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The Generator Coordinate Approach to $\Delta \to N\gamma$ Decay within a Non-Relativistic Deformed Quark Model

There are many theoretical and experimental papers concerning the electric quadrupole (E2) and magnetic dipole (M1) transitions amplitudes in the process $\Delta \to N\gamma$ because the mixing ratio δ(E2/M1) may be a good test for the quark models of hadron structure [10]. In the simple SU(6) quark model the E2 amplitude for the electromagnetic Δ resonance decay vanishes [11], but experiments indicate a non-zero δ(E2/M1) mixing ration [6]. Our model which is a non-relativistic approach predicts M1 as well as E2 non-zero transitions amplitudes for the process $\Delta \to N\gamma$ suggesting a possible deformation of the Δ resonance.

The papers [1,2] calculations of the electric quadrupole (E2) and magnetic dipole (M1) transition amplitudes in the process $\Delta \to N\gamma$ in the longwave approximation ($qr < 1$) have been done. The obtained results encourage us to treat that problem in a more precise way i.e. to obtain the transitions amplitudes and their mixing ratio δ(E2/M1) for the decay without longwave approximation.

The main assumption of the model is that barions can be deformed in their ground state configurations and this is the reason of non-vanishing mixing ratio δ. In order to calculate the M1 and E2 transition amplitudes we use the generator coordinate method developed in [1]. The basic assumption of the present considerations is that the photon absorption by the nucleon is due to the collective (rotational) excitations of three constituent quarks. This assumption allows to construct a proper class of generator functions. The interaction between the hadron and emitted photon is taken in the standard form [3, 4]

$$H_{\text{int}} = \frac{1}{c}\int \vec{\mathrm{j}}(\vec{\mathrm{r}}) \cdot \vec{\mathrm{A}}(\vec{\mathrm{r}}) d^3r, \tag{1}$$

where $\vec{\mathrm{j}}(\vec{\mathrm{r}})$ denotes the current of the quarks in the hadron operator which is defined by the following formula [2,7]

$$\vec{j}(\vec{r}) = \sum_{k=1}^{3} e(T_3 + \frac{1}{6})^{(k)}\frac{1}{2}[\vec{v}^{(k)}\delta(\vec{r} - \vec{r}^{(k)}) + \delta(\vec{r} - \vec{r}^{(k)})\vec{v}^{(k)}] + + \frac{e\hbar}{2m}\sum_{k=1}^{3} \nabla^{(k)} \times \vec{s}^{(k)}\delta(\vec{r} - \vec{r}^{(k)}), \quad (2)$$

and $\vec{A}(\vec{r})$ is the electromagnetic field of the photon with the momentum $\vec{q}$ and polarization $\vec{\varepsilon}$. The field $\vec{A}(\vec{r})$ have the following expansion

$$\vec{A}(\vec{r}) = \sqrt{4\pi/2q}\vec{\varepsilon}\left[a_q^+ e^{i\vec{q}\cdot\vec{r}} + a_q e^{-i\vec{q}\cdot\vec{r}}\right]. \quad (3)$$

Taking advantage of the formulas (1), (2), (3) one obtains [3,4] the following form of the interaction hamiltonian:

$$H_{\text{int}} = \frac{e}{2m_q}\sum_{k=1}^{3}(T_3 + \frac{1}{6})^k \left[-2i\gamma\vec{s}^{(k)} \cdot \vec{q} \times (\vec{A}(\vec{r}^{(k)}) + (\vec{p}^{(k)} + \vec{p'}^{(k)}) \cdot \vec{A}(\vec{r}^{(k)})\right],$$

where $e(T_3 + \frac{1}{6})$ denotes the quark electric charge operator (T_3 — the third component of quark isospin operator), m_q is the quark mass, γ is the quark gyromagnetic ratio, $\vec{s}$ denotes the quark spin operator and $\vec{p}$ and $\vec{p}'$ are the initial and final momentum of the emitting or absorbing photon quark, respectively.

Making use of the symmetry $SU(6) \otimes O(3)$ of the overall hadron wave function and expressing the interaction operator in the new coordinate frame by taking the photon momentum direction q as the quantization axis and restricting attention to the photon with right-handed polarization $\vec{\varepsilon} = -1/\sqrt{2}$ (1,i,0) one obtains a simpler form of the interaction operator [3]

$$H_{\text{int}} = 6\sqrt{\pi/q}\mu(T_3 + \frac{1}{6})^{(3)}\exp(-iqz^{(3)})[qS_+^{(3)} - P_+^{(3)}/\gamma],$$

where $S_+ = s_x + is_y$ and $P_+ = p_x + ip_y$.

The photoexcitation or the electromagnetic decay of the Δ resonance can be completely specified by the helicity amplitudes [3,5] defined as matrix elements of interaction operator between states with different spin projections on the quantization axis ($M = \frac{3}{2}, \frac{1}{2}$):

$$A_M = \langle\frac{3}{2}M\frac{3}{2}\frac{1}{2}; \lambda|H_{\text{int}}|\frac{1}{2}(M-1)\frac{1}{2}\frac{1}{2}; \lambda\rangle. \quad (4)$$

The states $|JMTM_T;\lambda\rangle$ are the wave functions describing deformed barions either Δ or N (J — spin, M — spin projection, T — isospin, M_T — isospin projection in the isospin space, λ — deformation parameter).

By means of the helicity amplitudes one can express the electromagnetic transition amplitudes in the following way [5]

$$\begin{aligned} M(\mathrm{E2}) &= -(\sqrt{3}A_{1/2} - A_{3/2})/2\sqrt{3}, \\ M(\mathrm{M1}) &= -(A_{1/2} + \sqrt{3}A_{3/2})/2. \end{aligned} \tag{5}$$

In order to calculate matrix elements (4), one needs the wave functions $|JMTM_T;\lambda\rangle$ expressed in the laboratory frame. These wave functions can be obtained in the way analogous to the one described in [1]

$$|JMTM_T;\lambda\rangle = d(j,T;\lambda)\int d\Omega d\Xi D^{J*}_{M1/2}(\Omega)D^{T*}_{M_T 1/2}(\Xi)|\Omega\Xi;\lambda\rangle, \tag{6}$$

where $d(j,T;\lambda) = (\dim(J)\dim(T)/\Lambda(J,T;\lambda))^{1/2}$
and

$$\Lambda(J,T;\lambda) = (3\delta_T)^{-1}\int_0^\pi d\beta \sin\beta \frac{d^J_{mm}(\beta)d^T_{mm}(\beta)}{2(b^2 g^{1/2})^3}, \quad \beta \equiv \Omega_2,\ m \equiv \frac{1}{2}.$$

$|\Omega\Xi;\lambda\rangle$ is the generator function, $\Omega = (\Omega_1,\Omega_2,\Omega_3)$, $\Xi = (\Xi_1,\Xi_2,\Xi_3)$ are Euler angles in the position and isospin space, respectively, and $D^j_{mk}(\alpha,\beta,\gamma) = \exp(-im\alpha)d^j_{mk}(\beta)\exp(-ik\gamma)$ are the SU(2) Wigner functions. g. and δ_T are defined in the Appendix [see also 1,2]. To construct the generator function first of all we introduce the scaling operator and an undeformed state $|\Phi_0\rangle$ defined below

$$K(\lambda) := \exp(-\lambda\hat{K}) \tag{7}$$

with $\hat{K} = \sum_{n=1}^3(z_n\frac{\partial}{\partial z_n} + \frac{1}{2})$ and $n = 1,2$ and 3 denoting z coordinate of the first, second and third quark in a nucleon. Using (7) and an undeformed ($\lambda = 0$) state $|\Phi_0\rangle$ one can write the deformed state as

$$|\Phi_0(\lambda)\rangle = K(\lambda)|\ \Phi_0\rangle. \tag{8}$$

For simplicity we choose the following group of motion $G = \mathrm{SU}_T(2) \times \mathrm{SU}_J(2)$ which is a symmetry group of our quark system. To obtain a realistic space of states the vector $|\Phi_0\rangle$ should contain the $\mathrm{SU}_F(3)$ octet and it should belong to 56-dimensional irreducible representation of the SU(6) group. Such a state is of the following form

$$|\Phi_0\rangle = |\mathrm{color}||\mathrm{singlet}\rangle|\mathrm{spin-isospin}\rangle|\mathrm{space}\rangle,$$

where

$$|\mathrm{spin-isospin}\rangle = (3)^{-1/2}(u_\uparrow u_\uparrow d_\downarrow + u_\uparrow d_\downarrow u_\uparrow + d_\downarrow u_\uparrow u_\uparrow)$$

and

$$|\mathrm{space}\rangle = \phi_0(1)\phi_0(2)\phi_0(3).$$

This form is much simpler in applications than the known proton function, see e.g. [8]. In our calculations $\phi_0(k)$, $k = 1,2,3$ is assumed to be the 3-dimensional harmonic oscillator ground state function. Let $R(\Omega) \in \mathrm{SU}_J(2)$ and $\mathcal{G}(\Xi) \in \mathrm{SU}_T(2)$

be the rotational operators in coordinate-spin space and in the isospin space, respectively. Then the full generator function

$$|\Omega, \Xi; \lambda\rangle = R(\Omega)\mathcal{G}(\Xi)|\Phi_0(\lambda)\rangle. \tag{9}$$

The matrix elements of the interaction operator between states (6) can be reduced to the following form:

$$\begin{aligned} A_{1/2} &= \langle\frac{3}{2}\frac{1}{2}\frac{3}{2}\frac{1}{2}; \lambda|H_{\rm int}|\frac{1}{2} - \frac{1}{2}\frac{1}{2}\frac{1}{2}; \lambda =\rangle \\ &= -\frac{1}{9}\sqrt{\pi q}\mu\left(\Lambda(\frac{3}{2}, \frac{3}{2}; \lambda)\Lambda(\frac{1}{2}, \frac{1}{2}; \lambda)\right)^{-1/2} \times \\ &\times \int_0^\pi d\Omega_2' \sin\Omega_2' d^{3/2}_{1/2 1/2}(\Omega_2') d^{3/2}_{1/2 1/2}(\Omega_2') \times \\ &\times \int_0^\pi d\Omega_2 \sin\Omega_2 d^{1/2}_{-1/2 1/2}(\Omega_2) d^{1/2}_{-1/2 1/2}(\Omega_2) \times \\ &\times (b^2 g^{1/2})^{-3} \exp\{-(g_x^2 g_{zz} + g_z^2 g_{xx} - 2 g_x g_z g_{xz})/4g\} \end{aligned} \tag{10}$$

and

$$\begin{aligned} A_{3/2} &= \langle\frac{3}{2}\frac{3}{2}\frac{3}{2}\frac{1}{2}; \lambda|H_{\rm int}|\frac{1}{2}\frac{1}{2}\frac{1}{2}\frac{1}{2}; \lambda\rangle = \\ &= -1/3\sqrt{3}\sqrt{\pi q}\mu\left(\Lambda(\frac{3}{2}, \frac{3}{2}; \lambda)\Lambda(\frac{1}{2}, \frac{1}{2}; \lambda)\right)^{-1/2} \times \\ &\times \int_0^\pi d\Omega_2' \sin\Omega_2' d^{3/2}_{1/2 1/2}(\Omega_2') d^{3/2}_{1/2 1/2}(\Omega_2') \times \\ &\times \int_0^\pi d\Omega_2 \sin\Omega_2 d^{1/2}_{1/2 1/2}(\Omega_2) d^{1/2}_{1/2 1/2}(\Omega_2) \times \\ &\times (b^2 g^{1/2})^{-3} \exp\{-(g_x^2 g_{zz} + g_z^2 g_{xx} - 2 g_x g_z g_{xz})/4g\}. \end{aligned} \tag{11}$$

The abbreviations used in these formulae are explained in the Appendix. The above integrals have been calculated numerically with the following parameters:

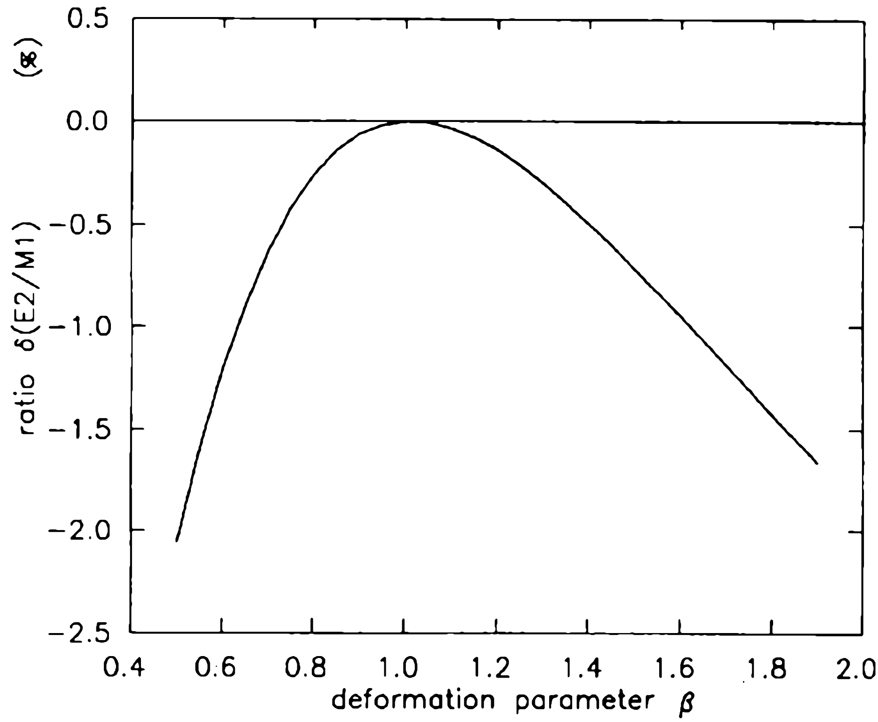
— momentum of photon γ : $q = 300$ MeV,

— width of Gaussian distribution of quark in the single quark function $\Phi_0(k)$: $b = 0.46$ fm see [9],

— quark magnetic moment (which is taken equal to the proton magnetic moment): $\mu = 0.13$ GeV^{-1}.

The numerical calculations lead to the dependence of ratio δ(E2/M1) on the deformation parameter shown in Fig. 1. The deformation parameter $\beta = e^\lambda$ represents a ratio of z-axis elongation of the nucleon elipsoide in respect to the perpendicular one.

The experimental value of the ratio δ(E2/M1) is $(1.3 \pm 0.5)\%$ [6]. There are two deformation parameters corresponding to the experimental data: $\beta = 0.6$ and $\beta = 1.75$. The first value ($\beta\langle 1$) suggests the oblate and second ($\beta\rangle 1$) — prolate



The mixing ratio $\delta(E2/M1)$ as a function of the deformation parameter $\beta = e^{\lambda}$ is shown

deformation of nucleon. The obtained value of prolate deformation is only slightly larger comparing with that one obtained from minimum of energy ($\beta = 1.55$) [1] and from the longwave approximation ($\beta = 1.2 - 1.3$) [2]. The results of the above calculations confirm qualitatively that hadrons are deformed in their ground states and that the deformation is rather large.

APPENDIX

In the Appendix we list the abbreviations used in the formulae (6), (10) and (11):

$$\begin{aligned}
\delta_T &= \delta_{T1/2} + 4\delta_{T3/2}, \\
g_{xx} &= (1 + \cos^2\beta + e^{-2\lambda}\sin^2\beta)/2b^2, \\
g_{xz} &= (e^{-\lambda} - e^{\lambda})\cos\beta\sin\beta/2b, \\
g_{zz} &= (1 + e^{2\lambda}\sin^2\beta + \cos^2\beta)/2b^2, \\
g &= g_{xx}g_{zz} - (g_{xz})^2.
\end{aligned}$$

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