functions $\bar{\bar{f}}$, $\tilde{h}_0$ and $\tilde{b}_0$ are continuous with respect to variables $\Lambda'$ and $\Lambda$, then the identities (8) cannot be maintained for all functions $\Lambda$ and all regions $\Upsilon$ unless the expressions under the integrals are antisymmetric with respect to replacing $\bar{x}$ by $\bar{x}'$ and $\bar{x}'$ by $\bar{x}$. After some calculations one obtains the following formulae:

$$\begin{aligned}
\bar{\bar{f}}(\Lambda',\Lambda) &= -\bar{\bar{f}}(\Lambda,\Lambda'),\\
\bar{\chi}\times\bar{\bar{f}} &= 0,\\
\tilde{h}_0 &= \gamma(\Lambda',\Lambda) + \frac{1}{2}\dot{\bar{\chi}}\bullet\bar{\bar{f}},\\
\tilde{b}_0 &= \beta(\Lambda',\Lambda) + \frac{1}{2}\cdot(\frac{1}{\theta'} - \frac{1}{\theta})\cdot\gamma - \frac{1}{4}(\frac{1}{\theta'} + \frac{1}{\theta})\cdot\dot{\bar{\chi}}\bullet\tilde{\bar{f}},
\end{aligned} \tag{9}$$

where $\bar{\chi} = \bar{x}' - \bar{x}$, $\dot{\bar{\chi}} = \bar{v}(\bar{x}') - \bar{v}(\bar{x})$ and $\gamma$ and $\beta$ are antisymmetric under transposition $\Lambda' \longleftrightarrow \Lambda$. The first equation represents Newton's third law. The second one states that the nonlocal force acts along the relative position vector $\bar{\chi}$. The third and fourth ones give the general form of generating functions $\bar{h}_0$ and $\bar{b}_0$, respectively. As one can see, the nonlocal energetic effect at a point $\bar{x}$ consists of two processes: the transfer of energy from a particle $\bar{x}'$ and the production of energy due to the nonlocal force interaction between $\bar{x}'$ and $\bar{x}$. Similarly, the nonlocal entropy effect consists of three processes: the transfer of entropy from a particle $\bar{x}'$, the production of entropy due to the energy transfer from $\bar{x}'$ to $\bar{x}$, and the production of entropy due to the dissipation of energy caused by the nonlocal force between $\bar{x}'$ and $\bar{x}$.

The next condition to be satisfied is the invariance requirement (b). Under superposed rigid body motions represented by

$$\bar{x} \longrightarrow \bar{x}^* = \bar{\bar{Q}}(t)\cdot\bar{x} + \bar{c}(t), \tag{10}$$

where $\bar{\bar{Q}}$ is a proper orthogonal tensor function of time and $\bar{c}$ is a vector function of time, it is demanded that the functions $\theta$, $\gamma$, $\beta$ and $\bar{\bar{f}}$ all be unaltered apart from orientation in the case of vector:

$$\theta^* = \theta, \quad \gamma^* = \gamma, \quad \beta^* = \beta, \quad \bar{\bar{f}}^* = \bar{\bar{Q}}\cdot\bar{\bar{f}}. \tag{11}$$

From the quantities $\Lambda$ and $\Lambda'$, that is $\bar{x}$, $\bar{v}$, $\theta$, $\bar{x}'$, $\bar{v}'$ and $\theta'$, one can compose four scalars and only one vector:

$$\chi, \quad \dot{\chi}, \quad \theta, \quad \theta' \quad \text{and} \quad \bar{\chi}, \tag{12}$$

where $\chi^2 = \bar{\chi}\cdot\bar{\chi}$ and $\dot{\chi}\chi = \dot{\bar{\chi}}\cdot\bar{\chi}$. Therefore, the only possible representation of $\bar{\bar{f}}$, $\gamma$ and $\beta$ is:

$$\begin{aligned} \bar{\bar{f}} &= \varphi(\chi, \dot{\chi}, \theta, \theta')\,\bar{\chi}, \\ \gamma &= \gamma(\chi, \dot{\chi}, \theta, \theta'), \end{aligned} \tag{13}$$

$$\beta = \beta(\chi, \dot{\chi}, \theta, \theta') \quad . \qquad (13)$$

and $\varphi$, $\gamma$ and $\beta$ are arbitrary scalar-valued functions with respect to (10). Of course $\varphi$ is symmetric and $\gamma$ and $\beta$ are antisymmetric upon transformation $\dot{\chi} \leftrightarrow -\dot{\chi}$ and $\theta' \leftrightarrow \theta$. One can easily see that the functions $\bar{h}_0$ and $\bar{b}_0$ possess appropriate symmetries and invariance properties as well.

Let us now proceed to determine the consequence of the thermodynamic admissibility. Condition (a) ascertains that every solution of the equations of motion (1.1 - 1.4) must identically satisfy the Clausius-Duhem inequality (1.5). After substituting (13) to (1.5) we obtain:

$$- \frac{1}{\theta} \varrho \left(\frac{d\psi}{d\theta} + \eta\right) \dot{\theta} + \qquad (14)$$
$$+ \varrho^2 \int_{\Upsilon} \left( - \beta - \frac{1}{2} \cdot \left(\frac{1}{\theta'} - \frac{1}{\theta}\right) \cdot \gamma + \frac{1}{4} \cdot \left(\frac{1}{\theta'} + \frac{1}{\theta}\right) \cdot \dot{\chi} \chi \varphi \right) \cdot dV' \geq 0 .$$

This inequality must hold for arbitrary fields $\theta$, $\dot{\theta}$ and $\dot{\chi}$. Consider first a rigid motion of the body $\mathfrak{B}$. For such a motion we have $\dot{\chi} = 0$, and so the expression under the integral vanishes. Thus, we get the classical relation between entropy and free energy:

$$\eta = - \frac{d\psi}{d\theta} . \qquad (15)$$

Now the inequality (14) is reduced to the expression with the integral only. Upon using the same arguments as those exploited in the consideration of condition (c) we conclude that the inequality (14) must be satisfied locally, that is the expression under the integral must be non-negative. Having taken into account the symmetry properties of $\varphi$, $\gamma$ and $\beta$ we obtain finally

$$- \frac{1}{2} \left(\frac{1}{\theta'} - \frac{1}{\theta}\right) \cdot \gamma + \frac{1}{4} \cdot \left(\frac{1}{\theta'} + \frac{1}{\theta}\right) \chi \dot{\chi} \varphi \geq |\beta| \geq 0, \qquad (16)$$

which is the condition for thermodynamical admissibility of generating functions $\varphi$, $\gamma$ and $\beta$. In particular, if one assumes that within the body there is a nonlocal force only ($\varphi \neq 0$), other interactions being naught ($\gamma = \beta = 0$), one gets

$$\dot{\chi} \cdot \varphi(\chi, \dot{\chi}, \theta, \theta') \geqslant 0. \tag{17}$$

This inequality states that the nonlocal force is dissipative so one may call it the nonlocal viscosity. Of course, the nonlocal viscosity force gives rise both to nonlocal production of internal energy and to nonlocal production of entropy:

$$\begin{aligned} \tilde{h}_0 &= \tfrac{1}{2} \cdot \chi \dot{\chi} \varphi \geqslant 0, \\ \tilde{b}_0 &= \tfrac{1}{4} \cdot \left(\tfrac{1}{\theta'} + \tfrac{1}{\theta}\right) \chi \dot{\chi} \varphi \geqslant 0. \end{aligned} \tag{18}$$

Both of them are positive and each of them is the same at both interacting points $\bar{x}$ and $\bar{x}'$.

Finally, we must analyse the principle of correspondence (d). Nonlocality can be roughly characterized by a parameter $a$ which stands for a range (or a mean range or an effective range) of the nonlocal interaction. Similarly, every problem of motion is characterized by a parameter $\ell$ being a typical length in this problem. The specific nonlocal effects can only be detected in the range of $\ell/a \sim 1$. When $\ell/a \gg 1$, then the nonlocal theory should not differ from its local counterpart. In the limit $a/\ell \to 0$ the equation (1.2) should turn into the well known Navier-Stokes equation. Thus, one must have the following asymptotic behaviour of the generating function $\varphi \bar{\chi}$:

$$\rho^2 \int_{\mathcal{V}} \varphi(\chi, \dot{\chi}, \theta, \theta') \cdot \bar{\chi} \cdot dV(\bar{x}') \xrightarrow[a/\ell \to 0]{} \mu(\theta) \cdot \Delta \bar{v}(\bar{x}), \tag{19}$$

where $\mu(\theta)$ is a (local) viscosity coefficient.

Similar conditions must be imposed on the generating functions of energy and entropy residuals. In a classical theory limit one should obtain a classical expression for the energy production due to a heat flow, e.g. $-\varkappa \operatorname{div} \bar{g}$, where $\varkappa$ is a heat conductivity coefficient and $\bar{g} = \operatorname{grad} \theta$. For the entropy flux the situation is similar so we will not pay more attention to it. It is obvious that the ranges of $\gamma$ and $\beta$ may in general be different from that of $\varphi$.

Summing up the above considerations we conclude that within a nonlocal medium where the generating functions of residuals depend on the distance between particles, on their relative velocity and their temperatures there may occur a priori three groups of effects:

(I) Any two particles interact with each other with a long-range force. This interaction gives rise to dissipation of energy and to production of entropy.

(II) Any two particles exchange energy in a long-range way. The energy flows from the particle of higher temperature to the particle of lower temperature. That flow is accompanied by a production of entropy.

(III) Any two particles exchange entropy by a long-range entropy transfer. The entropy can flow both from the particle of higher temperature to the particle of lower temperature and the other way round. This flow of entropy can only appear when the productions of entropy named above under (I) and (II) take place and its absolute value cannot exceed them.

The processes presented above are described by the generating functions $\varphi\bar{\chi}$, $\chi$ and $\beta$ which satisfy condition (15) and appropriate conditions resulting from postulate (d). In the next section we exploit them to solve an example of a flow of nonlocal fluid.

## IV. POISEUILLE FLOW THROUGH A CAPILLARY

Here the results of calculations of velocity profiles for a stationary laminar flow of a nonlocal fluid through a circular capillary are presented. In order to perform the calculations one must choose a shape of generating function $\varphi\bar{\chi}$. Let it be, within the first approximation limit, linear in $\dot{\chi}$ and independent of temperature:

$$\bar{f}(\bar{x}) = \rho^2 \int_{\Upsilon} A(\chi)\cdot \frac{\bar{\chi}\bar{\chi}\cdot(\bar{v}(\bar{x}') - \bar{v}(\bar{x}))}{\chi^2}\cdot dV(\bar{x}'). \tag{20}$$

Conditions (16) and (19) lead to the following formulae:

$$\begin{gathered} A(\chi) \geqslant 0, \\ \int_0^{\infty} A(\chi)\cdot\chi^4\cdot d\chi = \frac{15}{2\pi}\,\frac{\mu}{\rho^2}. \end{gathered} \tag{21}$$

The flow is induced by a constant pressure gradient and an interaction with the boundary. Remaining within the frames of a nonlocal velocity-dependent model we assume that the interaction with the boundary, viz. the external region $C\Upsilon$, is also nonlocal and has the same form as the interaction within the fluid. Thus, we choose

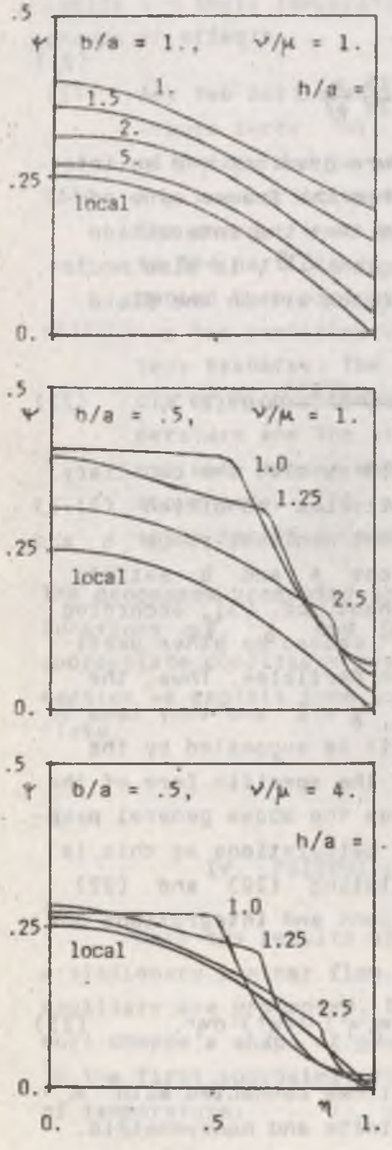
$$\bar{f}(\bar{x}) = \rho^2 \cdot \int_{C\Upsilon} B(\chi)\cdot \frac{\bar{\chi}\bar{\chi}\cdot(\bar{V}(\bar{x}') - \bar{v}(\bar{x}))}{\chi^2}\cdot dV(\bar{x}'), \tag{22}$$

where $\bar{V}(\bar{x})$ is a definite velocity field outside the capillary. Further we put $\bar{V} = 0$. Function B satisfies condition (21.1) and condition (21.2) with some different nonlocal range b and viscosity coefficient $\nu$. Both functions A and B satisfy additionally the postulate of neighbourhood, cf. [5], according to which the influence on a particle $\bar{x}$ caused by other particles decreases with the distance between particles. Thus, the functions A and B have maximum for $\chi = 0$ and they tend to zero quickly enough with $\chi \to \infty$. As it is suggested by the results of other calculations, cf. [5], the specific form of the generating functions is not as crucial as the above general properties. We shall not go into details of calculations as this is not the aim of this paper. After substituting (20) and (22) into (1.2) and performing transformations and integrations one obtains only one equation

$$0 = 1 - \delta(\eta)\cdot\psi(\eta) + \int_0^1 \varkappa(\eta,\eta')\cdot\psi(\eta')\cdot d\eta'. \tag{23}$$

Here $\delta$ and $\varkappa$ are some definite functions connected with A and B. The kernel $\varkappa$ is positive-definite and nonsymmetric.

$\psi(\eta) = (\mu/(g_0 h^2))\, v(r)$ is the dimensionless velocity, $\eta = r/h$ is the dimensionless radial coordinate in the appropriate cylindrical system of coordinates, $-g_0 = \partial p/\partial z$ is the pressure gradient in the direction of the flow, $h$ is the capillary radius.




The results of the calculations in the form of velocity profiles are shown in the figures as functions of three parameters: the capillary radius $h/a$, the range of interaction with the boundary $b/a$, and the intensity of the interaction with the boundary $\nu/\mu$. As can be seen in the figures, for wide capillaries ($h/a > 1$) the profile lies near the parabolic curve of the classical flow. The interaction with the boundary causes a strong resistance to the flow near the wall within a layer of thickness of about the range of interaction with the boundary. The effect is very conspicuous for short and strong interaction with the wall ($b/a = .5$, $\nu/\mu = 4$). Of course the deviations of the profile give rise to changes of the volume rate of flow as compared with that of the local fluid. The estimation of the form of influence functions A and B must be obtained upon consideration of more complicated problems (e.g. time-depemdent flows, turbulent flows, flows through capillaries of a finite length or of non circular section, etc.) and upon comparison with experiment.

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STRESZCZENIE

Przedstawiono model cieczy nielokalnej. W cieczy występują zależące od prędkości i temperatury zjawiska siły dalekiego zasięgu, dalekozasięgowego przekazu energii i dalekozasięgowego przekazu entropii. Otrzymano warunki dopuszczalności dla funkcji tworzących residuów. Profile prędkości dla przepływu Poiseuilla przez kapilarę wykazują różnice w stosunku do profilów klasycznych.

Р Е З Ю М Е

Приводится модель нелокальной жидкости. В жидкости проявляются, зависящие от скорости и температуры, эффекты сил дальнего действия, дальнедистанционные передачи энергии и энтропии. Получены условия применимости для функции образующих резидуов. Профили скоростей для потока Пуазейля через капилляр проявляют различие по отношению к классическим профилям.