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Regularization of Quantum Field Theories

Regularyzacja kwantowych teorii pola

Регуляризация теории квантовых полей

Quantum field theories suffer from the well known convergence difficulties being consequences of (an explicite or implicite) assumption of a point-like character of particles and their interactions. In consequence of the investigations by Tomonaga, Schwinger, Feynman, and others these difficulties could be partly removed, at least in the case of a class of field theories called renormalizable. In renormalizable theories (like electrodynamics, or other gauge theories, or in the case of Yukawa-type interactions) all infinities are reducible to a finite number of infinite constants like self-mass or self-charge arising in consequence of interactions of a particle with itself (self-action) in consequence of emission and reabsorption of quanta or of particle-antiparticle pairs. Assuming that only the "dressed" constants ("dressing" consists in changing the value in consequence of self-interaction) but not the "bare" constants appearing from the very beginning in the Lagrangian possess a physical meaning it is possible to renormalize them, i.e. to assume that the bare constants but not the dressed ones are infinite. In other words: the infinite effects of self-interaction may cancel the infinities (of an opposite sign) appearing from the very beginning as the bare constants in the Lagrangian so that, by subtraction of the two infinities, there remains a finite result representing the dressed (i.e. physical) mass or charge.

However, as stressed by Pauli, the renormalization procedure applied to infinite integrals is mathematically not correct, is ambiguous and in order to make it satisfactory and unique we need to regularize first the formalism of quantum field theory so that all terms appearing in the course of the calculations become finite, then perform renormalization and only afterwards remove the regularization. In view of the necessity of taking off the regularization at the end of calculation it is seen that regularization is only an auxiliary procedure of making some expressions unambiguous.

Two regularization procedures were found to be particularly efficient: one of them called usually Pauli-Villars regularization introduces auxiliary masses, the other, developed much later by 't Hooft and Veltman, is called dimensional regularization.

The Pauli-Villars regularization consists in introducing auxiliary fields with very high values of their masses and suitable coupling constant which yield cancellations of infinities. However, some of these fields are unphysical so that finally they have to be removed by a limit transitions so that the auxiliary masses go to infinity. On the other hand, the dimensional regularization consists in calculating the integrals over space-time variables so as if the dimension of space-time were not 4 but $4 - \varepsilon$ with an arbitrary non-integer ε. Such integrals are convergent. Then it is possible to perform renormalization of the constants appearing in the original Lagrangian in an unambiguous way, and finally we have, of course, to perform the limit transition $\varepsilon \rightarrow 0$ to obtain a physically meaningful theory.

It is difficult to say which of the two regularizations is practically superior. It depends upon the particular problem under investigation, but for the sake of discussion of some fundamental questions (e.g. in order to analyse the orders of different divergences, whether they are logarithmic, or quadratic, or whether the final take off of regularization is unavoidable) the regularization by means of auxiliary masses is certainly superior. Therefore we shall discuss in what follows only the regularization by means of auxiliary masses.

Let us begin with a short historical introduction. The idea of regularization by means of big auxiliary masses is due to Stuckelberg in the early forties. Let replace the usual Coulomb term by the following difference

$$\frac{e^2}{r} \longrightarrow e^2\left(\frac{1}{r} - \frac{e^{-Mr}}{r}\right) \qquad (1)$$

If M is large the second term (of Yukawa form) decreases quickly to zero so that at distances large in comparison with M^{-1} we are left with the usual Coulomb interaction but close to the origin, instead of tending to infinity, the resulting potential tends to a finite value e^2M. Such result may be obtained in field theory in a two-fold way: One possibility is to supplement the usual electrostatic interaction by an additional interaction with a scalar or pseudoscalar massive field with mass M, where the opposite sign of the Yukawa term appears automatically from the formalism. Indeed, Yukawa interaction is attractive whereas Coulomb interaction between charges of equal sign is repulsive.

The other possibility is to assume an interaction with a Proca field with a large mass values, but with an imaginary coupling constant. The quantized Proca field describes massive particles with spin 1 (the same as photon) and gives rise also to repulsive force between particles of equal charge unless their real charge e is replaced by an (unphysical) imaginary charge $e \longrightarrow ie$, whence $e^2 \longrightarrow -e^2$.

The first possibility, i.e. compensations of infinities by supplementary physical fields with different spins may be regarded as the first step towards the supersymmetric theories which became very fashionable nowadays, but they are unable to remove all the infinities plaguing quantum field theories. In order to remove the remaining infinities it is necessary to perform a cut off or another regularization by means of auxiliary fields describing particles with some unphysical properties.

It was Stuckelberg together with Rivier [1] who first applied the regularization by auxiliary masses to quantum electrodynamics, but they regularized only the electromagnetic field, i.e. photons by means of subsidiary masses μ_n which was sufficient to remove an infinite self-energy and self-mass of electrons. In order to regularize also the electric charge, to remove photon self-energy as well as the infinite terms of the vacuum polarization type it was necessary to regularize also the charged field (electronic field).

At the early stage of development of the regularization one used to regularize (instead of introducing some imaginary coupling constants) the causal delta functions D_c or Δ_c playing the role of Green functions for the electromagnetic as well as for the electronic field according to the prescriptions

$$D_c \longrightarrow \tilde{D}_c = \sum_n a_n \Delta_c^{(n)} \tag{2}$$

and

$$\Delta_c \longrightarrow \tilde{\Delta}_c = \sum_m b_m \Delta_c^{(m)} \tag{2'}$$

where D_c is the Gren function for the massless field and $\Delta_c^{(n)}$ are the Green functions for massive fields with masses M_n. The regularized functions (denoted by a wavy line) are regularized (i.e. free of singularities at the light cone) if the following two conditions are satisfied

$$\sum_n a_n = 0 \qquad \sum_m a_m M_m^2 = 0 \tag{3}$$

It appeared, however, soon that the consequences of such regularization are not satisfactory if there appear products of such regularized delta-functions for the charged fields. Instead of taking products of regularized functions one has rather to regularize their products

$$\Delta \cdot \Delta \longrightarrow \widetilde{\Delta \cdot \Delta} \tag{4}$$

where

$$\widetilde{\Delta \cdot \Delta} = \sum_n a_n \, \Delta^{(n)} \, \Delta^{(n)} \qquad (4')$$

At this moment allow me for a personal reminiscence. In 1948-49 when I was in Zurich with Pauli I was lucky to contribute to the regularization procedure by formulating a prescription: To regularize the products instead of taking products of regularized delta-functions for the charged fields. The importance of this prescription was acknowledged by Pauli [2] himself in several footnotes to his fundamental paper with F. Villars in Reviews of Modern Physics (1949). Also in the well known book entitled "Theory of Photons and Electrons" by Jauch and Rohrlich [3] there appears the following footnote concerning the regularization known under the name of "Pauli-Villars regularization": "W. Pauli and F. Villars, Rev. Mod. Phys.; (...) This work grew out of earlier investigations by J. Rayski, Phys. Rev. 75, 1961 (1949)".

Regularization of the delta-functions by means of auxiliary masses could be regarded either as a consequence of ficitious fields and particles with negative squared masses (or existence of the so called "tachions" with spacelike energy-momenta) or as a result of appearance of charged fields with real masses but imaginary coupling constants (charges). This last possibility is simpler and consists of the smallest deviation from the generally acknowledged physical principles.

In this formulation the Lagrangian of the theory consists of a sum of ordinary Lagrangians for free Dirac fields endowed with masses M_n ($M_o = m$ is the electron mass), and a set of vector fields $A_u^{(n)}$ with masses $\mu^{(n)}$ whereby $\mu^{(o)} = 0$ and $A_u^{(o)}$ is the usual electromagnetic field, and $A_u^{(n)}$ for n 0 are the massive Proca fields, with the following interaction Lagrangian

$$L' = \sum_{m,n} e_m^n \cdot j_{(n)}^n \cdot A_u^{(m)} \qquad (5)$$

where

$$e_m^n = i^{m+n} \cdot e$$

where e denotes the elementary charge and $j^{u}_{(n)}$ is the usual bilinear expression for the charge and current fourvector for the spinorial field $\psi_{(n)}$. The prescription of regularizing bilinear products follows automatically from the assumption (5).

It is to be noticed that the interaction-free fields are realistic but only their interactions exhibit some unrealistic features: are described by non-hermitian operators.

Just the lack of hermiticity is recompensated by an improved convergence in the higher orders of the perturbation calculus. By introducing a set of r spinor fields and s vector fields it is possible to dispose of the masses of the auxiliary fields and particles so that in the Feynman graphs each segment of a closed loop composed exclusively from spinor lines (denoted by full straight lines) contributes to the integrals over d^4p in momentum space by a factor p^{-r} instead of p^{-1} for p tending to infinity, and each internal line visualising vector field (denoted by a wavy line) contributes by a factor p^{-2s} (see the fig. 1).

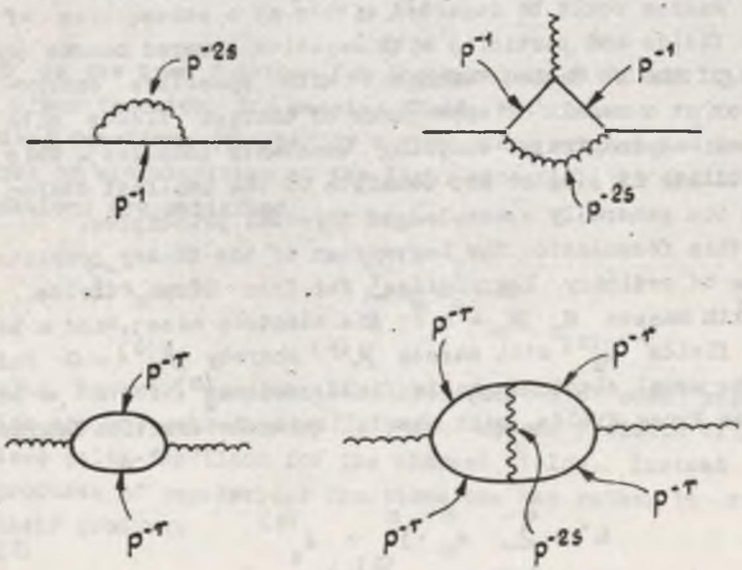

Fig. 1

Herefrom it is easily seen that if $r = s = 3$ then it is possible to achieve that all graphs (Feynman diagrams) yield finite contributions in the case of a fourdimensional space-time whereas in the case $r = s = 6$ all contributions are finite even in the case of an 11-dimensional space-time considered recently in the unified theories of Kaluza type.

Thus, it is possible to liberate electrodynamics from all infinities in arbitrary orders of perturbation calculus. Then the renormalization become well defined. The inconsistencies brought about by the introduction of imaginary coupling constants (charges) may be avoided if, at the very end, after renormalization, we remove the regularization by letting the auxiliary masses M_n and $\mathcal{M}_n$ tend to infinity. In this way the auxiliary particles will never appear in experiment: they play merely the role of auxiliary mathematical tools, and a transition with the auxiliary masses to infinity restores the unitarity of the formalism.

Pauli was highly interested in the following question: Will it be possible not to go to infinity with the auxiliary masses but to attach to them a certain physical meaning? It is equivalent to the following question: Is it possible to dispense the operator of evolution in time from the requirement of its unitarity? It seems to be the case, and it may be achieved in two different ways. One is straightforward: Inasmuch as a violation of unitarity brings about a non-conservation of the length of the state vector in the Hilbert space it might be simply assumed that only its direction but not its length possesses a physical meaning and renormalize the transition amplitudes so that the sum of their squared absolute values denoting probabilitities becomes equal one.

The other possibility is more sophisticated: We might abandon the assumption (higherto always assumed tacitly) that all possible physical events are (if not deterministic then at least statistically) predictable. Since a violation of unitarity comes into play only at high energies sufficient to produce the heavy particles with imaginary couplings, it might be assumed that in the domain of sufficiently high energy concentrations there might happen catastrophies, i.e. something even statistically quite unpredictable. Still, there may be estimated a "degree"of unpredictability" - a probability that something unexpected happens: Its measure is the difference between unity and the length of the state vector at the final time instant.

If unitarity is violated, the length of the state vector either decreases or increases with time. This means that the two directions along the time axis cease to be equivalent and it is possible to define as "future" that direction in which the length decreases, the other direction as pointing towards the past. It seems to be the first example of a dynamical theory preferring future against past, and involving irreversibility.

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STRESZCZENIE

Przedstawiono ogólną ideę regularyzacji i renormalizacji w kwantowej teorii pola. Wyrażono pogląd, że dodatkowe masy regularyzacyjne mogą posiadać skończone wartości, co wiąże się z kolei z łamaniem unitarności operatora ewolucji w czasie. Wyjaśnienie tego mogłoby być podwójne: 1) tylko kierunek wektora stanu, a nie jego długość mógłby posiadać fizyczne znaczenie, 2) nie wszystkie możliwe fizyczne zjawiska są przewidywalne.

Р Е З Ю М Е

Представлена общая идея регуляризации и ренормализации в квантовой теории поля. Высказывается мнение, что добавочные регуляризационные массы могут иметь конечные значения, что в свою очередь, связано с нарушением унитарности оператора эволюции по отношению к времени. Выяснить это можно двумя способами: I) не длина вектора состояния, но его направление может иметь физический смысл, 2) нельзя предвидеть всевозможные физические явления.