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The Extended Core-coupling Scheme with Example for the ^{147}Gd

Rozszerzony schemat sprzężenia rdzenia z cząstką na przykładzie ^{147}Gd

Расширенная схема сопряженного остова с применением к ^{147}Gd

Dedicated to Professor
Stanisław Szpikowski on occasion
of his 60th birthday

1. INTRODUCTION

The core-coupling model of Thankappan and True [1], which is in fact extending de'Shalit's ideas [2], has been applied to a wide range of atomic nuclei. The early papers (e.g.[3],[4],[5]) demonstrated the utility of this model for the interpretation of energy levels, reduced transition probabilities and spectroscopic factors in the lighter nuclei: copper, strontium, ytterbium. The core states were coupled to the possible single-particle states of the outermost-odd nucleon in all these cases. The Hamiltonian contained both the dipole-dipole and the quadrupole-quadrupole forces.

In the present paper we try to apply the core-coupling model to the interesting region of $A\sim 150$. The results have preliminary character and further studies of the model are continued.

In the model there is a plethora of parameters. As the first order of approximation we took their values accepted in the neighbouring regions and then investigated the effect of a variation of the parameters. There were nine parameters in our calculation and some of them, in general, did not have strong effects on levels splitting. Thus, although we paid attention to the computation of the right values of the parameters, there can appear some ambiguities.

A number of experimental works have been done lately on the ^{147}Gd (e.g.[6],[7],[8],[9]). It was found that in this nucleus the strong single-character spectra and many members of multiplets connected with the 3^- and 2^+ core states were populated.

Many of the published theoretical works, however, have not shown a good agreement with the basic features of energy schemes and specially with reduced transition probability and magnetic moment values [10]. A more phenomenological investigation [11],[12] could be useful, in such a situation, for a systematic study of the general behavior of the core-odd particle interaction parameters.

2. THEORY

The Hamiltonian for an odd-A nucleus near closed shells can be taken in the model as

$$H = H_c + H_{sp} + H_{int} \tag{1}$$

where H_c is the core Hamiltonian, H_{sp} - the Hamiltonian of the single particle outside the even-even core, and H_{int} is the Hamiltonian of the interaction between the particle and core excitations (collective vibrations, rotational or multi-quasiparticle excitations). A ground foundation is the weakness of the odd-particle--core force in comparison with the interaction of nucleons of the core. Then, H_{int} may be formally expanded as a series of multipole--multipole interactions

$$H_{int} = -\sum \eta_k T_c^{(k)} \cdot T_{sp}^{(k)} \qquad (2)$$

In eq. (2) η_k is an unknown strength for the k-range multipole--multipole force, and $T_c^{(k)} \cdot T_{sp}^{(k)}$ means a scalar product of the core multipole tensor

$$T_c^{(k\varkappa)} = \sum^{A-1} r_i^k Y_k(\vartheta_i, \varphi_i) \qquad (3)$$

and the odd-particle multipole operator

$$T_{sp}^{(k\varkappa)} = r_{sp}^k Y_{k\varkappa}(\vartheta_{sp}, \varphi_{sp}) \qquad (4)$$

The sum in eq. (3) is extended on all nucleons in the core.

In all the reported applications of the model the authors had taken into account two terms of eq. (2) only. We included, additionally, the third term which is necessary for the right interpretation of spectroscopic characteristics of the ^{147}Gd. We have therefore, H_{int} in the form

$$H_{int} = -\xi J_c \cdot j_{sp} - \eta Q_c \cdot Q_{sp} - \zeta T_c^{(3)} \cdot T_{sp}^{(3)} \qquad (5)$$

named "extended". J_c and j_{sp} in eq. (5) are the angular momentum operators for the core and the odd nucleon respectively; Q_c and Q_{sp} are quadrupole and $T_{sp}^{(3)}$, $T_c^{(3)}$ octupole operators. Only an explicit form of the single-particle operators will be needed. The matrix element of the Q_c and $T_c^{(3)}$ are parametrized in the manner of paper [1].

The eigenstates $|(j_{sp}J_c)IM\rangle$ of the Hamiltonian (1) can be taken as a linear combination of the angular momentum coupled eigenfunctions of the $H_c + H_{sp}$ part.

$$\left|(j_{sp}J_c)IM\right\rangle = \sum_{j_{sp}J_c} C^I_{j_{sp}J_c} \left|j_{sp}\right\rangle \left|J_c\right\rangle \qquad (6)$$

I is the total angular momentum for a system of the core (with the angular momentum J_c) and the odd particle (with angular momentum j_{sp}). The $C^I_{j_{sp}J_c}$ are, thus, the probability amplitudes describing the states (6) in terms of the simpler basis functions.

The ordinary procedure for finding energy levels involves diagonalization of the Hamiltonian (1) to compute its eigenvalues. The single particle energies of H_{sp} are treated as further parameters of the model. Their initial values are taken to be equal to the experimental single particle energies [13].

In the calculations we adopt the harmonic-oscillator wave functions with $\hbar\omega = 41\ A^{-1/3}$ as the single-particle functions.

Further details of the model are straightforward and the final forms of the needed matrix elements can be obtained by the standard methods of the angular momentum technique [14].

The computer code allows for a self-acting fit of the energy levels and transition probabilities to the experimental ones. As a result we obtain the coefficients $C^I_{j_{sp}J_c}$ and, therefore, the structure of the eigenfunctions (6).

This information is sufficient for an extraction of some additional interesting nuclear properties of the A+1 nucleus; e.g. quadrupole moments, magnetic moments, spectroscopic factors for the transfer of a single nucleon to the core can be calculated. All these quantities are good tests for the utility of the model.

The core-coupling model is of the feature that very little assumption is made about core states. The reduced matrix elements of multipole operators are taken to be parameters fitted to the experiment. In the model they are not derived from any model.

The approximation in the model arises from the assumed antisymmetry of the wave function. The states $\left|(j_{sp}J_c)IM\right\rangle$ in eq. (6) are taken as vector-coupling states. The states $\left|J_c\right\rangle$, however, contain an amplitude from single nucleon in the j_{sp} orbit and thus, strictly speaking the state $\left|j_{sp}\right\rangle$ is partially blocked according to the Pauli principle. Our approximation will, in consequence, be working well only if the blocking effects are relatively small.

3. RESULTS AND DISCUSSION

The preliminary results we have reported concern the ^{147}Gd nucleons, which belongs to the interesting region A $\sim$ 150. In the presented calculation we demonstrate the possibility of the extended core-coupling model application to the nucleus under discussion.

The ^{147}Gd has 64 protons and 83 neutrons. The even-even core is taken to be active by only collective excitations (in the sense we wrote about in the THEORY). The eighty-third neutron is treated as single nucleon which is distributed between single particle states: $f_{7/2}$, $i_{13/2}$, $h_{9/2}$, $p_{3/2}$ and $p_{1/2}$. Higher configurations are not included in the model space because of their large excitation energy. In the paper we adopted single-particle energies [13] as a first step of the fit procedure and the core excitations shown in Table 1.

Table 1.

^{146}Gd J	Energy (keV)
0^+	0
3^-	1579
2^+	1972
5^-	2658

This choice provides as set of parameters the matrix elements of the core multipole operators:

$$\chi_{02} = \eta \langle 0^+ \| Q_c \| 2^+ \rangle; \qquad \chi_{33} = \eta \langle 3^- \| Q_c \| 3^- \rangle;$$

$$\chi_{22} = \eta \langle 2^+ \| Q_c \| 2^+ \rangle; \qquad {}_{35} = \eta \langle 3^- \| Q_c \| 5^- \rangle;$$

$$\chi_{55} = \eta \langle 5^- \| Q_c \| 5^- \rangle; \qquad {}_{03} = \xi \langle 0^+ \| T_c^{(3)} \| 3^- \rangle;$$

$$\chi_{23} = \zeta\langle 2^+ \| T_c^{(3)} \| 5^- \rangle; \qquad \chi_{25} = \zeta\langle 2^+ \| T_c^{(3)} \| 5^- \rangle.$$

The best fit of level scheme was attained for the parameter values presented in Table 2. The energies of the calculated states are

Table 2.

Values of parameters								
$MeV \cdot fm^{-2}$					$MeV \cdot fm^{-3}$			MeV
02	33	35	22	55	03	23	25	
0.000	-0.139	0.097	0.049	0.114	0.009	0.009	0.014	0.017

reproduced well (Table 3). The $^1/2^+$ (1292 keV), $^3/2^+$ (1412 keV), $^7/2^+$ (1628), $^9/2^+$ (1642), $^{11}/2^+$ (1701) states are members of the ($3^- \otimes 7/2^-$) multiplet. Two of these states ($^1/2^-$ and $^3/2^+$) are other spectroscopic characteristics in the literature [13]. Our results were confirmed on the grounds of the transition probabilities and magnetic moments analyses. The levels: $^7/2^-$ (g.s.), $^3/2^-$ (1.152 keV), $^1/2^-$ (1.847) have a strong single-particle character and we obtain their "unperturbed" energies: $f_{7/2^-}$ (7 keV) $p_{3/2^-}$ (151 keV), $g_{9/2^-}$ (1404 keV), $p_{1/2^-}$ (1842 keV) which are very closed to those deduced from experiment [13].

More detailed analyses based on the reduced transition probabilities and quadrupole and magnetic moments are not possible in the wider scale because of very limited data on the ^{147}Gd nucleus. In the case of the best fit we can calculate only the value listed in Table 4. Comparison of the E3 reduced transition probability proves that the state $^{13}/2^+$ (998 keV) has very strong collective admixture. A coherent interpretation of the experimental data needs to place the $i_{13/2^+}$ single-particle state about 2116 keV above the $f_{7/2}$.

Concluding, we can confirm the utility of the model as a way of data interpretation (energy levels, spectroscopic factors, reduced probability transitions). We are able to reproduce the

Table 3.

State J	Energy (keV)	
	Theory	Experiment [13]
7/2⁻ g.s.	-0.08	0.0
13/2⁺	1014	998
3/2⁻	1151	1152
1/2⁺	1273	1292
9/2⁻	1401	1398
3/2⁺	1402	1412
7/2⁺	1645	1628
9/2⁺	1656	1643
11/2⁺	1664	1701
9/2⁻	1799	1797
1/2⁻	1842	1847
17/2⁺	2480	2488

Table 4.

	Theory	Experiment [15]
Quadrupole moment $13/2^+$ (998 keV) (b)	$-0.70^{a)}$	$\pm(0.73\pm7)$
B(E3: $13/2^+ \longrightarrow 7/2^-$ g.s.) (e^2fm^6)	$55\ 10^3$ b)	$(56\pm4)\cdot10^3$
Magnetic moment (u_N)	-0.24 c)	(-0.24 ± 0.07)

a) effective charge $e^{(2)} = 1.99e$

b) effective charge $e^{(3)} = 2.5e$

c) $g_c = Z/A = 0.44$; $g_l = 0.0$; $g_s^{eff} = 0.6$; $g_s^{free} = -2.29\,\mu_N$

strength of the core-odd particle interaction and unpeturbed single-particle energies.

We have not discussed, however, higher power effects in the core-single nucleon interaction and its influence on the change of the core structure. All these problems may be included in the model in a simple way.

Some other extensions of the core-particle scheme are easy to reach [16]. Especially, an addition of more than one particle and a definition of a structure of the core Hamiltonian are promising cases. We develop the searches with the IBM-1 Hamiltonian for the A=148 core.

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STRESZCZENIE

W pracy dyskutuje się hamiltonian oddziaływania cząstka-rdzeń z siłami multipolowymi zawierającymi człony do rzędu $\lambda \leqslant 3$. Dzięki zastosowaniu parametryzacji elementów macierzowych operatorów charakteryzujących rdzeń nie wymagana jest znajomość jawnej ich postaci. W teorii traktuje się je jako swobodne parametry dopasowywane do eksperymentalnych danych.

Model został zastosowany w jądrze atomowym ^{147}Gd. Pokazano użyteczność wprowadzonych przybliżeń, a w szczególności niezbędność włączenia do hamiltonianu członów zawierających oddziaływanie oktupol-oktupol.

Р Е З Ю М Е

В работе рассматривается гамильтониан взаимодействия частица-остов с мультипольными силами содержащими члены ранга $\lambda \leqslant 3$. Благодаря применению параметризации матричных элементов операторов описывающих остов - не требуется их явной формы. В теории рассматривается их как свободные параметры подгоняемые к экспериментальным данным.

Эта модель применяется к атомному ядру ^{147}Gd. Демонстрируется применимость введенных приближений, а в частности необходимость включить в гамильтониан членов учитивающие октуполь-октупольное взаимодействие.