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Nonadiabatic and Dissipative Effects in Low Energy Nuclear Fission

Nieadiabatyczne i dyssypatywne efekty w niskoenergetycznym rozszczepieniu jądra

Неадиабатические и диссипативные явления в низкоэнергетическом делении

Dedicated to Professor
Stanisław Szpikowski on occasion
of his 60th birthday

1. INTRODUCTION

In this paper we consider two types of phenomena which arise in the theory of low energy fission due to the finite velocity of a fission process: diabatic mode coupling (DMC) for collective coordinates and energy loss of the fission mode via dissipative processes: If the fission motion were infinitely slow one could describe the wave function of a fissioning nucleus within the zero order Born-Oppenheimer approximation (BOA), i.e. the wave function would separate into the simple product

$$\psi(\alpha, q, x) = U(\alpha)\, W(q, x; \alpha) \qquad (1)$$

Here α is the fission coordinate, q stands for the collective degrees of freedom of the fissioning nucleus and x for the remaining internal coordinates. $U(\alpha)$ is the probability amplitude for finding the fission coordinate between α and $\alpha + d\alpha$, while $W(x,q;\alpha)$ is the wave function in the space of all other coordinates; it depends parametrically on α. In the case of BOA the wave function $W(x,q;\alpha)$ belongs to the lowest eigenvalue of the internal Hamiltonian being compatible with the prescribed conserved quantum numbers. The first step towards the inclusion of effects of the finite fission velocity consists in expanding the wave function in a basis which allows for excited states in the x,q-space and for subsequent modifications of the fission wave function itself:

$$\psi(\alpha, q, x) = \sum\!\!\!\!\!\!\int_{m,k} C_{mk}\, U_{mk}(\alpha)\, W_m(q, x; \alpha) \qquad (2)$$

Here $W_m(x,q;\alpha)$ is an eigenfunction of the internal Hamiltonian $H_{int}(x,q;\alpha)$ which depends parametrically on α:

$$H_{int}(q, x; \alpha)\, W_m(q, x; \alpha) = \mathcal{E}_m(\alpha)\, W_m(q, x; \alpha) \qquad (3)$$

The fission wave function depends on the quantum numbers m of the internal state via the energy $\mathcal{E}_m(\alpha)$ and on the quantum numbers k which characterize the fission mode (energy, angular momentum etc.). The coupling between the excited modes $\psi_{mk}(\alpha, q, x)$ results from the operator $T^{nad}(\alpha)$ which is obtained from the part of the kinetic energy operator which acts on the parametric dependence of W_m on α. Since this parametric dependence of W_m on α is directly related to the deviation from the simple adiabatic wave function of Eq.(1), we call the coupling leading to the form of Eq.(2) diabatic mode coupling.

The DMC would adequately describe the fission process, if in Eq.(2) we could really perform the sums over all relevant quantum numbers m, i.e. sums over quantum numbers related to collective and intrinsic coordinates. Since this is mathematically unfeasable one can divide the problem into two distinct parts: In the first step one ignores the intrinsic degrees of freedom and performs a DMC calculation in q-space[x]. In the second step one takes account of the coupling between the intrinsic coordinates x and the collective coordinates α and q. The most prominent effect of this coupling is the thermalization of the energy of the collective motion. Two mechanisms can be distinguished: (i) The coupling between the intrinsic coordinates x and the collective coordinates q leads via intrinsic excitations to a damping of the collective mode (e.g. the asymmetry vibration). This is the same mechanism that leads to the damping of excited collective states of fission-stable nuclei, as for example in giant resonances. This damping depends on the fission motion indirectly via the available energy and the energy dependence of the damping width. (ii) The coupling between the intrinsic coordinates x and the fission coordinate α leads to a direct excitation of intrinsic degrees of freedom which in turn results in a direct dissipation of the fission energy. The dependence on the fission mode comes via the velocity of the fission process and via the fission energy which determines the phase space for intrinsic excitations.

In our treatment we ignore the change of nuclear structure parameters due to the heating up of the fissioning nucleus, though this will be probably important for the inertia parameters. These problems will be considered in further investigations.

Furthermore our treatment is limited to small fission energies; otherwise diabatic level crossing would become an important effect which upsets the basis of our description.

2. BASIC DEFINITIONS AND NOTATIONS

In recent years many calculations of potential surfaces and inertia tensors for fission configurations of nuclei have been

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[x] In the numerical application we deal with only one collective coordinate belonging to the mass asymmetry oscillation. In principle more collective coordinates could be included.

performed [1-5] and methods have been developed which allow the determination of adiabatic paths to fission [3]. To describe the fission process in the adiabatic approximation one generally introduces a number of collective coordinates: The fission coordinate α related to the mean elongation, a coordinate q_{as} describing a possible mass asymmetry, a necking coordinate q_{neck} and eventually further coordinates depending on the degree of sophistication one is aiming at.

In this paper we use the collective coordinates of ref. [5], i.e.

$$\alpha = (a_1 + a_2 + c_2 + c_1)/2R_0 \, ,$$

$$q = (a_1 b_1^2 - a_2 b_2^2)/(a_1 b_1^2 + a_2 b_2^2) \approx (A_1 - A_2)/(A_1 + A_2) \, .$$

With these coordinates the classical energy of a system of particles becomes

$$H_{cl} = \frac{1}{2} \sum_{i,j} M_{ij} \dot{q}^i \dot{q}^j + \sum_i V(q^i) \, , \tag{2.1}$$

where M_{ij} and $V(q)$ denote the mass tensor and the potential energy. After quantization the Hamiltonian takes the form

$$H = -\frac{\hbar^2}{2} \sum_{i,j} |M|^{-1/2} \frac{\partial}{\partial q^i} |M|^{-1/2} M^{ij} \frac{\partial}{\partial q^j} + \sum_i V(q^i) \, , \tag{2.2}$$

where the determinant of the mass tensor is $|M| = \det(M_{ij})$ and $M_{ij} M^{jk} = \delta_i^k$.

In the two dimensional space with the coordinates α , q the mass tensor is

$$M_{ij} = \begin{pmatrix} M_{\alpha\alpha} & M_{\alpha q} \\ M_{q\alpha} & M_{qq} \end{pmatrix} \, . \tag{2.3}$$

We chose a coordinate system $q_i = (\alpha, q)$ in which the mass tensor is diagonal $M_{\alpha q} = M_{q\alpha} = 0$.
We denote the diagonal elements of the mass tensor

$$M_{\alpha\alpha} = M_\alpha ,$$
$$M_{qq} = M_q . \tag{2.4}$$

They depend on α and q.

To describe the fission process we have to solve the eigen-problem with the collective Hamiltonian

$$H = -\frac{\hbar^2}{2} \frac{1}{\sqrt{M_\alpha M_q}} \left[\frac{\partial}{\partial\alpha} \sqrt{\frac{M_q}{M_\alpha}} \frac{\partial}{\partial\alpha} + \frac{\partial}{\partial q} \sqrt{\frac{M_\alpha}{M_q}} \frac{\partial}{\partial q} \right] + V \tag{2.5}$$

The variables α and q describe the fission and asymmetry modes respectively. The masses $M_\alpha(\alpha,q)$ and $M_q(\alpha,q)$ as well as the potential $V(\alpha,q)$ are obtained from the asymmetric two center shell model [5].

The scalar product of the eigenfunctions of the Hamiltonian H is defined with the metric

$$D(\alpha,q) = \sqrt{M_\alpha(\alpha,q)\, M_q(\alpha,q)} \; ; (\varphi_1/\varphi_2) = \iint \varphi_1^*(\alpha,q)\, \varphi_2(\alpha,q)\, D(\alpha,q)\, d\alpha\, dq$$

To obtain a metric which does not depend on masses and coordinates and gives orthogonal functions

$$(\tilde\varphi_1/\tilde\varphi_2) = \iint \tilde\varphi_1^*(\alpha,q)\, \tilde\varphi_2(\alpha,q)\, d\alpha\, dq$$

we perform now an unitary transformation of the Hamiltonian and its eigenfunctions $\varphi(\alpha,q)$ with the help of the function $\sqrt{D(\alpha,q)}$

$$\tilde\varphi(\alpha,q) = \sqrt{D(\alpha,q)}\; \varphi(\alpha,q) ,$$
$$\tilde H(\alpha,q) = \sqrt{D(\alpha,q)}\; H(\alpha,q) \sqrt{D(\alpha,q)}^{-1} . \tag{2.6}$$

Then the Hamiltonian takes the form

$$\tilde H = -\frac{\hbar^2}{2} \left[\frac{\partial}{\partial\alpha} \frac{1}{M_\alpha} \frac{\partial}{\partial\alpha} + \frac{\partial}{\partial q} \frac{1}{M_q} \frac{\partial}{\partial q} \right] + V(\alpha,q) + V_G(\alpha,q) . \tag{2.7}$$

Neglecting the small scalar term $V_G(\alpha,q)$ (Appendix A) we split up the Hamiltonian in the following way:

$$\tilde{H} = \tilde{T}_\alpha + \tilde{H}_q \,, \tag{2.8}$$

where

$$\tilde{T}_\alpha = -\frac{\hbar^2}{2}\frac{\partial}{\partial\alpha}\frac{1}{M_\alpha(\alpha,q)}\frac{\partial}{\partial\alpha} \,, \tag{2.8a}$$

$$\tilde{H}_q = -\frac{\hbar^2}{2}\frac{\partial}{\partial q}\frac{1}{M_q(\alpha,q)}\frac{\partial}{\partial q} + V(\alpha,q) \,. \tag{2.8b}$$

In the adiabatic approximation one assumes that M_q depends only weakly on the fission cordinate α. We therefore define an adiabatic Hamiltonian for the internal system

$$H_q^{ad}(\alpha;q) = -\frac{\hbar^2}{2}\frac{\partial}{\partial q}\frac{1}{M_q(q)}\frac{\partial}{\partial q} + V(q;\alpha). \tag{2.9}$$

where $M_q(q)$ is an average mass for the q-motion. H^{ad} defines the eigenfunctions of the q-mode

$$H_q^{ad}(q;\alpha)\, W_m(q;\alpha) = \mathcal{E}_m(\alpha)\, W_m(q;\alpha)$$

In order to define the adiabatic part of $\tilde{T}_\alpha$ we introduce the average mass

$$\bar{M}_\alpha(\alpha) = \int W^*(q;\alpha) M_\alpha(\alpha,q) W(q;\alpha) / |W(q;\alpha)|^2 dq$$

Then T_α^{ad} is defined by

$$T_\alpha^{ad} = -\frac{\hbar^2}{2}\frac{\partial}{\partial\alpha}\frac{1}{\bar{M}_\alpha(\alpha)}\frac{\partial}{\partial\alpha} \tag{2.10}$$

The eigenfunctions of the total adiabatic Hamiltonian

$$H^{ad}(\alpha,q) = T_\alpha^{ad} + H_q^{ad} \tag{2.11}$$

can be written as:

$$\varphi_{nE}^{ad}(\alpha,q) = u_{nE}(\alpha)\, W_n(q;\alpha) \tag{2.12}$$

The adiabatic Schrödinger equation is

$$\left(T_\alpha^{ad} + H_q^{ad}\right) U_{nE}(\alpha)\, w_n(q;\alpha) = E\, U_{nE}(\alpha)\, w_n(q;\alpha) \qquad (2.13)$$

which yields the following equation for the fission wave function:

$$\left(T_\alpha^{ad} + \mathcal{E}_n(\alpha)\right) U_{nE}(\alpha) = E\, U_{nE}(\alpha) \qquad (2.14)$$

The energies $\mathcal{E}_1(\alpha)$, $\mathcal{E}_2(\alpha)$, $\mathcal{E}_3(\alpha)$,... define the fission potential when the fission takes place with the internal system W_n in the states n=1,2,3,...

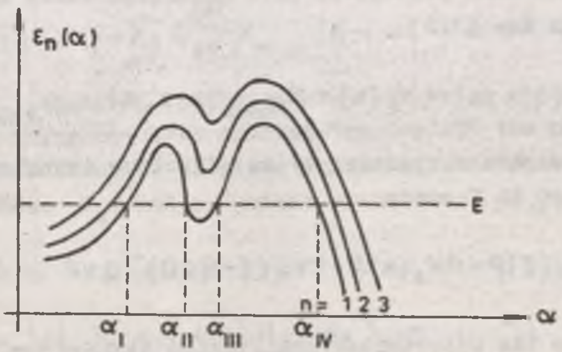

Fig. 1. Effective potential $\mathcal{E}_n(\alpha)$ for different internal states w_1, w_2, w_3.

3. DESCRIPTION OF THE MODEL .

Without dissipation the Hamiltonian in Eq. (2) is equal to $H = H^{ad} + T^{nad}$. The energies $\mathcal{E}_m(\alpha)$ are a sum of two terms: $\mathcal{E}_m(\alpha) + W_m(\alpha)$ where $V_f(\alpha)$ represents the fission potential and $W_m(\alpha)$, m = 0,1,2, is the energy of the m-th asymmetry phonon. When we include dissipation the asymmetry mode is no longer stable because it can now thermalize. In principle this thermalization could be treated by the explicit introduction of all the degrees of freedom into which the phonon can decay. However, in practice this procedure is quite unfeasable. The well known recipe to get around this problem is Feshbach's effective operator method [10]: One continues to work in the space defined by the unperturbed phonons (P-space) and takes account of the remaining space into which the phonons decay (Q-space) by the introduction of an effective energy dependent Hamiltonian. In this way the phonon energies become complex quantities and lead ultimately to a complex fission potential which is, of course, the analog of the complex optical potential for nucleon-nucleus scattering: in the course of the fission motion the amplitudes $|\phi_{m,E}\rangle$ must be damped because they are partly scattered into more complicated states.

To consider this picture in more detail we write for the operator H in Eq. (1.3)

$$H_{int}(q,x;\alpha) = V_f(\alpha) + H_{Phonon}(q,x;\alpha)$$

and replace the phonon operator by an effective operator $PH_{eff}(E)P$ which is defined in P-space

$$PH_{eff}(E)P = PH_0(\alpha)P + PVQ(E-QHQ)^{-1}QVP \tag{3.1}$$

Here $H_0(\alpha)$ is the unperturbed Hamiltonian defined by

$$H_0(\alpha)\, w_m = \Omega_m(\alpha)\, w_m$$

and V is the interaction which couples the phonon to more complicated configurations. There are two distinct contributions to this coupling:

(i) For fixed α the finite velocity of the asymmetry oscillation leads to a coupling $V_q = V(q,x)$ between the coordinate q and the internal coordinates x; if the asymmetry motion is approximated by an octupole vibration the form of this coupling is well known from the physics of low lying collective states (see e.g. ref. [11]). It leads to the usual damping of the collective motion as it is seen in another context in the width of giant resonances.

(ii) Since the velocity of the fission motion is finite, there will be also a direct coupling $V_\alpha = V(\alpha,x)$ between the fission coordinate α and the internal degrees of freedom x. The damping associated with this type of coupling has some similiarity to friction because it is directly related to the velocity of the fission motion.

The determination of the precise form of the coupling terms V_α and V_q and the evaluation of $H_{eff}(E)$ is a difficult problem which is far from being solved. For our purposed where the effective interaction enters only in an average way the following procedure appears to be reasonable:

The operators V_α and V_q are sums of single particle operators. We apply them to the wavefunction $W_m(x,q,\alpha)$ and obtain

$$V_\lambda W_m(q,x;\alpha) = \sum_{\mu\nu} C^{(\lambda)}_{\mu\nu} W_{m;\mu\nu}(q,x;\alpha) \quad ; \quad \lambda = \alpha, q \tag{3.2}$$

where $W_{m;\mu\nu}(q,x;\alpha)$ is the wavefunction of a state where either a ph-pair μ, ν has been created "on top of" the phonon state W_m or of a noncollective ph state which is orthogonal to the collective state W_m. Taking matrix elements of $H_{eff}(E)$ we obtain

$$\langle W_i | H_{eff} | W_k \rangle_\alpha = \langle W_i | H_0 | W_k \rangle_\alpha +$$

$$+ \sum_{\substack{\mu,\nu \\ \mu',\nu'}} \langle W_i | V_\alpha | W_{i;\mu'\nu'} \rangle_\alpha \langle W_{i;\mu'\nu'} | \frac{1}{E - QHQ + i\delta} | W_{k;\mu\nu} \rangle \langle W_{k;\mu\nu} | V_\alpha | W_k \rangle_\alpha$$

$$+ \sum_{\substack{\mu,\nu \\ \mu',\nu'}} \langle W_i | V_q | W_{i;\mu'\nu'} \rangle_\alpha \langle W_{i;\mu'\nu'} | \frac{1}{E - QHQ + i\delta} | W_{k;\mu\nu} \rangle \langle W_{k;\mu\nu} | V_q | W_k \rangle_\alpha \tag{3.3}$$

The matrix elements in this equation are defined by integrations over x and q; they depend parametrically on α.

In the following we shall consider only diagonal elements of the effective operator. Clearly in principle there will be also nondiagonal matrix elements, so that one would have to perform a diagonalization of $H_{eff}(E)$ in the space $|W_m\rangle$. Due to phase cancellation effects the nondiagonal matrix elements should be on the average much smaller than the diagonal ones. Therefore the main effect of the second an third term on the r.h.s. of Eq. (3.3) will be on the diagonal matrix elements. Also for phase concellation reasons we have neglected the interference terms between V_α and V_q.

In order to evaluate the matrix elements of the resolvent $(E-QHQ)^{-1}$ we proceed as follows:

(i) We put $\langle W_{i;\mu\nu} | W_{k;\mu'\nu'} \rangle = \delta_{ik}\,\delta_{\mu\mu'}\,\delta_{\nu\nu'}$

which is familiar from shell model calculations.

(ii) We replace the sum over μ and ν by an integration over the corresponding energy variable introducing the density of states $\varrho(w)$. We can then write Eq. (3.3) in the form

$$\langle W_i | H_{eff}(E) | W_i \rangle = \Omega_i(w) + \int dw\, \varrho(w;\alpha)\, \frac{|V_\alpha(i;w;\alpha)|^2 + |V_q(i;w;\alpha)|^2}{E - \varepsilon_i(\alpha) - w + i\delta} \tag{3.4}$$

$V_\alpha(i,w,\alpha)^2$ and $V_q(i,w,\alpha)^2$ are averages of the squares of the matrix elements.

For the real and imaginary part of (3.4) we obtain

$$Re\langle W_i | H_{eff}(E) | W_i \rangle = \Omega_i(\alpha) + \int dw\, Re\, \frac{\varrho(w,\alpha)\left[|V_\alpha(i;w;\alpha)|^2 + |V_q(i;w;\alpha)|^2\right]}{E - \varepsilon_i(\alpha) - w} \tag{3.5a}$$

$$\Im_m \langle W_i | H_{eff}(E) | W_i \rangle = -\pi\, \varrho(E - \varepsilon_i(\alpha);\alpha)\left[|V_\alpha(i;w;\alpha)|^2 + |V_q(i;w;\alpha)|^2\right] \tag{3.5b}$$

The real part implies a renormalization of the phonon energies which can be discarded for the following. The imaginary part makes the phonon energies complex or, in other words, leads to the appearence of an imaginary part in the fission potential; it describes the weakening of the amplitude $U_{iE}(\alpha)\, W_i(q,x,\alpha)$ in the course of the fission process.

It is a difficult problem to estimate the coupling matrix elements V_q and V_α. We propose the following approximate treatment: For fixed coordinate q and the constituent particles see a mean potential $U(\alpha_o, q_o; \vec{r}_n)$ acting on particle n. A displacement $\alpha_o \rightarrow \alpha$ or $q_o \rightarrow q$ leads to a change of the potential $(\alpha - \alpha_o) \frac{\partial U}{\partial \alpha} \Big|_{\alpha = \alpha_o}$ and $(q - q_o) \frac{\partial U}{\partial q} \Big|_{q = q_o}$ respectively.

For the asymmetry coordinate q the perturbation $(q - q_o) \frac{\partial U}{\partial q} \Big|_{q = q_o}$ leads to the conventional form of the particle-phonon coupling and the resulting damping width will be the width of the asymmetry phonon.

For small deformations, i.e. for small values of q, the quantity $|V_q|^2$ can be estimated rather reliably from the widths of giant resonances. The matrix elements are typically of the order of 0.1 MeV. For large deformations there may be considerable deviations from these values; at the moment we do not consider such deviations but they should be taken into account in future more refined studies.

The change of the fission coordinate α leads to dissipation of the fission kinetic energy via onebody and twobody dissipation. The resulting friction force $F = -\gamma\dot{\alpha}$ is related to the imaginary part of the optical potential via the damping time: The friction force leads to a damping of the energy of the fission motion with the relaxation time $\tau_\gamma = m/2\gamma$ where m is the corresponding inertia parameter. On the other hand an imaginary part W in the optical potential implies a damping of the energy with the relaxation time $\tau_W = \hbar/2W$. The counter part of the friction force in the optical potential is therefore an imaginary part

$$W = \hbar\gamma/m \tag{3.6}$$

For low energy fission the motion is slow enough that the friction coefficient can be calculated in the framework of linear response theory as discussed in refs.[12 , 13]. A calculation of γ in this spirit has been performed in ref.[14] for moderate deformations (around the second minimum of the fission potential) and extended to large deformations by Marcev (priv. communication). The calculated friction coefficient is an energy dependent quantity $\gamma\ (\omega)$. The large width of the energy distribution - of the order of 10 MeV - implies a memory time of the order of 10^{-22} sec which is short compared to characteristic times of the fission motion. Therefore memory effects can be safely neglected and $\gamma\ (\omega \to 0)$ defines the friction coefficient in the ordinary sense.

4. RESULTS AND DISCUSSIONS

4.1 In order to separate the effects of mode-coupling and of dissipation we have first made a calculation where dissipation was completely neglected. The calculations are done for the nucleus ^{236}U. The ground state fission barrier is double humped; in the region of the first maximum and the second minimum the system follows a symmetric path and nonadiabatic effects are believed to be small before the second maximum. For this reason we assume the interaction operator T^{nad} to vanish to the left of the turning point α_{III} (see Fig. 1) belonging to ground state fission. The wave function Ψ_E we are looking for contains in addition to the adiabatic wave function ϕ^{ad}_{1E} other contributions $\phi^{ad}_{\alpha E}$:

$$\Psi_E(\alpha,q)=\varphi^{ad}_{1E}(\alpha,q)+\sum_{n=1}^{N}\int dE'\, C_n(E,E')\,\varphi^{ad}_{nE'}(\alpha,q). \tag{4.1}$$

The coefficients $c_n(E,E')$ are determined from a variational principle in the following way: We make the ansatz

$$C_n(E,E')=\sum_{k=1}^{k_{max}} a_{nk}\, h_k\left(\frac{E-E'}{\Delta_E}\right)\exp\left[-\left(\frac{E-E'}{\Delta_E}\right)^2\right], \tag{4.2}$$

which allows the system to go off shell in the fission energy up to $|E-E'| \approx \Delta_E$; the function h_k is the Hermite polynomial of order k. Since we go up to $k_{max} = 10$ the form(4.2) garantees a great flexibility for the functional dependence of c_n on the off-shell energy $E' - E$. The coefficients a_{nk} are determined from the requirement

$$\frac{\partial}{\partial \alpha_{nk}} \left[\int \Psi_E^* (H-E)^2 \Psi_E \, n^2(\alpha) \, d\alpha \, dq \right] = 0 \tag{4.3}$$

which leads to the following system of linear equations for the coefficients a_{nk}:

$$\sum_{k=1}^{k_{max}} \sum_{n=1}^{N} A_{kn;k'n'} \, a^*_{k'n'} = b_{kn} \tag{4.4}$$

The normalisation factor $n^2(\alpha) = |U_{1E}(\alpha)|^{-2}$ is introduced for the following reason: In the classically forbidden region the fission wave function decays very rapidly with increasing α. Therefore, without the factor $|U(\alpha)|^2$, the integral in eq. (4.3) receives practically no contribution from regions of α which are far away from the turning point α_{III} and the variational method is consequently only sensitive to details of the wavefunction in the vicinity of α_{III}. Multiplication by $n^2(\alpha)$ corrects for this defect and leads to equal weights of the internal wavefunction in the entire intervall of the α-integration.

The matrix A is determined as

$$A_{kn;k'n'} = \int d\alpha \, dq \; g^*_{kn}(\alpha,q) \, g_{k'n'}(\alpha,q) \, n^2(\alpha)$$

with

$$g_{kn}(\alpha,q) = (H-E) \int dE' \, h_k\left(\frac{E-E'}{\Delta_E}\right) \exp\left[-\frac{(E-E')^2}{\Delta_E}\right] \varphi^{ad}_{nE}(\alpha,q)$$

where H-E acts on $\varphi^{ad}_{nE}(\alpha,q)$.

The vector b is given by

$$b_{kn} = -\int d\alpha \, dq \; \varphi^*_{1E}(\alpha,q) \, g_{kn}(\alpha,q) \, n^2(\alpha)$$

The width Δ_E is determined from the requirement that the norm of $(H-E)\,\Psi_E(\alpha,q)$ should be as small as possible. We found $\Delta_E \sim 0.6$ MeV.

The probability distributions for the masses of the fission fragments are obtained from the solution of eq. (4.1) for $\alpha \simeq \alpha_{sc}$:

$$|P_E(q)|^2 = |\Psi_E(\alpha = \alpha_{sc}, q)|^2 \tag{4.5}$$

Unfortunately the scission point $\alpha \cong 2.5$ can not be reached in our calculation. But we found that beyond $\alpha \sim 1.75$ the mass distributions do not change significantly (see fig. 2); we therefore calculated the mass distribution from eq. (4.5) with $\alpha = 1.85$.

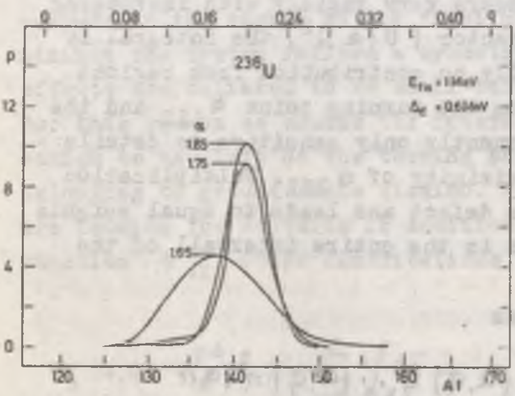

Fig. 2. Dependence of mass distribution on the fission coordinate for ^{236}U.

In order to give an idea of the level spacings of the phonon states associated with the asymmetry mode we show in fig. 4. the energies of the 10 lowest phonons as a function of α. It is seen that they follow without much fluctuations the potential surface belonging to the groundstate. It is clear from the figure that the number of phonons that one has to include will increase almost linearly with the fission energy.

In fig. 3 we show for the nucleus ^{236}U the mass distribution probability following from the adiabatic wave function - which is of course independent of the fission energy - and the mass distribution probability following from nonadiabatic mode

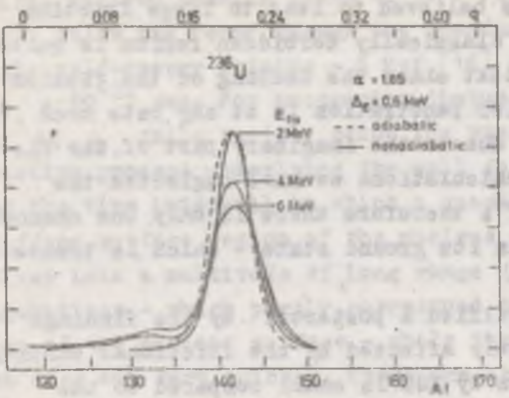

Fig. 3. Dependence of mass distribution on the fission coordinate for ^{236}U

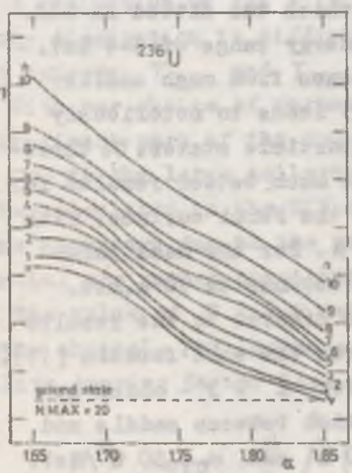

Fig. 4. Effective fission potential for the 10 lowest symmetry phonon states.

coupling for E = 2, 4 and 6 MeV. It is seen that with increasing fission energy the curves become broader and develope a shoulder towards more symmetric fission. The small bump for $A \sim 133$ is probably a numerical effect being due to a still too small phonon space.

The conclusion from these findings is that nonadiabatic coupling is unimportant for $E < 3$ MeV but becomes quite important for higher fission energies. However, for energies above 4 MeV our approximation begins to fail because diabatic level crossing becomes a very important effect which dominates the process of energy dissipation [15].

4.2 A second type of calculations was done in order to isolate the effects of the dissipative terms in eq. (2.8). They were done only in the region $\alpha > \alpha_{IV}$ since the increasing fission velocity in this region is believed to lead to large friction effects. In addition, the classically forbidden region is quite uninteresting in this context since the damping of the fission wave function due to barrier penetration is at any rate much stronger than the damping due to the imaginary part of the fission potential. In these calculations we have neglected the nonadiabatic coupling T^{nad}, therefore there is only one channel - in which the q-mode is in its ground state - which is treated explicitely.

This procedure is justified a posterori by the findings that: a) DMC is only slightly affected by the frictional damping of the α-mode. b) The loss by DMC is small compared to the friction induced energy loss of the fission motion.

In ref.[14] the friction coefficient was calculated with a coupling parameter $\Gamma_0 = 0.03\ \mathrm{MeV}^{-1}$ which was fitted in ref. [16] to optical potential data in an energy range of 5-0 MeV. However the main contributions to γ comes from much smaller energies for which the value $\Gamma_0 = 0.03$ leads to notoriously too small spreading width of the quasiparticle states. We therefore have chosen $\Gamma = 0.15$ which yields much better results for the spreading width of states close to the Fermi surface. With $\Gamma_0 = 0.15\ \mathrm{MeV}^{-1}$ we obtain $\gamma_{LR} \approx 50\ \hbar$. For the mass parameter we took $m = 100\ \hbar^2/\mathrm{MeV}$ which corresponds to 70 a.m.u.

It is interesting to compare these figures to the results of pure onebody dissipation obtained from the wall formula [17]. Taking a mean value for the multipolarity $I = 3$ - which appears reasonable for the large deformation between saddle and scission point - one obtains $\gamma_{WF} \approx 300\ \hbar$ and $m \approx 80\ \hbar^2/\mathrm{MeV}$.

The value of $\gamma_{LR} = 50\ \hbar$ comprises the effects of the moving walls and of residual interactions, i.e. one- and twobody dissipation. Therefore it might look surprising that onebody dissipation within the wall formula yields a much larger friction coefficient. However, one must realize that the wall formula is based on a single particle picture for the motions of the nucleons where all nucleons participate in the same manner in the interaction with the moving walls. In reality the single particle like states are quasiparticle states. They possess only a finite lifetime $\Delta\tau_{Q.P.}$. Only for states close to the Fermi energy is this lifetime large compared to a typical nucleon passing times of 10^{-22} sec. For states lying 5-10 MeV below the Fermi energy the imaginary part of the quasiparticle self-energy attains ~ 5 MeV [16] corresponding to $\Delta\tau_{Q.P.} = 10^{-22}$ sec. For increasing distance from the Fermi energy $\Delta\tau_{Q.P.}$ falls rapidly below the passing time. For the interaction process underlying the wall formula this means that during the time intervall in which a quasiparticle enters into the diffuse surface region of the nucleus and leaves it again it dissolves into a multitude of long range (n + 1) particle -n hole configurations - which partly correspond to dynamical degress of freedom of the nuclear surface - while the single particle component dies out very quickly. Therefore the effective number of particles which have to be counted in the application of the wall must be considerably smaller than the nucleon number A. A quantitative evaluation of the effect of the quasiparticle decay on one onebody dissipation is difficult but qualitatively the discrepancy between Υ_{LR} and Υ_{WF} is not astonishing.

With our choice of parameters we obtain W = 0.8 MeV for the imaginary part of the optical potential.

Due to the large collective mass the fission wavefunction can be calculated in the WKB approximation. The changes which become necessary due to the presence of an imaginary part in the potential are rather straightforward. We discuss them in App. C.

The value of W = 0.8 MeV implies a strong damping of the elastic channel. For instance, for ground state fission the probability damping factor (see eqs. (C5) and (C6))

$$P_{Damp} = \exp\left[-\frac{2}{\hbar}\int_{\alpha_{IV}}^{\alpha_{sc}} |q(\alpha')|\, d\alpha'\right] \tag{4.6}$$

becomes $P_{Damp} \approx 6\cdot10^{-4}$; consequently there are practically no cold fission fragments left when the system arrives at the scission configuration.

The energy absorption is governed by the relation

$$E(\alpha+\delta\alpha) = E(\alpha)\exp\left[-\frac{2W}{\hbar v_\alpha}\delta_\alpha\right] = E(\alpha)\exp\left[-\frac{2\Upsilon}{m v_\alpha}\delta\alpha\right], \tag{4.7}$$

where v_α is the collective velocity. With $E = \frac{m}{2} v^2$ one obtains

$$\frac{dE}{d} = - \Gamma v_\alpha \tag{4.8}$$

which is the classical relation following from the Rayleigh dissipation function. Our optical model treatment together with a quasiclassical treatment of the motion leads us back to the classical problem of the motion of a particle which moves in an external field and feels viscous damping. The quantum nature of the problem enters only via the determination of the friction coefficient.

The numerical treatment of the fission motion leads to the following results (for ground state fission):

(i) The time required to go from α_{IV} to α_{sc} is $\tau_{\Delta\alpha} \approx 22 \cdot 10^{-22}$ sec.

(ii) The gain in kinetic energy of the fission mode in going from α_{IV} to α_{sc} is ~ 10 MeV. It is seen that the motion is not as crrepy as predicted by the wall formula.

(iii) The intervall from α_{IV} to $\alpha \sim 1.85$ where DMC is most effective is passed within $7 \cdot 10^{-22}$ sec. The collective velocity at $\alpha = 1.85$ is $\dot{\alpha} = v_\alpha = 0.039 \cdot 10^{22}$ sec^{-1} compared to $v_\alpha = 0.050 \cdot 10^{22}$ sec^{-1} for $\Gamma = 0$. The collective energy loss at $\alpha = 1.85$ is ~ 3.5 MeV. The small relative change which results from switching on friction shows that the damping of the fission mode has only a small effect on DMC.

(iv) The typical period of the asymmetry oscillations τ_{AS} $15\ 10^{-22}$ sec is only slightly smaller than $\tau_{\Delta\alpha}$, so that there is just enough time for one period while the system goes from α_{IV} to α_{sc}. During this time the asymmetry phonons do not have sufficient time to fully thermalize because with a typical width of 0.1 to 0.2 MeV - estimated from speading widths of low lying collective states - the thermalization time lies between 30 and 60 10^{-22} sec.

APPENDIX A

Estimation of the geodetic potential

On the Hamiltonian defined in Eq. (2.5) we perform a transformation with the help of the function

$$\sqrt{D} = \sqrt{M_\alpha(\alpha,q)\,M_q(\alpha,q)} \tag{A.1}$$

in the following way:

$$H\varphi = E\varphi \longrightarrow \sqrt{D}\,H\left(\sqrt{D}\right)^{-1}\sqrt{D}\,\varphi = E\sqrt{D}\,\varphi \tag{A.2}$$

The new Hamiltonian and wavefunctions are $\tilde{H} = \sqrt{D}\,H\left(\sqrt{D}\right)^{-1}$ and $\tilde{\phi} = \sqrt{D}\,\varphi$ respectively. Apparently the transformation conserves the energy eigenvalues. The transformation of the first term

$$T_\alpha = -\frac{\hbar^2}{2}\frac{1}{\sqrt{M_\alpha M_q}}\frac{\partial}{\partial\alpha}\sqrt{\frac{M_q}{M_\alpha}}\frac{\partial}{\partial\alpha} \tag{A.3}$$

gives

$$\tilde{T}_\alpha = -\frac{\hbar^2}{2}\frac{1}{\sqrt[4]{M_\alpha M_q}}\frac{\partial}{\partial\alpha}\left[\sqrt{\frac{M_q}{M_\alpha}}\frac{\partial}{\partial\alpha}\frac{1}{\sqrt[4]{M_\alpha M_q}}\right]$$

$$= -\frac{\hbar^2}{2}\frac{\partial}{\partial\alpha}\cdot\frac{1}{M_\alpha}\frac{\partial}{\partial\alpha} + \mathcal{V}_G^{(\alpha)} \tag{A.4}$$

where

$$\mathcal{V}_G^{(\alpha)} = -\frac{\hbar^2}{2}\frac{1}{M_\alpha}\sqrt{D}\frac{\partial^2}{\partial\alpha^2}\frac{1}{\sqrt{D}} + \frac{1}{\sqrt{D}}\left[\frac{\partial}{\partial\alpha}\frac{1}{\sqrt{D}}\right]\left[\frac{\partial}{\partial\alpha}\frac{1}{M_\alpha}\sqrt{D}\right] \tag{A.5}$$

From the second term $\tilde{T}_q = -\frac{\hbar^2}{2}\frac{1}{\sqrt{M_\alpha M_q}}\frac{\partial}{\partial q}\sqrt{\frac{M_\alpha}{M_q}}\frac{\partial}{\partial q}$ (A.6) one obtains $$\tilde{T}_q = -\frac{\hbar^2}{2}\frac{\partial}{\partial q}\frac{1}{M_q}\frac{\partial}{\partial q} + \mathcal{V}_G^{(q)} \tag{A.7}$$

where
$$\mathcal{V}_G^{(q)} = -\frac{\hbar^2}{2}\frac{1}{M_q}\sqrt{D}\frac{\partial^2}{\partial q^2}\frac{1}{\sqrt{D}} + \frac{1}{\sqrt{D}}\left[\frac{\partial}{\partial q}\frac{1}{\sqrt{D}}\right]\left[\frac{\partial}{\partial q}\frac{1}{M_q}\sqrt{D}\right] \tag{A.8}$$

Finally one gets for the transformed Hamiltonian

$$\tilde{H} = -\frac{\hbar^2}{2}\frac{\partial}{\partial\alpha}\frac{1}{M_\alpha}\frac{\partial}{\partial\alpha} - \frac{\hbar^2}{2}\frac{\partial}{\partial q}\frac{1}{M_q}\frac{\partial}{\partial q} + \mathcal{V} + \mathcal{V}_G^{(\alpha)} + \mathcal{V}_G^{(q)} \tag{A.9}$$

The quantity $V_G(\alpha, q) = V_G^{(\alpha)} + V_G^{(q)}$ is the last term appearing in Eq. (2.6) on the r.h.s. Numerical estimates show that it can be neglected compared to the other terms in $\tilde{H}$.

APPENDIX B

Evaluation of the fission wave function in WKB approximation

The collective Hamiltonian connected with the coordinate describing the fission mode is assumed in the following form:

$$\left\{-\frac{\hbar^2}{2}\frac{\partial}{\partial\alpha}\frac{1}{M(\alpha)}\frac{\partial}{\partial\alpha}+\left[U(\alpha)-E\right]\right\}u(\alpha)=0 \qquad \text{(B.1)}$$

where $M(\alpha)$ is the collective mass parameter and $U(\alpha)$ is the collective potential which consists of the two parts:

$$U(\alpha)=V(\alpha)+iW(\alpha) \qquad \text{(B.2)}$$

The real part V of the potential is the fission barrier and the imaginary part W is responsible for nuclear dissipation.

The first order WKB approximation gives the following general solution for $U(\alpha)$:

$$U(\alpha)=C_1\frac{1}{\sqrt{v(\alpha)}}\exp\left[-\frac{1}{\hbar}\int q(\alpha)\,d\alpha\right]\exp\left[\frac{i}{\hbar}\int p(\alpha)\,d\alpha-\frac{i}{2}\operatorname{arctg}\frac{q(\alpha)}{p(\alpha)}\right]$$
$$+C_2\frac{1}{\sqrt{v(\alpha)}}\exp\left[\frac{1}{\hbar}\int q(\alpha)\,d\alpha\right]\exp\left[-\frac{i}{\hbar}\int p(\alpha)\,d\alpha+\frac{i}{2}\operatorname{arctg}\frac{q(\alpha)}{p(\alpha)}\right] \qquad \text{(B.3)}$$

as will be shown in Appendix C.

Here

$$p(\alpha)=\sqrt{M(\alpha)\left[E-V(\alpha)+\sqrt{(E-V(\alpha))^2+W^2(\alpha)}\right]} \qquad \text{(B.3a)}$$

and

$$q=-\frac{M(\alpha)\,W(\alpha)}{p(\alpha)} \qquad \text{(B.3b)}$$

The collective velocity v is given by

$$v(\alpha) = \frac{|p(\alpha)|}{M(\alpha)} \qquad \text{(B.3c)}$$

Let us make the following assumption about the real part of collective potential

$$V(\alpha) = \begin{cases} \text{const}=V(\alpha_{gr}) & \text{for} \quad \alpha < \alpha_{gr} \\ V_m(\alpha) & \text{for} \quad \alpha_{gr} \leqslant \alpha \leqslant \alpha_{sc} \\ \text{const}=V(\alpha_{sc}) & \text{for} \quad \alpha > \alpha_{sc} \end{cases} \qquad \text{(B.4)}$$

where α_{gr} corresponds to the ground state configuration and α_{sc} to the scission point. The potential (4) is shown in Fig. 1. The last assumption in (4) (for $\alpha > \alpha_{sc}$) means that the omit the Coulomb tail.

The imaginary part of the potential is assumed in the following form

$$W(\alpha) = \begin{cases} 0 & \text{for } \alpha < \alpha_{r} \\ W_m(\alpha) & \text{for } \alpha_{r} \leqslant \alpha \leqslant \alpha_{sc} \\ 0 & \text{for} \quad \alpha > \alpha_{sc} \end{cases} \qquad \text{(B.5)}$$

It means that we assume the presence of nuclear dissipation in the region from the right turning point α_T (see Fig. 1) to the scission configuration α_{sc}. $V_m(\alpha)$ and $W_m(\alpha)$ are the microscopic estimates of the real and imaginary parts of the collective potential respectively. This choice of the potential ensures that the incoming (for $\alpha < \alpha_{gr}$) and outgoing ($\alpha > \alpha_{sc}$) wave functions are plane waves. The WKB wave function which corresponds to the potential (4) and (5) is given by (see e.g. Landau and Lifshitz, Quantum Mechanics)

$$
U(\alpha) = \begin{cases}
\dfrac{2}{\sqrt{v}} \cos\left(\dfrac{1}{\hbar} \displaystyle\int_{\alpha_c}^{\alpha} p\, d\alpha + \dfrac{\pi}{4}\right) & \text{for } \alpha < \alpha_c \\[2ex]
\dfrac{1}{\sqrt{v}} \exp\left(-\dfrac{1}{\hbar} \left| \displaystyle\int_{\alpha_c}^{\alpha} p\, d\alpha \right|\right) & \text{for } \alpha_c < \alpha < \alpha_r \\[2ex]
\dfrac{\exp\left(-\frac{1}{\hbar} \int_{\alpha_r}^{\alpha} q\, d\alpha\right.}{\sqrt{v}} \exp\left(-\dfrac{1}{\hbar} \left| \displaystyle\int_{\alpha_c}^{\alpha_r} p\, d\alpha \right| + \dfrac{i}{\hbar} \displaystyle\int_{\alpha_r}^{\alpha} p\, d\alpha - \dfrac{i}{2} \operatorname{arctg} + i\dfrac{\pi}{\psi}\right\} & \text{for } \alpha > \alpha_r
\end{cases}
\qquad (B.6)
$$

APPENDIX C

WKB for complex potential and variable mass

We start from the Schrödinger equation

$$\left[-\frac{\hbar^2}{2}\frac{\partial}{\partial\alpha}\frac{1}{M(\alpha)}\frac{\partial}{\partial\alpha}+\left(U(\alpha)-E\right)\right]u(\alpha)=0 \tag{C.1}$$

with the complex potential

$$U(\alpha)=V(\alpha)+iW(\alpha)$$

Taking the ansatz

$$u(\alpha)\sim\exp\left(i\sigma(\alpha)/\hbar\right) \tag{C.2a}$$

with

$$\sigma(\alpha)=\sigma'(\alpha)+i\sigma''(\alpha) \tag{C.2b}$$

one obtains the following equation for $\sigma(\alpha)$

$$\frac{1}{2M(\alpha)}\left(\frac{d\sigma}{d\alpha}\right)^2-\frac{i\hbar}{2M(\alpha)}\left[\frac{d^2\sigma}{d\alpha^2}-\frac{d\ln M(\alpha)}{d\alpha}\frac{d\sigma}{d\alpha}\right]=E-U(\alpha) \tag{C.3}$$

Making the usual expansion

$$\sigma=\sigma^{(0)}+\frac{\hbar}{i}\sigma^{(1)}+\left(\frac{\hbar}{i}\right)^2\sigma^{(2)}+\dots$$

we obtain after a lengthy but straightforward calculation the following expressions for $\sigma^{(0)}$ and $\sigma^{(1)}$:

$$\sigma^{(0)\prime}(\alpha)=\pm\int p(\alpha)\,d\alpha+C_1$$

$$\sigma^{(0)\prime\prime}(\alpha)=\pm\int q(\alpha)\,d\alpha+C_2 \tag{C.4}$$

$$\sigma^{(1)\prime}(\alpha)=-\frac{1}{2}\ln\frac{\sqrt{p^2(\alpha)+q^2(\alpha)}}{M(\alpha)}+C_3$$

$$\sigma^{(1)\prime\prime}(\alpha)=-\frac{1}{2}\operatorname{arctg}\frac{q(\alpha)}{p(\alpha)}+C_4$$

where C_1, C_2, C_3, C_4 are arbitrary integration constants and $p(\alpha)$ and $q(\alpha)$ are defined as

$$p(\alpha)=\sqrt{M(\alpha)\left[E-V(\alpha)+\sqrt{(E-V(\alpha))^2+W^2(\alpha)}\right]}$$
$$q(\alpha)=-\frac{M(\alpha)W(\alpha)}{p(\alpha)} \tag{C.5.}$$

Thus one obtains the following form for $\sigma(\alpha)=\sigma^{(0)}(\alpha)+\frac{\hbar}{i}\sigma^{(1)}(\alpha)$

$$\sigma(\alpha)=\pm\int p(\alpha)d\alpha-\frac{\hbar}{2}\operatorname{arc\,tg}\frac{q(\alpha)}{p(\alpha)}+C_1+\hbar C_4$$
$$+i\left[\pm\int q(\alpha)d\alpha+\hbar\ln\sqrt{\frac{\sqrt{p^2(\alpha)+q^2(\alpha)}}{M(\alpha)}}+C_2-\hbar C_3\right]$$

This gives the WKB wave function

$$u(\alpha)=\sqrt{\frac{M(\alpha)}{\sqrt{p^2(\alpha)+q^2(\alpha)}}}\left\{D_1\exp\left[-\frac{1}{\hbar}\int q(\alpha)d\alpha\right]\exp\left[\int p(\alpha)d\alpha-\frac{\hbar}{2}\operatorname{arctg}\frac{q(\alpha)}{p(\alpha)}\right]\right.$$
$$\left.+D_2\exp\left[\frac{1}{\hbar}\int q(\alpha)d\alpha\right]\exp\left[-\frac{i}{\hbar}\int p(\alpha)d\alpha+\frac{\hbar}{2}\operatorname{arctg}\frac{q(\alpha)}{p(\alpha)}\right]\right\}$$

where D_1 and D_2 are arbitrary constants which have to be chosen in accordance with the boundary conditions. In our case where we have a plane incident wave coming from the left we put $D_2 = 0$ (recall that $W < 0$).

(v) Comparing the effects coming from DMC and damped fission motion one sees that the energy loss is primarily determined by the frictional type of damping of the fission mode; comparatively little energy goes into the excitation of asymmetry phonons. On the other hand the latter process governs the fission fragment distribution. It is formed in an early stage of the decent towards scission where the motion is slow and the energy loss via friction is small so that the frictional energy dissipation has little influence on mass distributions.

Everyghing that has been said is valid only for small fission energies probably not higher than 3 or 4 MeV. Above this limit nonadiabatic level crossing becomes so important that it invalidates the whole approach so that a completely different theory must be envisaged.

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STRESZCZENIE

Praca dotyczy oceny rozkładu mas jąder fragmentów rozszczepienia i dyssypacji energii jądra ^{236}U. Obliczenia są wykonane w przybliżeniu adiabatycznym, ale dyskutowane są także: a) poprawki do rozkładu mas ze względu na nieadiabatyczne sprzężenie z fononami asymetrycznych wibracji, b) tłumienie ruchu do rozszczepienia poprzez dyssypację jedno- i dwuciałową ze współczynnikami tarcia obliczonymi na drodze mikroskopowej.

РЕЗЮМЕ

Работа посвящена оценке распределения масс осколков деления и диссирации энергии делящегося ядра U 36. Расчёт основан на адиабатическим приближении но мы учитываем тоже: а) добавки к распределении масс из за неадиабатического спаривания фононов ассиметрических вибраций; б) задержание деления из за 1- и 2-частичной диссипации, где параметр трения определён по микроскопической теории.