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Instytut Fizyki Teoretycznej
Uniwersytet Warszawski

Z. SZYMAŃSKI

Possible Reasons for the Rigid-Rotor Like Behaviour of the Fast Rotating Nuclei

O możliwych przyczynach zachowywania się szybko rotujących jąder jak sztywnego rotatora

Возможные причины поведения быстровращающихся ядер как жестких роторов

Many interesting features of the atomic nucleus do not seem to manifest themselves until the nucleus undergoes a fast rotation. When the rotational frequency exceeds a certain critical value the well-known static superfluid correlations existing in a cold nonrotating nucleus may be destroyed. In this state the system seems to be mostly governed by the interplay between the rotational couplings and the single-particle structure. Recent experiments (see e.g.refs.[1-4]) have shown that angular momentum I of the rotating nucleus is proportional to its rotational frequency ω in the region of high values of I and . This has all the features of the

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& in collaboration with J.Dudek and W.Nazarewicz rigid rotation since both moments of inertia kinematical $\mathcal{J}^{(1)}$ and dynamical $\mathcal{J}^{(2)}$ are equal to each other [5]

$$\mathcal{J}^{(1)} = \mathcal{J}^{(2)} = \mathcal{J}_{rigid} \tag{1}$$

and independent of ω .Let us analyse this behaviour assuming that the influence of the superfluid correlations can be disregarded.

It has become customary to describe the very high spin states by means of an external rotation of the deformed nuclear potential (cranking model). It has been also established (see e.g.refs.[6,7]) that the average moment of inertia $\mathcal{J}^{(1)}$ in the normal nuclear phase should be equal to the rigid-body value.

This picture, however, encounters some difficulties that could be predicted even without performing the detailed numerical calculations in the framework of the cranking model [8]. In fact, in the independent-particle description of nuclear rotation the energies e_ν^ω in the rotating frame (the routhians) can be plotted as functions of rotational frequency ω . When two levels cross at the Fermi surface the angular momentum I (or, more precisely, its components on the rotation axis) of the system undergoes a discontinuity equal to difference in the slopes of the two crossing levels. In between any two crossings the curve of angular momentum must, therefore, increase less steeply as to provide an average slope corresponding to the rigid-body value $\mathcal{J}^{(1)} = \mathcal{J}_{rigid}$. Thus the dynamical moment of inertia $\mathcal{J}^{(2)}$ which is determined by the local slope $dI/d\omega$ should be lower than $\mathcal{J}_{rigid}$. This contradicts the experiment that requires eq. (1) to be valid.

In order to analyse this contradiction in more details let us look closely what happens with nuclear deformation when the nucleus in cranked. Usually, the nuclear shape changes are disregarded in the calculation. This may seem justified, as the corresponding changes appear to be rather small. One has to bear in mind, however, that even small changes in nuclear shape may cause considerable variation in energy or angular momentum of the nucleus.

Let us adopt the rotating harmonic oscillator as a simple model to illustrate better the situation. Fig.1 represents the approximate [6,9,10] independent-particle solutions to the rotating h.o. (= harmonic oscillator) potential plotted versus rotational frequency ω .In this model the existence of the nucleonic spin is ignored. It results only in the double degeneracy of levels. The single-particle routhians are labelled by the three h.o. quantum numbers n_λ, n_κ and n_μ leading to

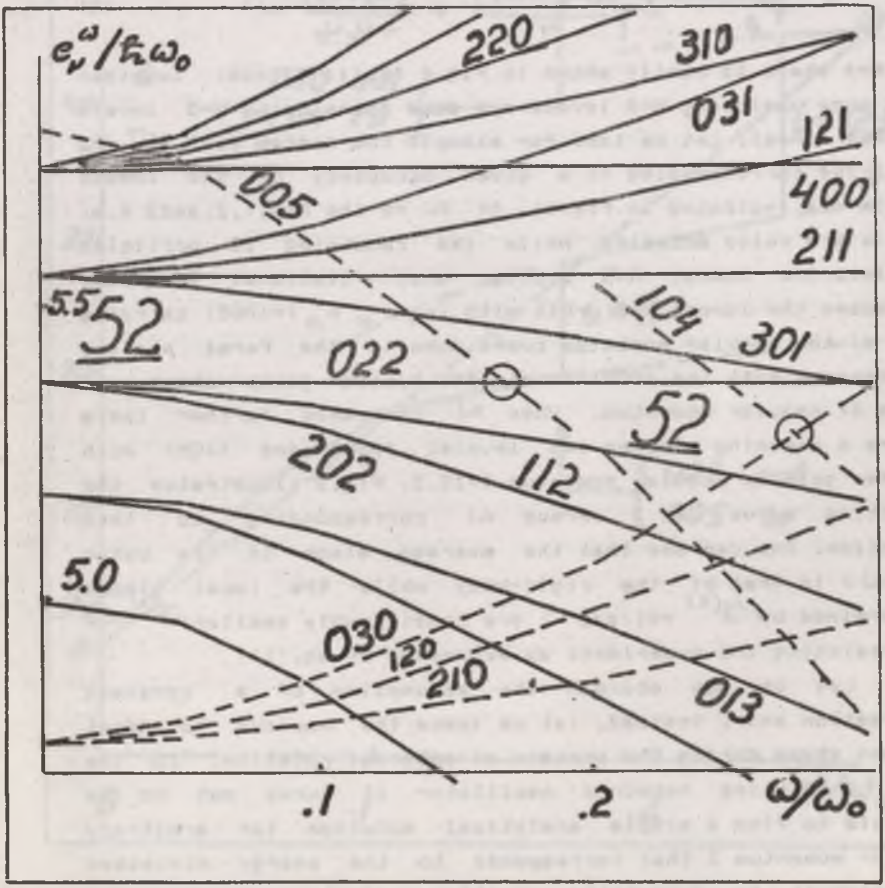

Fig.1. Single-particle energies e_ν^ω in a rotating potential of of harmonic oscillator (routhians) plotted versus rotational frequency ω at fixed value of the deformation parameter $\varepsilon = (\omega_1 - \omega_3)/\omega_0 = 0.2$. Both e and ω are given in units of $\hbar\omega_0$.

$$e^{\omega}_{n_1 n_\alpha n_\beta} = (n_1+\tfrac{1}{2})\hbar\omega_1 + (n_\alpha+\tfrac{1}{2})\hbar\omega_\alpha + (n_\beta+\tfrac{1}{2})\hbar\omega_\beta \tag{2}$$

with normal mode frequencies ω_1, ω_α and ω_β depending on the original h.o. frequencies $\omega_1, \omega_2, \omega_3$ (which fulfil the volume conservation condition $\omega_1\omega_2\omega_3 = \omega_0^3$) and on rotational frequency ω

$$\omega_{\alpha/\beta} = \frac{\omega_2+\omega_3}{2} \pm \sqrt{\frac{(\omega_2-\omega_3)^2}{4} + \frac{(\omega_2+\omega_3)^2}{4\omega_2\omega_3}\omega^2} \tag{3}$$

The N=4 shell is mostly shown in Fig.1 (solid lines) together with some upsloping N=3 levels and some downsloping N=5 levels (dashed lines). Let us take for example the system with $\mathcal{N}$ =52 particles corresponding to a given occupancy of the lowest levels (as indicated in Fig.1). At ω =0 the N=0,1,2,and3 h.o. shells are fully occupied while the remaining 12 particles populate the lowest N=4 orbits. When rotational frequency increases the lowest N=5 orbit with (n_1 n_α n_β)=(005) carrying most of the angular momentum comes down to the Fermi surface and crosses with the (002) level. The system gains about i=9 units of angular momentum. When ω increases further there occurs a crossing between two levels: (030) and (104) with further gain in angular momentum i=12.5. Fig.2 illustrates the resulting curve of I versus ω corresponding to this situation. One can see that the average slope in the curve I=I(ω) is that of the rigid-body while the local slopes (determined by $\mathcal{J}^{(2)}$ =dI/dω) are considerably smaller: $\mathcal{J}^{(2)} \ll \mathcal{J}^{(1)}$ contradicting the experiment as expressed by eq.(1).

Let us now abandon the assumption of a constant deformation and , instead, let us leave the nucleus to adapt its own shape during the process of external rotation. In the model of rotating harmonic oscillator it turns out to be possible to find a simple analytical solution for arbitrary angular momentum I that corresponds to the energy minimised with respect to deformation (selfconsistent solution). The energy E in the laboratory system turns out [9,10] to be given by a simple formula

$$E = 3\hbar\omega_0 \left\{ \Sigma_1 \left(\Sigma_\alpha \Sigma_\beta + \tfrac{1}{4} I^2 \right) \right\}^{1/3} \tag{4}$$

where Σ_1, Σ_α and Σ_β define the occupancy of the h.o. quanta along the three axes

$$\Sigma_{\mathcal{H}} = \sum_{\nu-occ.} \left(n_{\mathcal{H}} + \frac{1}{2}\right) \quad (5)$$

with $\mathcal{H}$ = 1, α, or β.

It has to be emphasized that this formula gives the energy E that is already minimised with respect to deformation.

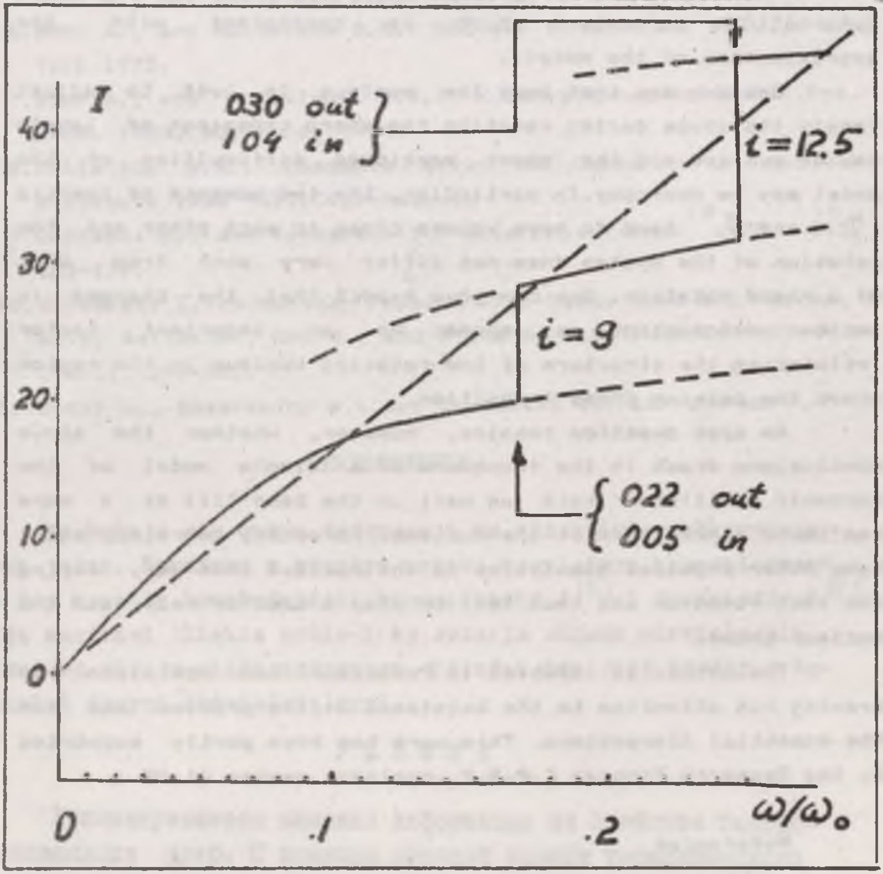

Fig.2. Angular momentum I plotted versus rotational frequency ω in units of $\hbar\omega_0$ (solid line). The two crossings of levels mentioned in the text are indicated by arrows. Long-dashed line indicates the rotation with rigid moments of inertia shown for the sake of comparison.

The lowest energy for the system of $\mathcal{N}$ =52 particles corresponds to $\Sigma_1 = \Sigma_\alpha$ =64 and Σ_β =98. It is essential to observe that this configuration remains yrast in the large range of angular momentum (i.e. there is no crossing of orbits if we follow the path of minimised energy). The two moments of inertia $\mathcal{J}^{(1)}$ and $\mathcal{J}^{(2)}$ following from eq.(4) turn out to be almost equal. They also appear to be close to the rigid moment of inertia (actually, $\mathcal{J}^{(1)} = \mathcal{J}_{rigid}$ up to quadratic terms in deformation parameter which is consistent with the approximation of the model).

One can see that once the nucleus is left to adjust freely its shape during rotation the sharp crossings of levels may be avoided and the above mentioned difficulties of the model may be overcome.In particular, the two moments of inertia $\mathcal{J}^{(1)}$ and $\mathcal{J}^{(2)}$ tend to have values close to each other and the rotation of the system does not differ very much from that of a rigid rotation. One can thus expect that the changes in nuclear deformation may appear as an important factor influencing the structure of the rotating nucleus in the region above the pairing phase transition.

An open question remains, however, whether the above conclusions drawn in the framework of a simple model of the harmonic oscillator hold as well in the case [11] of a more realistic description of the nucleus. Moreover, there may exist some other physical quantities in the nucleus that vary during the fast rotation and thus tend to play a similar role as the nuclear shape.

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STRESZCZENIE

Dyskutuje się wpływ deformacji na własności szybkorotujących jąder. Pokazano w prostym modelu oscylatora harmonicznego, że dwa momenty bezwładności: kinematyczny $J^{(1)}$ i dynamiczny $J^{(2)}$ mają wartości bliskie sobie i że rotacja układu niewiele się różni od obrotu ciała sztywnego. Wniosek ten jest zgodny z ostatnimi danymi doświadczalnymi.

Р Е З Ю М Е

Рассматривается влияние деформации на свойства быстровращающихся ядер. С помощью простой модели гармонического осциллятора доказано, что значения кинематического $J^{(1)}$ и динамического $J^{(2)}$ моментов инерции близки друг другу и что ротация системы почти не отличается от вращения жесткого тела. Этот вывод согласуется с последними экспериментальными данными.