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On the Rate of Convergence of Functions of Sums of Infima of Independent Random Variables

Abstract. Let $\{Y_n, n \geq 1\}$ be a sequence of independent and positive random variables, defined on a probability space $(\Omega, \mathcal{A}, P)$, with a common distribution function F. Put $Y_m^* = \inf(Y_1, Y_2, \dots Y_m)$, $m \geq 1$, and $S_n = \sum_{m=1}^n Y_m^*$, $n \geq 2$, $S_1 = 0$.

Let g be a real function such that g' satisfies the Lipschitz condition, and let $\{\alpha_n, n \geq 1\}$ be a sequence of positive real numbers.

In this paper the convergence rates in the central limit theorem and in the invariance principle for $\{g(S_n/\alpha_n), n \geq 1\}$ are obtained.

1. Introduction and notations. Let $\{Y_n,\ n \geq 1\}$ be a sequence of independent and positive random variables (i.p.r.v's.) with a common distribution function F such that

$$\int_0^1 |F(x) - x/\ell| x^{-2}\, dx < \infty \quad \text{for some} \quad \ell\,,\ 0 < \ell < \infty\,. \tag{1}$$

Let us put

$$Y_m^* = \inf(Y_1, Y_2, \dots Y_m)\,,\ m \geq 1\,, \text{and}\quad S_n = \sum_{m=1}^{n} Y_m^*\,,\ n \geq 2\,,\ S_1 = 0\,.$$

There is a large literature on properties of Y_n^*, $n \geq 1$ (cf. [3]-[11]).

Now, let $\mathcal{G}$ be the class of real and differentiable functions g, such that g' satisfies the Lipschitz condition, i.e.

$$|g'(x) - g'(y)| \leq L|x - y|\,, \tag{2}$$

where L is a positive constant.

The asymptotical normality for functions of the average of independent random variables is considered, for instance, in [1], [2], [12], [13] and [16]. In this paper we examine the rate of weak convergence of $\{g(S_n/\alpha_n)\,,\ n \geq 1\}$, where S_n is the sum of infima of independent random variables, g belongs to $\mathcal{G}$, and $\{\alpha_n\,,\ n \geq 1\}$ is a sequence of positive numbers.

2. Results. Let $\{Y_n \ , \ n \geq 1\}$ be a sequence of i.p.r.v's. with a common distribution function F such that (1) holds for some ℓ , $0 < \ell < \infty$.

Let us define

$$(3) \qquad Z_n = \frac{\alpha_n}{\ell\sqrt{2\log n}\ g'(\ell\log n/\alpha_n)}\left[g\left(\frac{S_n}{\alpha_n}\right) - g\left(\frac{\ell\log n}{\alpha_n}\right)\right], \ n \geq 1,$$

where $g \in \mathcal{G}$ and $\{\alpha_n \ , n \geq 1\}$ is a sequence of positive real numbers such that $\alpha_n \to \infty$, as $n \to \infty$. Let F_{Z_n} denotes the distribution function of Z_n and Φ the standard normal distribution function.

Theorem 1. *Under the assumption* (1) *and* (2), *we get*

$$(4) \ \sup_{x\in\mathbb{R}} |F_{Z_n}(x) - \Phi(x)| = O\left(\max\left\{\frac{\log_2 n}{(\log\ n)^{1/2}},\ \epsilon_n,\ \frac{1}{B_n^{1/2}}\ e^{-B_n/2}\right\}\right), \ \textit{as } n \to \infty \ ,$$

where $\{\epsilon_n \ , n \geq 1\}$ *is any sequence of positive real numbers decreasing to zero such that*

$$(5) \qquad \epsilon_n\alpha_n/(2\log n)^{1/2} \to \infty \ , \quad \textit{and} \quad \epsilon_n(2\log n) \to \infty \ , \quad \textit{as } n \to \infty \ ,$$

and

$$(6) \qquad B_n = \frac{\epsilon_n\alpha_n|g'(\ell\log n/\alpha_n)|}{L\theta\ell\sqrt{2\log n}} \ , \quad 0 < \theta < 1 \ .$$

Putting $\epsilon_n = \log_2 n/(\log n)^{1/2}$, and $\alpha_n = \log n$ where $\log_2 n = \log(\log n)$, from Theorem 1 we easily get the following:

Corollary 1. *Under the assumptions of* Theorem 1 *we have*

$$\sup_{x\in\mathbb{R}} |F_{Z_n}(x) - \Phi(x)| = O\left(\frac{\log_2 n}{(\log n)^{1/2}}\right), \ \textit{as } n \to \infty \ .$$

Now, let us define random functions $\{Z_n(t) \ t \in <0,1>\}$, $n \geq 1$, as follows:

$$(7) \qquad Z_n(t) = \frac{\alpha_n}{\ell\sqrt{2\log n}g'(\ell t\log n/\alpha_n)}\left\{g\left(\frac{S_{[e^t \log n]}}{\alpha_n}\right) - g\left(\frac{\ell t\log n}{\alpha_n}\right)\right\},$$

$Z_1(t) = 0$, $t \in (0,1>$, $Z_n(0) = 0$, $n \geq 1$, where $[x]$ denotes the integral part of x .

One can note that $\{Z_n(t) \ , t \in <0,1>\}$ is a sequence of $\mathcal{D}_{<0,1>}$ - valued random elements, where $\mathcal{D}_{<0,1>}$ is the space of functions defined on $[0,1]$ that are right-hand side continuous and have left-hand side limits.

Let us denote

$$(8) \qquad T(x) = P[\sup_{0\leq t\leq 1} |W(t)| \leq x] = \frac{4}{\pi}\sum_{k=0}^{\infty}\frac{(-1)^k}{2k+1}\exp\{-(2k+1)^2\pi^2/8x^2\} \ ,$$

where $\{W(t)\ ,\ t \in <0,1>\}$ is a standard Wiener process on $\mathcal{D}_{<0,1>}$.

Theorem 2. *If* $g \in \mathcal{G}$ *and* $\inf_x |g'(x)| > 0$, *then under the assumptions* (1), (2) *and* (5) *we get*

$$\sup_x |P[\sup_{0\le t\le 1} |Z_n(t)| \le x] - T(x)| \tag{9}$$
$$= O\left(\max\left\{(\log n)^{-1/3},\ \epsilon_n\ ,\ \frac{1}{C_n^{1/2}} e^{-C_n/2}\right\}\right)\ ,\ \textit{as } n \to \infty\ ,$$

where

$$C_n = \epsilon_n \alpha_n \inf_{0\le t\le 1} |g'(t\log n/\alpha_n)|/L\theta\sqrt{2\log n}\ ,\quad 0 < \theta < 1\ . \tag{10}$$

From Theorem 2, putting $\epsilon_n = (\log n)^{-1/3}$, $\alpha_n = \log n$, we obtain

Corollary 2. *Under the assumptions of* Theorem 2 *we have*

$$\sup_x |P[\sup_{0\le t\le 1} |Z_n(t)| \le x] - T(x)| = O((\log n)^{-1/3})\ .$$

3. Proofs of the results. In the proof of Theorems 1 and 2 we apply Theorem 1 [9] and Theorem 1 [8], respectively.

Proof of Theorem 1. At the beginning suppose that $\{X_n\ , n \ge 1\}$ is a sequence of independent random variables uniformly distributed on $[0,1]$ (i.r.v's.u.d.). In this case $\ell = 1$.

Put $X_m^* = \inf(X_1, X_2, \ldots, X_m)\ , m \ge 1\ ,\ \widetilde{S}_n = \sum_{m=1}^n X_m^*\ ,\ n \ge 2\ ,\ \widetilde{S}_1 = 0$, and define

$$\widetilde{Z}_n = \frac{\alpha_n}{\sqrt{2\log n}\ g'(\log n/\alpha_n)}\left[g\left(\frac{\widetilde{S}_n}{\alpha_n}\right) - g\left(\frac{\log n}{\alpha_n}\right)\right]\ , \tag{11}$$

where g and α_n are as in (3).

Let us denote

$$h_n(x) = \begin{cases} \frac{g(x)-g(\log n/\alpha_n)}{(x-\log n/\alpha_n)g'(\log n/\alpha_n)} & ,\ x \ne \log n/\alpha_n\ , \\ 1 & ,\ x = \log n/\alpha_n\ . \end{cases}$$

One can observe that

$$\widetilde{Z}_n = Z_n^{(1)} + Z_n^{(2)}\ , \tag{12}$$

where

$$Z_n^{(1)} = \frac{\widetilde{S}_n - \log n}{\sqrt{2\log n}}\ ,\quad Z_n^{(2)} = Z_n^{(1)}\left[h_n\left(\frac{\widetilde{S}_n}{\alpha_n}\right) - 1\right]\ .$$

Let $F_{\widetilde{Z}_n}$ and $F_{Z_n^{(1)}}$ denote the distribution functions of $\widetilde{Z}_n$ and $Z_n^{(1)}$, respectively.

By (11), for any $\epsilon_n > 0$, we get

$$F_{Z_n^{(1)}}(x-\epsilon_n) - P[|Z_n^{(2)}| \geq \epsilon_n] \leq F_{\widetilde{Z}_n}(x) \leq F_{Z_n^{(1)}}(x+\epsilon_n) + P[|Z_n^{(2)}| \geq \epsilon_n] ,$$

hence

$$\text{(13)} \quad \begin{aligned} \sup_x |F_{\widetilde{Z}_n}(x) - \Phi(x)| &\leq \sup_x |F_{Z_n^{(1)}}(x) - \Phi(x)| \\ &\quad + \sup_x |\Phi(x+\epsilon_n) - \Phi(x-\epsilon_n)| + P[|Z_n^{(2)}| \geq \epsilon_n] . \end{aligned}$$

By Theorem 1 [9], we have

$$\text{(14)} \quad \sup_x |F_{Z_n^{(1)}}(x) - \Phi(x)| = O\left(\frac{\log_2 n}{(\log n)^{1/2}}\right), \text{ as } n \to \infty ,$$

and, moreover, by the inequalities presented in [14], p.143,

$$\text{(15)} \quad \sup_x |\Phi(x+\epsilon_n) - \Phi(x-\epsilon_n)| \leq 2(2\pi)^{-1/2}|\epsilon_n| .$$

Now, we shall estimate the last term of the right-hand side of inequality (13). By simple evaluation, using (2), we obtain

$$\begin{aligned} P[|Z_n^{(2)}| \geq \epsilon_n] &= P\left[|Z_n^{(1)}|\left|\frac{1}{g'(\log n/\alpha_n)}\,\frac{g(\widetilde{S}_n/\alpha_n) - g(\log n/\alpha_n)}{\widetilde{S}_n/\alpha_n - \log n/\alpha_n} - 1\right| \geq \epsilon_n\right] \\ &= P\left[|Z_n^{(1)}|\left|\frac{g'(\log n/\alpha_n + \theta(\widetilde{S}_n/\alpha_n) - \log n/\alpha_n))}{g'(\log n/\alpha_n} - 1\right| \geq \epsilon_n\right] \\ &\leq P\left[|Z_n^{(1)}|^2 \left|\frac{L\theta}{|g'(\log n/\alpha_n)|}\,\frac{\sqrt{2\log n}}{\alpha_n}\right| \geq \epsilon_n\right] \\ &= P\left[|Z_n^{(1)}| \geq \left(\frac{\epsilon_n \alpha_n}{L\theta\sqrt{2\log n}}|g'(\log n/\alpha_n)|\right)^{1/2}\right] \\ &\leq 2\sup_x |F_{Z_n^{(1)}}(x) - \Phi(x)| + 2(1 - \Phi((B_n)^{1/2}) , \end{aligned}$$

where B_n is given by (6), L is a positive constant and $0 < \theta < 1$.

Hence, by (13)-(15), Theorem 1 [9] and the inequality $1 - \Phi(x) \leq 1/\sqrt{2\pi x}\, e^{-x^2/2}$ for $x > 0$, we get (4). Thus the proof, in this case is ended.

Now, let $\{Y_n, n \geq 1\}$ be a sequence of i.p.r.v's. with the same distribution function F satisfying (1), and let, as previous, $\{X_n\ , n \geq 1\}$ be a sequence of i.r.v's.u.d. on [0,1].

Put

$$G(t) = \inf\{x > 0 : F(x) \geq t\} .$$

Then, by [5], the sequences $\{G(X_n)\ ,n\geq 1\}$ and $\{Y_n\ ,n\geq 1\}$ are the same in law. Furthermore, the sum $S_n=\sum_{k=1}^n Y_k^*$, where $Y_k^*=\inf(Y_1,Y_2,\dots,Y_k)\ ,k\geq 1$ can be represented as

$$\overline{S}_n=\sum_{k=1}^n G(X_k^*)\ ,\text{ where } X_k^*=\inf(X_1,X_2,\dots,X_k)\ ,\ k\geq 1\ . \tag{16}$$

Let us define $\{\overline{Z}_n\ ,\ n\geq 1\}$ as follows:

$$\overline{Z}_n=\frac{\alpha_n}{\ell\sqrt{2\log n}g'(\ell\log n/\alpha_n)}\left[g\left(\frac{\overline{S}_n}{\alpha_n}\right)-g\left(\frac{\ell\log n}{\alpha_n}\right)\right],$$

and put

$$h_n^{(1)}(x)=\begin{cases}\frac{g(x)-g(\ell\log n/\alpha_n)}{(x-\ell\log n/\alpha_n)g'(log n/\alpha_n)} & ,\ x\neq \ell\log n/\alpha_n\ ,\\ 1 & ,\ x=\ell\log n/\alpha_n\ .\end{cases}$$

Analogously, as previous, we get

$$\overline{Z}_n=\overline{Z}_n^{(1)}+\overline{Z}_n^{(2)}\ , \tag{17}$$

where

$$\overline{Z}_n^{(1)}=\frac{\overline{S}_n-\ell\log n}{\ell\sqrt{2\log n}}\ ,\quad \overline{Z}_n^{(2)}=\overline{Z}_n^{(1)}\left[h_n^{(1)}\left(\frac{\overline{S}_n}{\alpha_n}\right)-1\right]\ .$$

By relation (23) [8] for all sequence $\{\ell_n\ ,n\geq 1\}$ of real numbers such that $\ell_n\to\infty$, as $n\to\infty$, we have

$$\frac{\overline{S}_n-\ell\widetilde{S}_n}{\ell_n}=O(1)\ ,\ \text{a.s.},$$

so that

$$F_{Z_n^{(1)}}\left(x-\frac{\ell_n}{\ell\sqrt{2\log n}}\right)\leq F_{\overline{Z}_n^{(1)}}(x)\leq F_{Z_n^{(1)}}\left(x+\frac{\ell_n}{\ell\sqrt{2\log n}}\right)\ ,$$

for sufficiently large n .

Putting $\ell_n=\ell\log_2 n$, by (14), we obtain

$$\sup_x|F_{\overline{Z}_n^{(1)}}(x)-\Phi(x)|=O\left(\frac{\log_2 n}{(\log n)^{1/2}}\right)\ . \tag{18}$$

Analogously, as previous, we get

$$P[|\overline{Z}_n^{(2)}|\geq\epsilon_n]=O\left(\max\left\{\frac{\log_2 n}{(\log n)^{1/2}}\ ,\epsilon_n\ ,\ \frac{1}{B_n^{1/2}}e^{-B_n/2}\right\}\right)\ , \tag{19}$$

where B_n is given by (6).

Using (17)-(19) we get (4) and the proof of Theorem 1 is completed.

Proof of Theorem 2. At first we assume that $\{X_n\ ,n\geq 1\}$ is a sequence of i.r.v's.u.d. on [0,1] and put

$$\widetilde{Z}_n(t)=\frac{\alpha_n}{\sqrt{2\log n}g'(t\log n/\alpha_n)}\left[g\left(\frac{\widetilde{S}_{[e^t\log n]}}{\alpha_n}\right)-g\left(\frac{t\log n}{\alpha_n}\right)\right]\ ,$$

for $t \in <0,1>$, $n \geq 2$, $\widetilde{Z}_1(t) = 0$.

We can write

(20) $$\widetilde{Z}_n(t) = Z_n^{(1)}(t) + Z_n^{(2)}(t) ,$$

where

$$Z_n^{(1)}(t) = \frac{\widetilde{S}_{[e^{t \log n}]} - t \log n}{\sqrt{2 \log n}} ,$$

$$Z_n^{(2)}(t) = Z_n^{(1)}(t) \left[h_n^t \left(\widetilde{S}_{[e^{t \log n}]} / \alpha_n \right) - 1 \right] ,$$

and

(21) $$h_n^{(t)}(x) = \begin{cases} \frac{g(x) - g(\ell t \log n/\alpha_n)}{(x - t \log n/\alpha_n) g'(t \log n/\alpha_n)} & , x \neq \ell t \log n/\alpha_n , \\ 1 & , x = \ell t \log n/\alpha_n . \end{cases}$$

We remind that in this case $\ell = 1$. We will show, that

(22) $$\sup_x |P[\sup_{0 \leq t \leq 1} |Z_n^{(1)}(t)| \leq x] - T(x)| = O(\max\{(\log n)^{-1/3} , \epsilon_n\}) ,$$

where $T(x)$ is given by (8), and ϵ_n satisfies (5).

Let us put

$$\widetilde{S}_{n,k} = \left(\widetilde{S}_k - \sum_{i=1}^{k} 1/i \right) / (2 \log n)^{1/2} ,$$

and define the random functions $\{X_n(t) , t \in <0,1>\}$ as follows:

$$X_n(t) = \widetilde{S}_{n,k} , \text{ for } t \in < t_k, t_{k+1}) , 1 \leq k \leq n ,$$
$$X_n(0) = 0 , \ n \geq 1 ,$$

where $t_k = \log k / \log n$, $1 \leq k \leq n$, $n \geq 2$.

One can note that

$$P[\sup_{0 \leq t \leq 1} |Z_n^{(1)}(t) - X_n(t)| \geq \epsilon_n]$$

$$= P\Big[\max_{1 \leq k \leq n} \sup_{t \in <t_k, t_{k+1})} \Big| \frac{\widetilde{S}_{[e^{t \log n}]}}{\sqrt{2 \log n}} - \frac{t \log n}{\sqrt{2 \log n}} - \frac{\widetilde{S}_k - \sum_{i=1}^k 1/i}{\sqrt{2 \log n}} \Big| \geq \epsilon_n \Big]$$

$$\leq P\Big[\max_{1 \leq k \leq n} \Big[\max\Big(\frac{\widetilde{S}_{k+1} - \log k}{\sqrt{2 \log n}} - \frac{\widetilde{S}_k - \sum_{i=1}^k 1/i}{\sqrt{2 \log n}} ,$$

$$\frac{\widetilde{S}_k - \sum_{i=1}^k 1/i}{\sqrt{2 \log n}} - \frac{\widetilde{S}_k - \log(k+1)}{\sqrt{2 \log n}} \Big) \Big] \geq \epsilon_n \Big]$$

$$= P\Big[\max_{1 \leq k \leq n} \max\Big(\frac{X_{k+1}^*}{\sqrt{2 \log n}} + \frac{\sum_{i=1}^k (1/i) - \log k}{\sqrt{2 \log n}} , \frac{\log(k+1) - \sum_{i=1}^k 1/i}{\sqrt{2 \log n}} \Big) \geq \epsilon_n \Big]$$

$$\leq P[X_1^* + \gamma \geq \epsilon_n \sqrt{2 \log n}] = 0$$

for sufficiently large n, as by (5) $\epsilon_n\sqrt{2\log n}\to\infty$, as $n\to\infty$, where γ is the Euler's constant ($\gamma\approx 0,577$).

Hence

$$P[\sup_{0\le t\le 1}|X_n(t)|\le x-\epsilon_n]\le P[\sup_{0\le t\le 1}|Z_n^{(1)}(t)|\le x]\le P[\sup_{0\le t\le 1}|X_n(t)|\le x+\epsilon_n],$$

and

$$\sup_x|P[\sup_{0\le t\le 1}|Z_n^{(1)}(t)|\le x]-T(x)| \le \sup_x|P[\max_{0\le t\le n}|\widetilde{S}_{n,k}|\le x]-T(x)+\sup_x|T(x+\epsilon_n)-T(x-\epsilon_n)|. \tag{23}$$

On the other hand

$$\sup_x|T(x+\epsilon_n)-T(x-\epsilon_n)|\le\sqrt{8/\pi}2\epsilon_n, \tag{24}$$

(cf.(3.1), [15]), so that by Theorem 1 [8], we get (22).

Now, let us observe that

$$\begin{aligned}
&P[\sup_{0\le t\le 1}|Z_n^{(2)}(t)|\ge\epsilon_n]\\
&=P\Big[\sup_{0\le t\le 1}|Z_n^{(1)}(t)|\Big|\frac{g'\big(t\log n/\alpha_n+\theta\big(\widetilde{S}_{[e^{t\log n}]}/\alpha_n-t\log n/\alpha_n\big)\big)}{g'(t\log n/\alpha_n)}-1\Big|\ge\epsilon_n\Big]\\
&\le P\Big[\sup_{0\le t\le 1}|Z_n^{(1)}(t)|\Big|\frac{L\theta|\widetilde{S}_{[e^{t\log n}]}-t\log n|}{\alpha_n g'(t\log n/\alpha_n)}\Big|\ge\epsilon_n\Big]\\
&\le\Big[\sup_{0\le t\le 1}|Z_n^{(1)}(t)|^2\le\frac{\epsilon_n\alpha_n\inf_{0\le t\le 1}|g'(t\log n/\alpha_n)|}{L\theta\sqrt{2\log n}}\Big]\\
&\le P[\sup_{0\le t\le 1}|Z_n^{(1)}(t)|\ge C_n^{1/2}],
\end{aligned}$$

where C_n is a positive constant given by (10).

Hence, by (22),

$$\begin{aligned}
P[\sup_{0\le t\le 1}|Z_n^{(2)}(t)|\ge\epsilon_n]&\le P[\sup_{0\le t\le 1}|Z_n^{(1)}(t)|\ge C_n^{1/2}]\\
&=P[\sup_{0\le t\le 1}|W(t)|\le C_n^{1/2}]-P[\sup_{0\le t\le 1}|Z_n^{(1)}(t)|\le C_n^{1/2}]\\
&+P[\sup_{0\le t\le 1}|W(t)|>C_n^{1/2}]\\
&\le\sup_x|P[\sup_{0\le t\le 1}|Z_n^{(1)}(t)|\le x]-T(x)|+4P[W(1)\ge C_n^{1/2}]\\
&=O((\log n)^{-1/3}+\frac{4}{\sqrt{2\pi}C_n^{1/2}}e^{-C_n/2}),
\end{aligned}$$

so by (23) and (24) we get (9).

Now, let $\{Y_n,\ n \geq 1\}$ be a sequence of i.p.r.v's. with the common distribution function F satisfying (1), and let us put

$$\overline{Z}_n(t) = \frac{\alpha_n}{\ell\sqrt{2\log n}g'(\ell t\log n/\alpha_n)}\left[g\left(\frac{\overline{S}_n}{\alpha_n}\right) - g\left(\frac{\ell t\log n}{\alpha_n}\right)\right],$$

where $\{\overline{S}_n,\ n \geq 1\}$ is given by (15).

Putting

$$\overline{Z}_n^{(1)}(t) = \frac{\overline{S}_{[e^{t}\log n]} - \ell t\log n}{\ell\sqrt{2\log n}},$$

we have

$$\overline{Z}_n(t) = \overline{Z}_n^{(1)}(t) + \overline{Z}_n^{(1)}(t)[h_n^t(\overline{S}_n/\alpha_n) - 1], \tag{25}$$

where $h_n^t(x)$ is defined by (21). If we denote

$$\overline{F}_n(x) = P[\sup_{0\leq t\leq 1}|\overline{Z}_n^{(1)}(t)| \leq x],$$

then by Theorem 1[8] we have

$$\sup_x|\overline{F}_n(x) - T(x)| = O((\log n)^{-1/3}).$$

Moreover, we obtain

$$\begin{aligned}
&P[\sup_{0\leq t\leq 1}|\overline{Z}_n^{(1)}(t)[h_n^t(\overline{S}_n/\alpha_n) - 1| \geq \epsilon_n]\\
&\leq P[\sup_{0\leq t\leq 1}|\overline{Z}_n^{(1)}(t)|^2 \geq C_n] \leq P[\sup_{0\leq t\leq 1}|\overline{Z}_n^{(1)}(t)| \geq C_n^{1/2}]\\
&\leq \sup_x|\overline{F}_n(x) - T(x)| + 4P[W(1) \geq C_n^{1/2}]\\
&= O((\log n)^{-1/3} + \frac{4}{\sqrt{2\pi}C_n^{1/2}}e^{-C_n/2}).
\end{aligned}$$

Usind (25), (26) and the above, we get (9) and the proof of Theorem 2 is completed.

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