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Octupole Degrees of Freedom in Nuclei

Oktupolowe stopnie swobody w jądrach

Октупольные степени свободы в ядрах

1. INTRODUCTION

Octupole degrees of freedom in nuclei have been the subject of many investigations. They were introduced by Bohr and Mottelson [1] within the context of a description of collective states in nuclei in terms of shape variables. Some of their properties were subsequently investigated by Strutinski [2] and Lipas and Davidson [3]. An alternative treatment of collective states in nuclei is in terms of interacting bosons. Octupole (or f) bosons were introduced in Refs.[4,5] and their properties studied in Ref.[6] and, more recently, by Han et. al [7], Barfield [8] and Engel [9]. Szpikowski, together with Góźdź and Zając [10], analyzed other aspects of the same problem and discussed in detail the spectra expected in some situations.

All the references mentioned above were concerned mainly with octupole vibrations. Meanwhile, other authors [11] had suggested that octupole deformations may occur in some nuclei. This suggestion arose from the study of some properties of odd-even nuclei in the Ra region [12]. Although the question of whether or not octupole deformations in this region do occur is still debated [13], it appears necessary to have a framework in which it can be investigated in a systematic and detailed way.

In constructing models capable of describing simultaneously both vibrations and rotations two approaches are possible. One is in terms of shape variables. Rohozinski [14] and others have developed the appropriate formalism. The other is in terms of interacting bosons. Engel [15] has developed the appropriate formalism here. In this article, written in honor of Stanisław Szpikowski who has contributed considerably to the subject, I will summarize the boson formalism and comment on the results obtained so far.

2. INTERACTING BOSON MODEL OF OCTUPOLE STATES

In previous treatments of this problem [5,6], f bosons were introduced in addition to s and d bosons. These can be thought of as the quantization of the shape variables $\alpha_{3\mu}$ ($\mu=0,\pm1,\pm2,\pm3$). However, it appears that this introduction is not sufficient to describe properly phenomenological [16] and microscopic [17] properties of the observed states. It has been suggested that a comprehensive treatment requires the introduction of both p and f bosons alongside the usual s and d. One is thus led to consider a system of N s,p,d and f bosons with angular momenta and parities $J^P=0^+,1^-,2^+$ and 3^- respectively. The introduction of p bosons here is somewhat similar to that of s bosons in the usual case. On one side they facilitate the phenomenological treatment [16] while on the other side they are dictated by microscopic considerations [17]. With s,p,d and f bosons the space spanned by single boson states becomes 1+3+5+7=16 dimensional and the corresponding algebraic structure is that of U(16). The algebra of U(16) is rather large since it is composed of $16^2=256$ generators. A general phenomenological study requires the introduction of many parameters which cannot be determined directly from experiments. There are thus two possible alternatives. One is to study the structure of the solutions corresponding to dynamic symmetries. The other is to do a numerical analysis but retaining in the Hamiltonian only those terms which are suggested by microscopic considerations.

I begin by briefly outlining some of the dynamic symmetries of U(16). The single boson states contained in this model are shown in Fig.1. From microscopic considerations

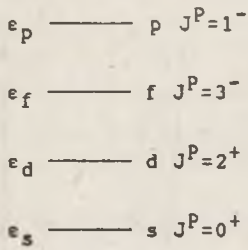

Fig.1. Schematic representation of the single boson states in U(16).

one expects that in many nuclei the energy of p-bosons, ε_p, is so large (≈4 MeV) that its effects on the low-lying states can, in lowest order, be neglected. This corresponds to breaking U(16) into

$$U(16) \supset U(13) \otimes U(3) \quad . \tag{2.1}$$

A model in terms of U(13) was studied by Góźdź, Szpikowski and Zając [10]. If the energy of f-bosons is also large (≈2 MeV) as compared with that of s and d bosons (≈0.5 MeV), U(13) can be further separated into

$$U(13) \supset U(6) \otimes U(7) \quad . \tag{2.2}$$

This is the situation in many nuclei [4-9]. Effects of p-bosons can be introduced either by a renormalization of operators [6] or explicitly [7]. Since U(6), U(7) and U(3) do not have any common subalgebra, except that of O(3), the situation corresponding to (2.1)-(2.2) can only be investigated numerically, except in the case in which there is no interaction between bosons of different species. This situation is semi-realistic only in vibrational nuclei where U(6) can be further separated into U(1)⊗U(5) leading to

$$U(16) \supset U(1) \otimes U(5) \otimes U(7) \otimes U(3) \quad . \tag{2.3}$$

States here can be characterized by the irreducible representations of the various groups appearing in (2.3) and of their subgroups,

$$\left|\begin{array}{l} U(16) \supset U(1) \otimes U(5) \otimes U(7) \otimes U(3) \supset \\ \quad\downarrow \qquad\qquad\quad \downarrow \qquad\quad \downarrow \qquad\quad \downarrow \\ \quad N \qquad\qquad\quad n_d \qquad\quad n_f \qquad\quad n_p \\ O(5) \otimes O(7) \otimes O_p(3) \supset \\ \downarrow \qquad\quad \downarrow \qquad\quad \downarrow \\ v_d, n_\Delta \quad v_f, v_1, v_2, v_3 \quad L_p \\ O_d(3) \otimes O_f(3) \otimes O_p(3) \supset O(3) \supset O(2) \\ \downarrow \qquad\quad \downarrow \qquad\qquad\qquad \downarrow \qquad\quad \downarrow \\ L_d \qquad L_f \qquad\qquad\qquad L_{df}, L \qquad M_L \end{array}\right\rangle \quad . \tag{2.4}$$

The quantum numbers $n_\Delta, v_1, v_2, v_3, L_{df}$ represent missing labels. A simple Hamiltonian with this symmetry is

$$H^{VIB} = E_0 + \epsilon_d \hat{n}_d + \epsilon_f \hat{n}_f + \epsilon_p \hat{n}_p \quad , \tag{2.5}$$

which yields an harmonic spectrum

$$E^{VIB}(N, n_d, n_f, n_p, v_d, v_1, v_2, v_3, L_p, L_d, L_f, L_{df}, L, M_L) =$$

$$= E_0 + \epsilon_d n_d + \epsilon_f n_f + \epsilon_p n_p \quad . \tag{2.6}$$

In (2.5) the energy of s-bosons has been taken as zero and included in E_0. The spectrum of (2.6) is shown in Fig.2. It is worthwhile noting that the total number of labels characterizing the totally symmetric irreducible representations of U(16) is 16.

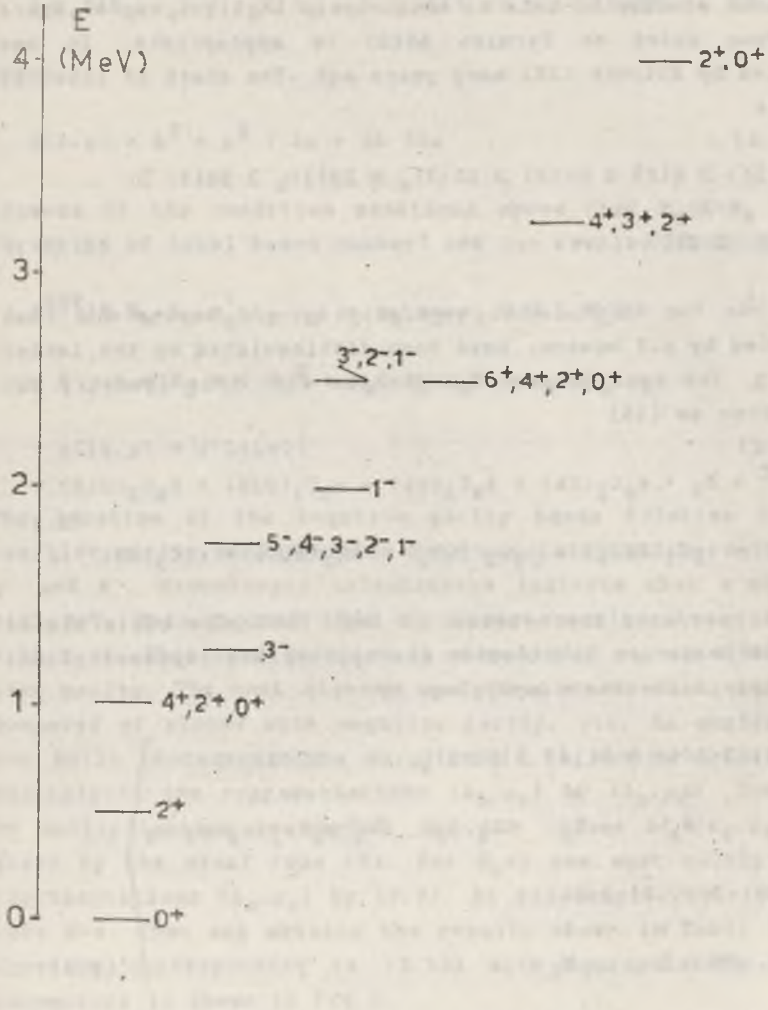

Fig.2. Vibrational spectrum with s,d,f and p bosons (N=2).

It may happen that in some nuclei the energy of p and f bosons becomes comparable. This leads to the possibility of another dynamic symmetry. This arises from the fact that s and d bosons transform as the representation (2,0) of SU(3), while p and f bosons transform as the representation (3,0). Indeed a situation similar to this is encountered in light nuclei where a scheme based on fermion SU(3) is appropriate. It was analyzed by Elliott [18] many years ago. The chain of interest here is

$$U(16) \supset U(6) \otimes U(10) \supset SU(3)_a \otimes SU(3)_b \supset SU(3) \supset O(3) \supset O(2) \quad , \tag{2.7}$$

where the two SU(3), that generated by s,d bosons and that generated by p,f bosons, have been distinguished by the letter a and b. The most general Hamiltonian with this symmetry can be written as [16]

$$H^{ROT} = E_0 + \alpha_a C_1(U6) + \beta_a C_2(U6) + \alpha_b C_1(U10) + \beta_b C_2(U10) + \kappa_a C_2(SU3_a) + \kappa_b C_2(SU3_b) + \kappa C_2(SU3) + \kappa' C_2(O3) \quad , \tag{2.8}$$

where I have used the notation of Ref. [19]. The basis states for this chain are labelled by the appropriate representations of groups in the chain and given by

$$\left| \begin{array}{ccccc} U(16) \supset & U(6) \otimes & U(10) \supset & SU(3)_a \otimes & SU(3)_b \supset \\ \downarrow & \downarrow & \downarrow & \downarrow & \downarrow \\ N & N_a & N_b & (\lambda_a,\mu_a) & (\lambda_b,\mu_b)\omega_1,\omega_2,\omega_3,\omega_4 \\ SU(3) \supset & O(3) \supset & O(2) & & \\ \downarrow & \downarrow & \downarrow & & \\ \omega,(\lambda,\mu)K & L & M_L & & \end{array} \right\rangle \quad . \tag{2.9}$$

Here $\omega_1,\omega_2,\omega_3,\omega_4,\omega,K$ represent missing labels. Although superfluous, I have also included the label $N_a=N-N_b$ since it appears in the following formula (2.10). The energy eigenvalues of H^{ROT} in the basis (2.9) are given by

$$E^{ROT}(N,N_a,N_b,\lambda_a,\mu_a,\lambda_b,\mu_b,\omega_1,\omega_2,\omega_3,\omega_4,\omega,\lambda,\mu,K,L,M_L) =$$

$$= E_0 + \alpha_a N_a + \beta_a N_a(N_a+5) + \alpha_b N_b + \beta_b N_b(N_b+9) +$$

$$+ \kappa_a C(\lambda_a,\mu_a) + \kappa_b C(\lambda_b,\mu_b) + \kappa C(\lambda,\mu) + \kappa' L(L+1), \qquad (2.10)$$

where

$$C(\lambda,\mu) = \lambda^2 + \mu^2 + \lambda\mu + 3\lambda + 3\mu \quad . \qquad (2.11)$$

Because of the condition mentioned above that $N_a=N-N_b$ (conservation of total boson number) one can rewrite (2.10) as

$$E^{ROT}(N,N_b,\lambda_a,\mu_a,\lambda_b,\mu_b,\omega_1,\omega_2,\omega_3,\omega_4,\omega,K,L,M_L) =$$

$$= E'_0 + \alpha' N_b + \beta' N_b^2 + \kappa_a C(\lambda_a,\mu_a) + \kappa_b C(\lambda_b,\mu_b) +$$

$$+ \kappa C(\lambda,\mu) + \kappa' L(L+1) \quad . \qquad (2.12)$$

The location of the negative parity bands relative to the positive parity bands depends here crucially on the values of α' and β'. Microscopic calculations indicate that $\alpha'\approx 1$ MeV, $\beta'\approx 1$ MeV. This implies that the lowest configuration is that with $N_b=0$. This configuration is composed of states with positive parity. The next highest configuration is that with $N_b=1$ composed of states with negative parity, etc. An analysis of the SU(3) representations (λ,μ) of (2.9) can be obtained by multiplying the representations (λ_a,μ_a) by (λ_b,μ_b). For $N_b=0$ no multiplication is needed, and the values of (λ_a,μ_a) are given by the usual rule [5]. For $N_b=1$ one must multiply the representations (λ_a,μ_a) by (3,0). As an example, consider the case N=4. Then one obtains the results shown in Table I. The spectrum corresponding to (2.13) with appropriately chosen parameters is shown in Fig.3.

Table I. SU(3) representations (λ,μ) contained in N=4 of U(16) when N_b=0 or 1.

N_a	N_b	(λ,μ)
4	0	(8,0)(4,2)(0,4)(2,0)
3	1	(9,0)(7,1)(5,2)(3,3)
		(5,2)(3,3)(1,4)(4,1)
		(2,2)(3,0)(1,1)(0,3)
		(3,0)

It is interesting and important to note that in Fig.3 there are two bands with $K^P=0^-$, two with $K^P=1^-$ and one with $K^P=2^-$ and 3^-. With only f bosons only one band of each K^P appears. Furthemore, the representation (5,2) or in general (λ=odd,2) contains bands with both $K^P=0^-$ and 2^-. There are several nuclei in the rare-earth and actinide region which show close-lying bands with $J^P=0^-,2^-$. In SU(3) with p and f bosons the two bands are expected to be degenerate. With f-bosons only one band can be obtained [8].

Although $\alpha',\beta'>0$ represents the situation normally expected on the basis of microscopic considerations, it could in principle happen that $\alpha'<0$. In this case the minimum of energy would occur for $N_b\neq 0$ (i.e. a condensate of octupole-dipole bosons leading to deformations). The spectrum corresponding to this case is illustrated in Fig.4 of Ref.[16]. This situation seems to be unrealistic since it would require that F-pairs be more bound than S and D pairs.

Before considering other, more realistic, situations it is worth noting that U(16) contains another, somewhat unusual chain. This is the chain [16]

$$U(16) \supset U(4)_a \otimes U(4)_b \supset Sp(4)_a \otimes Sp(4)_b \supset SU(2)_a \otimes SU(2)_b \approx$$

$$O(4) \supset O(3) \supset O(2) \quad . \tag{2.13}$$

As discussed in Ref.[16], the chain (2.13) arises from strong dipole-dipole interactions. These are the dominant forces in

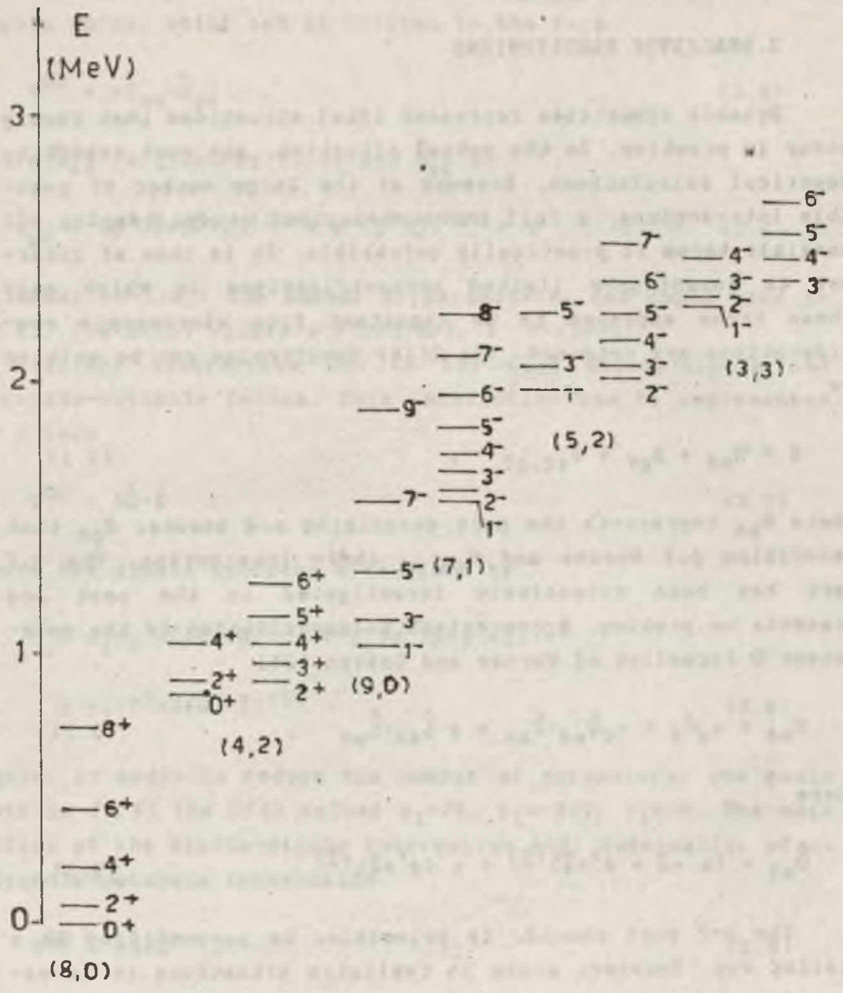

Fig.3. Schematic representation of the spectrum in the SU(3) limit of U(16). Only the lowest lying bands are shown.

molecules. In nuclei, where the dominant forces are pairing plus quadrupole, the chain (2.14) is quite unlikely to be realized.

3. REALISTIC HAMILTONIANS

Dynamic symmetries represent ideal situations that rarely occur in practice. In the actual situation, one must resort to numerical calculations. Because of the large number of possible interactions, a full phenomenological study, keeping all possible terms is practically unfeasible. It is thus of interest to investigate limited parametrizations in which only those terms expected to be important from microscopic considerations are retained. The U(16) Hamiltonian can be written as

$$H = H_{sd} + H_{pf} + V_{sd,pf} \quad , \tag{3.1}$$

where H_{sd} represents the part describing s,d bosons, H_{pf} that describing p,f bosons and $V_{sd,pf}$ their interaction. The s,d part has been extensively investigated in the past and presents no problem. A convenient parametrization is the consistent-Q formalism of Warner and Casten [20]

$$H_{sd} = \epsilon_d \hat{n}_d + \kappa_d \hat{Q}_{sd} \cdot \hat{Q}_{sd} + \kappa' \hat{L}_{sd} \cdot \hat{L}_{sd} \quad , \tag{3.2}$$

where

$$\hat{Q}_{sd} = (s^\dagger \times \tilde{d} + d^\dagger \times \tilde{s})^{(2)} + \chi \, (d^\dagger \times \tilde{d})^{(2)} \quad . \tag{3.3}$$

The p-f part should, in principle, be parametrized in a similar way. However, since in realistic situations it is expected that the energy of p and f bosons is much larger than that of s and d bosons and thus that, in low-lying states, only configurations with zero, one and two p-f bosons are important, it is sufficient to consider the Hamiltonian

$$H_{pf} = \epsilon_p \hat{n}_p + \epsilon_f \hat{n}_f + \beta(\hat{n}_p + \hat{n}_f)^2 \quad . \tag{3.4}$$

The important part is the interaction between s,d and p,f bosons. This has several contributions:

(i) the interaction due to the <u>strong</u> quadrupole-quadrupole force, which can be written in the form

$$V^{QQ} = \kappa \hat{Q}_{sd} \cdot \hat{Q}_{pf} \quad , \qquad (3.5)$$

where $\hat{Q}_{sd}$ is given by (3.3) and $\hat{Q}_{pf}$ by

$$\hat{Q}_{pf} = (p^\dagger \times \tilde{f} + f^\dagger \times \tilde{p})^{(2)} + \chi'(p^\dagger \times \tilde{p})^{(2)} + \chi''(f^\dagger \times \tilde{f})^{(2)}. \qquad (3.6)$$

In order to limit the number of parameters, one could take in (3.6), the SU(3) values $\chi'=-3\sqrt{3}/2\sqrt{7}$, $\chi''=-\sqrt{3}/\sqrt{2}$;

(ii)the interaction due to the <u>weak</u> dipole-dipole and octupole-octupole forces. This interaction can be represented by a term

$$V^{DD} = A\hat{D} \cdot \hat{D} \quad , \qquad (3.7)$$

where the dipole operator $\hat{D}$ is given by

$$\hat{D} = \alpha_1(p^\dagger \times \tilde{s} + s^\dagger \times \tilde{p})^{(1)} + \beta_1(d^\dagger \times \tilde{p} + p^\dagger \times \tilde{d})^{(1)}$$

$$+ \gamma_1(f^\dagger \times \tilde{d} + d^\dagger \times \tilde{f})^{(1)} \quad . \qquad (3.8)$$

Again, in order to reduce the number of parameters, one could take in (3.6) the O(4) values $\alpha_1=\sqrt{5}$, $\beta_1=-2\sqrt{2}$, $\gamma_1=\sqrt{7}$. The main effect of the dipole-dipole interaction and, eventually, of an octupole-octupole interaction

$$V^{UU} = B\hat{U} \cdot \hat{U} \quad , \qquad (3.9)$$

with

$$\hat{U} = \alpha_3(s^\dagger \times \tilde{f} + f^\dagger X \tilde{s})^{(3)} + \beta_3(d^\dagger \times \tilde{f} + f^\dagger \times \tilde{d})^{(3)} \quad , \qquad (3.10)$$

is that of mixing states with different numbers of <u>a</u> and <u>b</u> bosons. Since the Hamiltonian is a scalar, it can only mix states differing by two p,f bosons. If the minimum before the introduction of the mixing terms was for $N_b=0$, the interaction

V^{DD} will introduce some admixture in the ground state of $N_b=2$ states (octupole-dipole correlations). This situation was analyzed by Engel in his thesis [15], and appears to describe nuclei in the Ra region reasonably well.

Finally, in cases in which one wishes to do a calculation spanning an isotopic chain, one may introduce also terms that take into account the filling of the shells. These terms, exchange interactions, have been considered in great detail when treating the coupling of single particle degrees of freedom to s-d bosons, and discussed by Barfield [8] in her analysis of f bosons. The structure of the exchange interaction is

$$H_{exc} = \sum_{k=1}^{5} \Lambda_k \, (d^\dagger \times \tilde{f})^{(k)} \cdot (f^\dagger \times \tilde{d})^{(k)} + \sum_{k=1}^{3} \Lambda_k' \, (d^\dagger \times \tilde{p})^{(k)} \cdot (p^\dagger \times \tilde{d})^{(k)} + \sum_{k=1}^{3} \Lambda_k'' \, [(d^\dagger \times \tilde{f})^{(k)} \cdot (p^\dagger \times \tilde{d})^{(k)} + (d^\dagger \times \tilde{p})^{(k)} \cdot (f^\dagger \times \tilde{d})^{(k)}]. \tag{3.11}$$

4. CONCLUSIONS

Because of its complexity, the study of octupole degrees of freedom in nuclei still represents a major challenge to nuclear spectroscopy. A completely phenomenological study appears to be unfeasible and one must resort to a combination of phenomenology and semi-microscopic calculations. I have presented here a scheme in which this study can be done. Within this scheme and in view of the fact that the dominant interactions in nuclei are pairing and quadrupole with small dipole and octupole contributions, the most natural scheme to treat deformed nuclei appears to be that based on $\hat{Q}\cdot\hat{Q}$ interactions (SU(3)-like structure as discussed in Sect.2). With these interactions, a situation in which large and rigid octupole deformations occur seems to be quite unnatural [16]. A situation likely to occur is that in which some octupole and dipole correlations are introduced in low-lying states by octupole and dipole interactions (soft deformations). Further work is needed to clarify the situation both from the theoretical and experimental point of view. In particular, the experimental determination of more than one negative parity band is a crucial ingredient in order to distinguish the various possibilities, as discussed in detail in Ref. [16].

AKNOWLEDGEMENTS

It is a pleasure for me to dedicate this article to Stanisław Szpikowski in occasion of his 60th birthday and in honor of his many contributions to the development of nuclear spectroscopy.

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FOOTNOTES AND REFERENCES

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STRESZCZENIE

Przedstawiono opis oktupolowych stopni swobody w ramach przybliżenia oddziałujących bozonów. Grupę U(16) i jej podgrupy zinterpretowano jako możliwe symetrie dynamiczne układu zawierającego bozony typu s ($L = 0^+$), d ($L = 2^+$), f ($L = 3^-$) i p ($L = 1^-$).

Р Е З Ю М Е

Изложено описание октупольных степеней свободы в рамках приближения взаимодействующих бозонов. Группа U(16) и ее подгруппы интерпретируются как возможные динамические симметрии системы включающей бозоны типа s ($L = 0^+$), d ($L = 2^+$), f ($L = 3^-$) и p ($L = 1^-$).