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On Normality of Almost r-paracontact Structures

O normalności struktur prawie r-parakontaktowych

О нормальности почти r-параконтактных структур

An almost r-paracontact structure $\Sigma = (\varphi, \xi_{(i)}, \eta^{(i)})$ $i = 1, \ldots, r$ on a manifold M is normal if and only if: $\overset{1}{N}(X,Y) = N_\varphi(X,Y) - 2d\eta^i(X,Y)\xi_i = 0$, [2], where N_φ is the Nijenhuis tensor of φ. In this paper we give one more algebraic characterization of normal almost r-paracontact structures and define the notion of the weak-normality and give its geometric interpretation.

Definition 1. [2]. $\Sigma = (\varphi, \xi_1, \ldots, \xi_r, \eta^1, \ldots, \eta^r)$ on a manifold M is said to be an almost r-paracontact structure if :

$$\eta^i(\xi_j) = \delta^i_j\,, \qquad i,j = 1,2,\ldots,r\,, \tag{1}$$

$$\varphi(\xi_i) = 0\,, \qquad i = 1,2,\ldots,r\,, \tag{2}$$

$$\eta^i \circ \varphi = 0\,, \qquad i = 1,2,\ldots,r\,, \tag{3}$$

$$\varphi^2 = \mathrm{Id} - \eta^i \otimes \xi_i \tag{4}$$

where φ is a tensor field of type (1,1); $\xi_1, \ldots, \xi_r$ are vector fields and $\eta^1, \ldots, \eta^r$ 1-forms on M.

Put

$$\overset{1}{N}(X,Y) = N_\varphi(X,Y) - 2d\eta^i(X,Y)\xi_i\,, \tag{5}$$

$$\overset{2}{N^i}(X,Y) = (\alpha_{\varphi X}\eta^i)(Y) - (\alpha_{\varphi Y}\eta^i)(X)\,, \tag{6}$$

$$\overset{3}{N_i}(X) = -(\alpha_{\xi_i}\varphi)(X)\,, \tag{7}$$

$$\overset{4}{N}{}_i^j(X) = -(\alpha_{\xi_i}\eta^j)(X) \tag{8}$$

where α_X is the Lie derivative with respect to a vector field X.

Theorem 1. [2]. *An almost r-paracontact structure* $\Sigma = (\varphi, \xi_{(i)}, \eta^{(i)})_{i=1,\ldots,r}$ *on* M *is normal if and only if* $\overset{1}{N} = 0$.

Define:

$$\begin{aligned} D^+ &= \{X;\ \varphi X = X\}, \\ D^- &= \{X;\ \varphi X = -X\}, \\ D^0 &= \{X;\ \varphi X = 0\}\,. \end{aligned} \tag{9}$$

We also have:

Theorem 2. [2]. *An almost r-paracontact structure* $\Sigma = (\varphi, \xi_{(i)}, \eta^{(i)})_{i=1,\ldots,r}$ *on* M *is normal if and only if* $\overset{4}{N}{}_j^i = 0, [\xi_i, \xi_j] = 0, \quad i,j = 1,\ldots,r$ *and the distributions:* D^+, D^-, $D^+ \oplus D^0$, $D^- \oplus D^0$ *are integrable.*

Let M and $\overline{M}$ be manifolds and $\Sigma = (\varphi, \xi_{(i)}, \eta^{(i)})_{i=1,\ldots,r}$ and $\overline{\Sigma} = (\overline{\varphi}, \overline{\xi}_{(i)}, \overline{\eta}^{(i)})_{i=1,\ldots,r}$ be almost r-paracontact structures on M and $\overline{M}$ respectively. A vector field X on M will be identified with the vector field $\tilde{X}$ on $M \times \overline{M}$ as follows: $\tilde{X}_{(p,\overline{p})} = X_p + O_{\overline{p}}$ for $(p,\overline{p}) \in M \times \overline{M}$, where $O_{\overline{p}}$ denotes the zero vector of $\overline{M}$ at $\overline{p}$. Similary, we identify $\overline{X}$ on $\overline{M}$ with $\tilde{X}$ on $M \times \overline{M}$ as : $\tilde{X}_{(p,\overline{p})} = O_p + \overline{X}_{\overline{p}}$. Let $\tilde{X} = X + \overline{X}$ be a vector field on $M \times \overline{M}$ and put:

$$F(X + \overline{X}) = \varphi X + \overline{\eta}^i(\overline{X})\xi_i + \overline{\varphi}\overline{X} + \eta^i(X)\overline{\xi}_i\,. \tag{10}$$

It is easy to see that : $F^2 = \mathrm{Id}_{M\times\overline{M}}$, so F is a tensor of an almost product structure on $M \times \overline{M}$.

Remark 1. Observe that when $\overline{M} = \mathbf{R}^r$, $\overline{\varphi} = 0$, $\overline{\xi}_i = \frac{d}{dt^i}$, $\overline{\eta}_i = dt^i$ the definition (10) becomes (7) from [2].

If Σ is an almost r-paracontact structure on N then we define the following tensor field ψ of type (1,2) and differential 2-forms θ^i on M:

$$\begin{aligned} \psi(X,Y) &= \varphi[X,Y] - [\varphi X, Y] - [X, \varphi y] + \varphi[\varphi X, \varphi Y] + \\ &+ \{(\varphi X)(\eta^i(Y)) - (\varphi Y)(\eta^i(X))\}\,\xi_i\,, \end{aligned} \tag{11}$$

$$\theta^i(X,Y) = \eta^i[X,Y] - X(\eta^i(Y)) + Y(\eta^i(X)) + \eta^i[\varphi X, \varphi Y]\,. \tag{12}$$

Similary we define $\overline{\psi}$ and $\overline{\theta}^i$ for $\overline{\Sigma}$ on $\overline{M}$. Now we prove the following:

Lemma 1. *Let* Σ *and* $\overline{\Sigma}$ *be almost r-paracontact structures on* M *and* $\overline{M}$ *respectively. Then the induced on* $M \times \overline{M}$ *an almost product structure* F *given by* (10) *is integrable if and only if the following conditions are satisfied :*

$$\psi = 0 \quad and \quad [\xi_i, \xi j] = 0, \quad i,j = 1,2,\ldots,r\ , \tag{13}$$

$$\bar{\psi} = 0 \quad and \quad [\bar{\xi}_i, \bar{\xi}_j] = 0\ , \quad i,j = 1,2,\ldots,r\ . \tag{14}$$

Proof. All calculations are similar to those in [1], so they are omitted. The integrability condition of the induced almost product structure F on $M \times \bar{M}$ is the following:

$$\begin{aligned} &F[X+\bar{X}, Y+\bar{Y}] + F[F(X+\bar{X}), F(Y+\bar{Y})] = \\ &= [F(X+\bar{X}), Y+\bar{Y}] + [X+\bar{X}, F(Y+\bar{Y})]\ . \end{aligned} \tag{15}$$

The first term of the LHS of (15) is:

$$\begin{aligned} &F[X+\bar{X}, Y+\bar{Y}] = F([X,Y] + [\bar{X},\bar{Y}]) = \\ &\varphi[X,Y] + \bar{\eta}^i[\bar{X},\bar{Y}]\xi_i + \bar{\varphi}[\bar{X},\bar{Y}] + \eta^i[X,Y]\bar{\xi}_i\ . \end{aligned} \tag{16}$$

The second term of the LHS of (15) is:

$$\begin{aligned} F[F(X+\bar{X}), F(Y+\bar{Y})] &= \varphi\,[\varphi X + \bar{\eta}^i(\bar{X})\xi_i, \varphi Y + \bar{\eta}^i(\bar{Y})\xi_j] + \\ &+\bar{\eta}^i\,[\bar{\varphi}\bar{X} + \eta^j(X)\bar{\xi}_j, \bar{\varphi}\bar{Y} + \eta^j(Y)\bar{\xi}_j]\,\xi_i + \\ &+\bar{\varphi}\,[\bar{\varphi}\bar{X} + \eta^i(X)\bar{\xi}_i, \bar{\varphi}\bar{Y} + \eta^j(Y)\bar{\xi}_j] + \\ &+\eta^i\,[\varphi X + \bar{\eta}^j(\bar{X})\xi_j, \varphi Y + \bar{\eta}^j(\bar{Y})\xi_j]\,\bar{\xi}_i + \\ &+f^i(X,\bar{X},Y,\bar{Y})\xi_i + \bar{f}^i(X,\bar{X},Y,\bar{Y})\bar{\xi}_i\ , \end{aligned} \tag{17}$$

where

$$f^i(X,\bar{X},Y,\bar{Y}) = \bar{\eta}^j(\bar{X})\xi_j(\eta^i(Y)) + \varphi(X)(\eta^i(Y)) - \varphi(Y)(\eta^i(X)) - \bar{\eta}^j(\bar{Y})\xi_j(\eta^i(X))$$

$$\bar{f}^i(X,\bar{X},Y,\bar{Y}) = \eta^j(X)\bar{\xi}_j(\bar{\eta}^i(\bar{Y})) + \bar{\varphi}(\bar{X})(\bar{\eta}^i(\bar{Y})) - \bar{\varphi}(\bar{Y})(\bar{\eta}^i(\bar{X})) - \eta^j(Y)\bar{\xi}_j(\bar{\eta}^i(\bar{X})).$$

The first term of the RHS of (15) is :

$$\begin{aligned} &[F(X+\bar{X}), Y+\bar{Y}] = [\varphi X + \bar{\eta}^i(\bar{X})\xi_i, Y] + \\ &+ [\bar{\varphi}\bar{X} + \eta^i(X)\bar{\xi}_i, \bar{Y}] - \bar{Y}(\bar{\eta}^i(\bar{X}))\xi_i - Y(\eta^i(X))\bar{\xi}_i\ . \end{aligned} \tag{18}$$

The second term of the RHS of (15) is:

$$\begin{aligned} &[X+\bar{X}, F(Y+\bar{Y})] = [X, \varphi Y + \bar{\eta}(\bar{Y})\xi_i] + \\ &+[\bar{X}, \bar{\varphi}\bar{Y} + \eta(Y)\bar{\xi}_i] + X(\eta(Y))\bar{\xi}_i + \bar{X}(\bar{\eta}(\bar{Y}))\xi_i\ . \end{aligned} \tag{19}$$

Hence, (15) is equivalent to the following two identities:

$$\begin{aligned} &\varphi[X,Y] + \varphi[\varphi X + \bar{\eta}(\bar{X})\xi_i, \varphi Y + \bar{\eta}(\bar{Y})\xi_i] + \bar{\eta}[\bar{X},\bar{Y}]\xi_i + \\ &+\bar{\eta}[\bar{\varphi}\bar{X} + \eta(X)\bar{\xi}_j, \bar{\varphi}\bar{Y} + \eta^j(Y)\bar{\xi}_j]\xi_i + f^i(X,\bar{X},Y,\bar{Y})\xi_i = \\ &= [\varphi X + \bar{\eta}^i(\bar{X})\xi_i, Y] + [X, \varphi Y + \bar{\eta}^i(\bar{Y})\xi_i] + \bar{X}(\bar{\eta}^i(\bar{Y}))\xi_i - \bar{Y}(\bar{\eta}^i(\bar{X}))\xi_i\ , \end{aligned} \tag{20}$$

$$\overline{\varphi}[\overline{X},\overline{Y}] + \overline{\varphi}[\overline{\varphi}\overline{X} + \eta^i(X)\overline{\xi}_i, \overline{\varphi}\overline{Y} + \eta^i(Y)\overline{\xi}_i] + \eta^i[X,Y]\overline{\xi}_i +$$
$$+\eta^i[\varphi X + \overline{\eta}(\overline{X})\xi_j, \varphi Y + \overline{\eta}^j(\overline{Y})\xi_j]\overline{\xi}_i + \overline{f}^i(\overline{X},Y,\overline{Y})\overline{\xi}_i) = \qquad (21)$$
$$= [\overline{\varphi}\overline{X} + \eta^i(X)\overline{\xi}_i, \overline{Y}] + [\overline{X}, \overline{\varphi}\overline{Y} + \eta^i(Y)\overline{\xi}_i] + X(\eta^i(Y))\overline{\xi}_i - Y(\eta^i(X))\overline{\xi}_i \ .$$

Now, putting $\overline{X} = \overline{Y} = 0$ in (20) and (21) we get:

$$\psi(X,Y) + \eta^i(X)\eta^j(Y)\overline{\eta}^k[\overline{\xi}_i,\overline{\xi}_j]\xi_k = 0 \ , \qquad (22)$$

$$\theta^i(X,Y)\overline{\xi}_i + \eta^i(X)\eta^j(Y)\overline{\varphi}[\overline{\xi}_i,\overline{\xi}_j] = 0 \ . \qquad (23)$$

Putting $X = Y = 0$ in (20) and (21) we have:

$$\overline{\theta}^i(\overline{X},\overline{Y})\xi_i + \overline{\eta}^i(\overline{X})\overline{\eta}^j(\overline{Y})\varphi[\xi_i,\xi_j] = 0 \ , \qquad (24)$$

$$\overline{\psi}(\overline{X},\overline{Y}) + \overline{\eta}(\overline{X})\overline{\eta}(\overline{Y})\eta[\xi_i,\xi_j]\overline{\xi}_k = 0 \ . \qquad (25)$$

Putting $\overline{X} = Y = 0$ in (20) and (21) we have:

$$\varphi[\varphi X, \overline{\eta}^i(\overline{Y})\xi_i] + \overline{\eta}^i[\eta^j(X)\overline{\xi}_j, \overline{\varphi}\overline{Y}]\xi_i - [X, \overline{\eta}^i(\overline{Y})\xi_i] - \overline{\eta}^j(\overline{Y})\xi_j(\eta^i(X))\xi_i = 0 \ , \qquad (26)$$

$$\overline{\varphi}[\eta^i(X)\overline{\xi}_i, \overline{\varphi}\overline{Y}] + \eta^i[\varphi X, \overline{\eta}^j(\overline{Y})\xi_j]\overline{\xi}_i - [\eta^i(X)\overline{\xi}_i, \overline{Y}] + \eta^j(X)\overline{\xi}_j(\overline{\eta}^i(\overline{Y}))\xi_i = 0 \ . \qquad (27)$$

Inserting $X = \overline{Y} = 0$ into (20) and (21) we obtain:

$$\varphi[\overline{\eta}^i(\overline{X})\xi_i, \varphi Y] + \overline{\eta}^i[\overline{\varphi}\overline{X}, \eta^j(Y)\overline{\xi}_j]\xi_i - [\overline{\eta}^i(\overline{X})\xi_i, Y] + \overline{\eta}^j(\overline{X})\xi_j(\eta^i(Y))\xi_i = 0 \ , \qquad (28)$$

$$\overline{\varphi}[\overline{\varphi}\overline{X}, \eta^i(Y)\overline{\xi}_i] + \eta^i[\overline{\eta}^j(\overline{X})\xi_j, \varphi Y]\overline{\xi}_i - [\overline{X}, \eta^i(Y)\overline{\xi}_i] - \eta^j(Y)\overline{\xi}_j(\overline{\eta}^i(\overline{X}))\overline{\xi}_i = 0 \ . \qquad (29)$$

The system of identities: (22) through (29) is equivalent to (20), (21). We have the following identities:

$$\psi(X,\xi_i) = \varphi[X,\xi_i] - [\varphi X, \xi_i] \ , \quad i = 1,\ldots,r \ , \qquad (30)$$

$$\psi(\varphi X, Y) + \varphi\psi(X,Y) + \eta^i(X)\psi(\xi_i, \varphi Y) + \theta^i(X,Y)\xi_i = \eta^i(X)\eta^j(Y)[\xi_i,\xi_j] \ . \qquad (31)$$

We may write similar identities for the structure $\overline{\Sigma}$ on $\overline{M}$. It is easy to verify that the LHS of (26) may be expressed as:

$$\text{LHS of (26)} \quad = \overline{\eta}^i(\overline{Y})\eta^j(X)[\xi_i,\xi_j] + \overline{\eta}^i(\overline{Y})\psi(\varphi X, \xi_i)$$

and the LHS of (27) in the following way:

$$\text{LHS of (27)} \quad = \eta^i(X)\overline{\psi}(\overline{\xi}_i, \overline{\varphi}\overline{Y}) - \eta^i(X)\overline{\eta}^j(\overline{Y})[\overline{\xi}_i,\overline{\xi}_j] \ . \qquad (33)$$

Moreover, (26) is equivalent with (28) and (27) with (29). Now, if we assume that F is integrable, then acting with η^k on (24) and with $\overline{\eta}^k$ on (23) we obtain:

$$\theta^i(X,Y) = \overline{\theta}^i(\overline{X},\overline{Y}) = 0 \ . \qquad (34)$$

Hence and from (23) and (24) we get:

$$\varphi[\xi_i, \xi_j] = 0 \quad \text{and} \quad \overline{\varphi}[\overline{\xi}_i, \overline{\xi}_j] = 0 \ . \tag{35}$$

Putting $X = \xi_i$, $Y = \xi_j$ in (22) and $\overline{X} = \overline{\xi}_i$, $\overline{Y} = \overline{\xi}_j$ in (25) and making use of (30) and (35) we obtain:

$$\eta^k[\xi_i, \xi_j] = \overline{\eta}^k[\overline{\xi}_i, \overline{\xi}_j] = 0 \ . \tag{36}$$

Hence, from (22) and (25) we get:

$$\psi = 0 \quad \text{and} \quad \overline{\psi} = 0 \ . \tag{37}$$

Because of (35) and (36) we obtain:

$$[\xi_i, \xi_j] = 0 \quad \text{and} \quad [\overline{\xi}_i, \overline{\xi}_j] = 0 \tag{38}$$

and this means that (13) and (14) are satisfied. Now, if (13) and (14) are satisfied, then from (31) : $\theta^i = \overline{\theta}^i = 0$ and from (32) and (33) all identities (22) through (29) are fulfilled, and F is integrable.

In case of $M = \overline{M}$ and $\Sigma = \overline{\Sigma}$ we give:

Definition 2. An almost r-paracontact structure Σ on a manifold M is said to be integrable if an only if the product structure F given by (10) on $M \times M$ is integrable. We have the following:

Theorem 3. *Let $\Sigma = \left(\varphi, \xi_{(i)}, \eta^{(i)}\right)_{i=1,\dots,r}$ be an almost r-paracontact structure on M. Then Σ is integrable if and only if the following conditions are satisfied:*

$$\psi = 0 \quad \text{and} \quad [\xi_i, \xi_j] = 0 \ , \quad i, j = 1, \dots, r \ . \tag{39}$$

Combining Lemma 1 and Theorem 3 we get:

Theorem 4. *Let Σ and $\overline{\Sigma}$ be almost r-paracontact structures on M and $\overline{M}$ respectively. Then the induced by Σ and $\overline{\Sigma}$ almost product structure F on $M \times \overline{M}$ is integrable if and only if Σ and $\overline{\Sigma}$ are both integrable.*

If in Theorem 4 we take $\overline{M} = R^r$ and $\overline{\Sigma} = \left(0, \frac{d}{dt^i}, dt^i\right)$ then we get:

Theorem 5. *An almost r-paracontact structure Σ on M is integrable if and only if Σ is normal.*

In particular we have:

Corollary 1. *An almost r-paracontact structure Σ on M is normal if and only if the condition (39) is satisfied.*

Let:

$$F_1 = \varphi - \xi_i \otimes \eta^i \quad , \quad F_2 = \varphi + \xi_i \otimes \eta^i \ ,$$

then:

$$F_1^2 = F_2^2 = \mathrm{Id} \ .$$

Analogously as in [1] we can give the following:

Definition 8. An almost r-paracontact structure Σ on M is said to be weak-normal, if both almost product structures F_1 and F_2 are integrable. Similarly as in [1] we prove:

Theorem 6. *An almost r-paracontact structure Σ on M is weak-normal if and only if:*

$$\psi(\varphi X,\varphi Y)=0\ , \tag{40}$$

$$(\varphi\circ\psi)(X,\xi_i)=0\ ,\quad i=1,2,\ldots,r \tag{41}$$

for any vector fields X and Y on M.

We also have:

Theorem 7. *If an almost r-paracontact structure Σ on M is normal, then Σ is also weak-normal.*

Now, we give geometric interpretation of weak-normality of an almost r-paracontact structure Σ.

Theorem 8. *Let $\Sigma=\left(\varphi,\xi_{(i)},\eta^{(i)}\right)_{1=1,\ldots,r}$ be an almost r-paracontact structure on M. Then the following conditions are equivalent:*

(i) Σ is weak-normal.

(ii) The distributions D^+ , D^- , $D^+ + D^0$, $D^- + D^0$ are integrable.

Proof. We have:

$$D^+=\{X;\varphi X=X\}=\{X;F_1(X)=X\}=D^{+F_1}\ ;$$

$$D^-=\{X;\varphi X=-X\}=\{X;F_2(X)=-X\}=D^{-F_2}\ .$$

On account of Lemma 2 from [2] we have:

$$D^{+}+D^0=\{X;F_2(X)=X\}=D^{+F_2}\ ;\quad D^-+D^0=\{X;F_1(X)=-X\}=D^{-F_1}\ .$$

In virtue of the definition of the weak-normality and Remark 3 from [2] both conditions are equivalent. From Theorems 2 and 8 we obtain:

Theorem 9. *Weak-normal almost r-paracontact structure Σ on M is normal if and only if:* $\overset{4}{N}{}^i_j=0$ and $[\xi_i,\xi_j]=0,\ i=1,2,\ldots,r.$

We also have the following:

Theorem 10. *For an almost r-paracontact structure $\Sigma=\left(\varphi,\xi_{(i)},\eta^{(i)}\right)_{1=1,\ldots,r}$ on M the following conditions are equivalent:*

(i) $\psi(\varphi X,\varphi Y)=0$,

(ii) The distributions D^+ and D^- are integrable.

Proof. If Σ satisfies $\psi(\varphi X,\varphi Y)=0$, then for X , $Y\in D^+$ we have:

$$0=\psi(\varphi x,\varphi y)=\psi(X,Y)=2(\varphi[X,Y]-[X,Y])\ \text{ or }\ [X,Y]\in D^+\ .$$

For X , $Y\in D^-$ we have:

$$0=\psi(\varphi X,\varphi Y)=\psi(X,Y)=2(\varphi[X,Y]+[X,Y])$$

which means that D^- is integrable. Conversely, let D^+ and D^- be integrable. Then for $X,Y \in D^+$, $X,Y \in D^-$ and $X \in D^+, Y \in D^-$ we have: $\psi(\varphi X, \varphi Y) = 0$.

Now consider an almost r-paracontact ξ-structure:
$\Sigma = \left(\mathrm{Id} - \eta^i \otimes \xi_i, \xi_{(i)}, \eta^{(i)} \right)_{i=1,\dots,r}$. From Theorem 12 [2] we know that Σ is normal if and only if: $d\eta^i = 0$ and $[\xi_i . \xi_i] = 0$, $i = 1,\dots,r$. Now we prove the following:

Theorem 11. *An almost r-paracontact ξ-structure Σ on M is weak-normal if and only if:*

(i) $d\eta^i = \eta^i \wedge a^i$, *for some 1-form* a^i $, i = 1,\dots,r$

(ii) $[\xi_i, \xi_j] = \eta^k[\xi_i, \xi_j]\xi_k$, $i, j, k = 1,\dots,r$

Proof. In our case: $D^+ = \{X; \eta^i(X) = 0\}$; $D^- = 0$; $D^0 =$ Lin $\{\xi_1,\dots,\xi_r\}$. If Σ is weak-normal, then these distributions are integrable, and since D^+ is described by means of Pfaff's system $\eta^i = 0$, $i = 1,\dots,r$, then the integrability of this distribution, from Frobenius' Theorem, is equivalent to: $d\eta^i = \eta^i \wedge a^i$, for some 1-forms a^i, $i = 1,\dots,r$.

Theorem 12. *For any almost r-paracontact ξ-structure Σ the following conditions are equivalent:*

(i) $\psi(\varphi X, \varphi Y) = 0$,

(ii) $d\eta^i = \eta^i \wedge a^i$ *for some 1-forms* a^i.

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STRESZCZENIE

W pracy podajemy warunki algebraiczne charakteryzujące normalność struktury prawie r-parakontaktowej. Ponadto wprowadzamy pojęcie słabej normalności i podajemy jej interpretację geometryczną.

РЕЗЮМЕ

В данной работе введены алгебраические условия нормальности почти r-параконтактных структур. Введено также понятие слабой нормальности вместе с ее геометрической интерпретацией.