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Instytut Matematyki
Uniwersytet Marii Curie-Skłodowskiej

Andrzej BUCKI, Ryszard HOŁUBOWICZ, Andrzej MIERNOWSKI

Almost Paracontact Structures on a Lie Group

Prawie para-kontaktowe struktury na grupie Lie'go

Почти параконтактные структуры на группе Ли

1. Introduction. In [3] A. Morimoto has dealt with the left invariant almost contact, normal structure on a Lie group.

In the present paper we shall investigate some relations between the normality and the weak-normality [1] of almost paracontact structures [3] on a Lie group. It turns out that the problems can be reduced to purely algebraic ones in Lie algebras. Let $\Sigma = (\phi, \xi, \eta)$ be an almost paracontact structure on a differentiable manifold M. It is easy to see that $F_1 = \phi - \eta \otimes \xi$, $F_2 = \phi + \eta \otimes \xi$ are the tensor fields of almost product structures M.

Definition 1.1 [1]. An almost paracontact structure Σ on M is said to be weak-normal if the almost product structures F_1 and F_2 defined above are both integrable.

In [1] we have defined the tensor field:

$$\psi(X, Y) = \phi [X, Y] - [\phi X, Y] - [X, \phi Y] + \phi [\phi X, \phi Y] + \left\{(\phi X)(\eta(Y)) - (\phi Y)(\eta(X))\right\} \xi.$$

We also have:

Theorem 1.1 [1]. *An almost paracontact structure $\Sigma = (\phi, \xi, \eta)$ on M is weak-normal if and only if the following conditions are satisfied:*

$$(\phi \circ \psi)(X, \xi) = 0 \wedge \psi(\phi X, \phi Y) = 0\,,$$

for any vector fields X, Y on M.

J. Sato [3] considers the following tensors:

$N_1(X, Y) = [\phi, \phi](X, Y) - 2\, d\eta(X, Y) \cdot \xi\,,$

$N_2(X, Y) = (\mathcal{L}_{\phi X}\, \eta)(Y) - (\mathcal{L}_{\phi Y}\, \eta)(X)\,,$

$N_3(X) = (\mathcal{L}_\xi\, \phi)(X)\,,$

$N_4(X) = (\mathcal{L}_\xi\, \eta)(X)\,,$

where $[\phi, \phi]$ denotes the Nijenhuis tensor for ϕ, and $\mathcal{L}_X$ is the Lie derivative with respect to a vector field X. We know the following:

Theorem 1.2. [3]. *An almost paracontact structure* (ϕ, ξ, η) *on* M *is normal if and only if* $N_1 = 0$.

Theorem 1.3 [3]. *If* $N_1 = 0$ *then* $N_i = 0, i = 2, 3, 4$. *If* $N_2 = 0$ *or* $N_3 = 0$ *then* $N_4 = 0$.

Theorem 1.4 [1]. *An almost paracontact structure* $\Sigma = (\phi, \xi, \eta)$ *on* M *is normal if and only if* $\psi = 0$.

Theorem 1.5 [1]. *A weak-normal almost paracontact structure* Σ *on* M *is normal if and only if* $N_4 = 0$.

Combining above theorems we obtain:

Corollary 1.6. *A. weak-normal almost paracontact structure* Σ *on* M *is not normal if and only if* $N_1 \neq 0 \wedge N_2 \neq 0 \wedge N_3 \neq 0 \wedge N_4 \neq 0$.

Example 1.1. On a Riemannian manifold (M, g) let $\Sigma = (\phi, \xi, \eta)$ be an almost paracontact ξ-structure [1]. In [1] we have proved that ξ-structure is weak-normal if and only if $d\eta = \alpha \wedge \eta$ for some 1-form α and ξ-structure is normal if and only if $d\eta = 0$.

In virtue of the Corollary 1.6 we can observe that if $d\eta = \alpha \wedge \eta$ and $\alpha \neq 0$, then ξ-structure corresponding to η is weak-normal and $N_i \neq 0$, $i = 1, 2, 3, 4$, for this structure.

2. Almost Paracontact Structures on a Lie Group. Let G be a Lie group and for any element $a \in G$, we denote by L_a the left translation of G defined as follows:

$$L_a : G \to G \,/\, x \to ax\,.$$

Definition 2.1. An almost paracontact structure $\Sigma = (\phi, \xi, \eta)$ on G is said to be left invariant if the following conditions are satisfied:

$$\phi \circ (L_a)_* = (L_a)_* \circ \phi\,, \tag{2.1}$$

$$(L_a)_* (\xi) = \xi\,. \tag{2.2}$$

Corollary 2.1. *The following condition holds good:*

$$(L_a)^* (\eta) = \eta\,. \tag{2.3}$$

Let $(q, [\ ])$ be a Lie algebra, and $\phi_0 : q \to q$ be a linear map and $\xi_0 \in q$ and $\eta_0 \in q^*$.

Definition 2.2. The triple (ϕ_0, ξ_0, η_0) is called a paracontact structure on q if the following conditions are satisfied

$$\phi_0 (\xi_0) = 0\,, \qquad \eta_0 \circ \phi_0 = 0\,,$$
$$\eta_0 (\xi_0) = 1\,, \qquad \phi_0^2 = Id_q - \eta_0 \otimes \xi_0\,. \tag{2.4}$$

$$\phi_0 [X, Y] = [\phi_0 X, Y] + [X, \phi_0 Y] - \phi_0 [\phi_0 X, \phi_0 Y]\,, \tag{2.5}$$

where $X, Y \in q$

Remark. We shall call a Lie algebra with a paracontact structure a paracontact Lie algebra.

Now we shall prove:

Theorem 2.2. *If* (ϕ_0, ξ_0, η_0) *is a paracontact structure on a Lie algebra q, then the following condition is satisfied:*

$$\eta_0 [X, Y] + \eta_0 [\phi_0 X, \phi_0 Y] = 0, \tag{2.6}$$

where $X, Y \in q$.

Proof. Operating with ϕ_0 on both hand s ides of (2.5) we get:

$$\phi_0^2 [X, Y] = \phi_0 [\phi_0 X, Y] + \phi_0 [X, \phi_0 Y] - \phi_0^2 [\phi_0 X, \phi_0 Y].$$

Hence

$$\begin{gathered} [X, Y] - \eta_0 [X, Y] \xi_0 = \\ = \phi_0 [\phi_0 X, Y] + \phi_0 [X, \phi_0 Y] - [\phi_0 X, \phi_0 Y] + \eta_0 [\phi_0 X, \phi_0 Y] \xi_0 . \end{gathered} \tag{2.7}$$

Inserting $\phi_0 X$ instead of X into (2.5) we have:

$$\begin{gathered} \phi_0 [\phi_0 X, Y] = \\ = [X, Y] - \eta_0(X) [\xi_0, Y] + [\phi_0 X, \phi_0 Y] - \phi_0 [X, \phi_0 Y] + \eta_0(X) \phi_0 [\xi_0, \phi_0 Y] . \end{gathered} \tag{2.8}$$

Inserting ξ_0 instead of Y into (2.5) we obtain:

$$\phi_0 [X, \xi_0] = [\phi_0 X, \xi_0] . \tag{2.9}$$

From (2.8) and (2.9) we have:

$$\begin{gathered} \phi_0 [\phi_0 X, Y] = [X, Y] - \eta_0(X) [\xi_0, Y] + [\phi_0 X, \phi_0 Y] - \phi_0 [X, \phi_0 Y] + \eta_0(X) [\xi_0, \phi_0^2 Y] = \\ = [X, Y] + [\phi_0 X, \phi_0 Y] - \phi_0 [X, \phi_0 Y] . \end{gathered} \tag{2.10}$$

Inserting (2.10) into (2.7) we obtain:

$$\begin{gathered} [X, Y] - \eta_0 [X, Y] \xi_0 = \\ = [X, Y] + [\phi_0 X, \phi_0 Y] - \phi_0 [X, \phi_0 Y] + \phi_0 [X, \phi_0 Y] - [\phi_0 X, \phi_0 Y] + \eta_0 [\phi_0 X, \phi_0 Y] \xi_0 , \end{gathered}$$

which is equivalent to (2.6).

Now we can prove:

Theorem 2.3. *Let G be a connected Lie group. Then G admits a left invariant normal almost paracontact structure if and only if the Lie algebra* G *of the left invariant vector fields on G is a paracontact Lie algebra.*

Proof. Suppose that G admits a left invariant normal almost paracontact structure $\Sigma = (\phi, \xi, \eta)$. For any $X \in \mathbf{G}$ we have $\phi(X) \in \mathbf{G}$, since $L_{a*}(\phi X) = \phi(L_{a*}X) = \phi(X)$ for any $a \in G$. Hence the restriction ϕ_0 of ϕ to $\mathbf{G}$ is a linear map of a vector space $\mathbf{G}$ into itself. Take $X \in \mathbf{G}$, then $\eta(X)$ is a constant function on G, since η and X are left invariant. Hence the restriction η_0 of η to $\mathbf{G}$ is a linear form on $\mathbf{G}$. Moreover, it is clear that $\xi \in \mathbf{G}$. Hence, by putting $\xi_0 = \xi$, we obtain the structure $\Sigma_0 = (\phi_0, \xi_0, \eta_0)$ satisfying (2.4). Because Σ is normal, it follows that $\psi(X, Y) = 0$ in particular for $X, Y \in \mathbf{G}$. Since $\eta(X)$ is constant for $X \in \mathbf{G}$ and $\phi(X)(\eta(Y)) - \phi(Y)(\eta(X)) = 0$. Hence

$$\phi_0 [X, Y] - [\phi_0 X, Y] - [X, \phi_0 Y] + \phi_0 [\phi_0 X, \phi_0 Y] = 0$$

for $X, Y \in \mathbf{G}$ what means that (2.5) is satisfied and $\mathbf{G}$ is a paracontact Lie algebra. Conversely, suppose that $\mathbf{G}$ has a paracontact structure $\Sigma_0 = (\phi_0, \xi_0, \eta_0)$ satistying (2.4) and (2.5). Let $X_1, \dots, X_n$ be a basis of $\mathbf{G}$ over $\mathbf{R}$. Then for any vector field X on G we can find n functions: $\alpha^i : G \to \mathbf{R}$ such that X can be written uniquelly: $X = \alpha^i X_i$. Now define $\Sigma = (\phi, \xi, \eta)$ as follows:

$$\phi(X) = \alpha^i \phi_0(X_i), \quad \eta(X) = \alpha^i \eta_0(X_i), \quad \xi = \xi_0 .$$

Then we have:

$$\phi^2(X) = \phi(\phi X) = \alpha^i \phi(\phi_0 X_i) = \alpha^i \phi_0^2(X_i) = \alpha^i (X_i - \eta_0(X_i)\xi_0) = \alpha^i X_i - \eta_0(\alpha^i X_i)\xi_0 =$$
$$= X - \eta(X)\xi = (Id - \eta \otimes \xi)(X) ,$$

$$\phi(\xi) = \phi_0(\xi_0) = 0, \quad \eta(\xi) = \eta_0(\xi_0) = 1,$$

$$(\eta \circ \phi)(X) = \eta(\alpha^i \phi_0(X_i)) = \alpha^i \eta(\phi_0(X_i)) = \alpha^i (\eta_0 \circ \phi_0)(X_i) = 0 .$$

Hence Σ is an almost paracontact structure on G. Moreover we have:

$$\psi(X, Y) = \alpha^i \alpha^j \psi(X_i, X_j)$$

and in virtue of (2.5) we have:

$$\psi(X_i, X_j) = \phi [X_i, X_j] - [\phi X_i, X_j] - [X_i, \phi X_j] + \phi [\phi X_i, \phi X_j] +$$
$$+ \left\{(\phi(X_i))(\eta(X_j)) - (\phi(X_j))(\eta(X_i))\right\} \xi = \phi_0 [X_i, X_j] - [\phi_0 X_i, X_j] - [X_i, \phi_0 X_j] +$$
$$+ \left\{\phi_0 [\phi_0 X_i, \phi_0 X_j] + (\phi_0 X_i)(\eta_0(X_j)) - (\phi_0 X_j)(\eta_0(X_i))\right\} \xi = 0 .$$

Since ψ is a tensor field on G, ψ vanishes identically, which proves that Σ is normal.

Now we have:

Theorem 2.4. *Let $\Sigma = (\phi, \xi, \eta)$ be a paracontact structure on a Lie algebra q. Let $\widetilde{q} = q \oplus \widetilde{a}$ be the direct sum of q and 1-dimensional Lie algebra $\widetilde{a}$. Define the linear map F of $\widetilde{q}$ into itself by:*

$$F(X, a) = (\phi X + a\xi, \eta(X))$$

for $X \in q$ and $a \in \widetilde{a}$. Then F satisfies the following conditions:

$$F^2 = Id_{\widetilde{q}} . \tag{2.11}$$

$$F[\widetilde{X}, \widetilde{Y}] = [F\widetilde{X}, \widetilde{Y}] + [\widetilde{X}, F\widetilde{Y}] - F[F\widetilde{X}, F\widetilde{Y}] \tag{2.12}$$

for all $\widetilde{X}, \widetilde{Y} \in \widetilde{q}$.

Proof. Let $\widetilde{X} = (X, a) \in \widetilde{q}$. Then we have:

$$F^2(\widetilde{X}) = F(\phi X + a\xi, \eta(X)) = (\phi(\phi X + a\xi) + \eta(X)\xi, \eta(\phi(X) + a\xi) =$$
$$= (\phi^2 X + a\phi(\xi) + \eta(X)\xi, \eta(\phi(X)) + a\eta(\xi)) = (X - \eta(X)\xi + \eta(X)\xi, a) = (X, a) = \widetilde{X}.$$

Hence $F^2 = Id_{\widetilde{q}}$. For $\widetilde{X} = (X, a)$ and $\widetilde{Y} = (Y, b) \in \widetilde{q}$, we have:

$$[\widetilde{X}, \widetilde{Y}] = [(X, a), (Y, b)] = ([X, Y], 0)$$

and

$$\begin{aligned} F[\widetilde{X}, \widetilde{Y}] &= F([X, Y], 0) = (\phi[X, Y], \eta[X, Y]) = \\ &= ([\phi X, Y] + [X, \phi Y] - \phi[\phi X, \phi Y], \eta[X, Y]) . \end{aligned} \tag{2.13}$$

On the other hand, we have:

$$[F\widetilde{X}, \widetilde{Y}] = [(\phi X + a\xi, \eta(X)), (Y, b)] = ([\phi X + a\xi, Y], 0) \tag{2.14}$$

$$[\widetilde{X}, F\widetilde{Y}] = ([X, \phi Y + b\xi], 0) \tag{2.15}$$

$$F[F\widetilde{X}, F\widetilde{Y}] = F[(\phi X + a\xi, \eta(X)), (\phi Y + b\xi, \eta(Y))] = F([\phi X + a\xi, \phi Y + b\xi], 0) =$$
$$= (\phi[\phi X + a\xi, \phi Y + b\xi], \eta[\phi X + a\xi, \phi Y + b\xi]) .$$

Here we shall use the equalities (2.9) and (2.6) and obtain:

$$F[F\widetilde{X}, F\widetilde{Y}] = (\phi[\phi X, \phi Y] + \phi[\phi X, b\xi] + \phi[a\xi, \phi Y], \eta[\phi X, \phi Y] + \eta[\phi X, b\xi] + \eta[a\xi, \phi Y]) =$$
$$= (\phi[\phi X, \phi Y] + \phi^2[X, b\xi] + \phi^2[a\xi, Y], -\eta[X, Y]) =$$
$$= (\phi[\phi X, \phi Y] + [X, b\xi] - \eta[X, b\xi]\xi + [a\xi, Y] - \eta[a\xi, Y]\xi, -\eta[X, Y]) .$$

From (2.6) by putting $Y = \xi$, we have: $\eta[X, \xi] = 0$. Hence

$$F[F\widetilde{X}, F\widetilde{Y}] = (\phi[\phi X, \phi Y] + [X, b\xi] + [a\xi, Y], -\eta[X, Y]) \tag{2.16}$$

(2.14) + (2.15) − (2.16) gives (2.13), since:

$$([\phi X, Y] + [a\xi, Y] + [X, a\xi] + [X, \phi Y] - \phi[\phi X, \phi Y] - [X, b\xi] - [a\xi, Y], 0 + 0 + \eta[X, Y]) =$$
$$= ([\phi X, Y] + [X, \phi Y] - \phi[\phi X, \phi Y], \eta[X, Y]) .$$

which completes the proof.

Let F be a tensor field of an almost product structure on a Lie group G. F is said to be the left invariant if:

$$(L_a)_* \circ F = F \circ (L_a)_* .$$

This condition implies that F maps left invariant vector fields on G into left invariant vector fields. Let F_0 denotes the restriction of F to G, being the Lie algebra of the left invariant vector fields on G.An almost product structure F is integrable if $[F, F] = 0$ where $[F, F]$ denotes the Nijenhuis tensor for F. We have:

$$[F, F](X, Y) = F[X, Y] + F[FX, FY] - [FX, Y] - [X, FY] .$$

If an almost product structure F is integrable, then F_0 satisfies:

$$F_0[X, Y] + F_0[F_0X, F_0Y] - [F_0X, Y] - [X, F_0Y] = 0 \tag{2.17}$$

$$F_0^2 = Id_G . \tag{2.18}$$

Conversely, if a linear map $F_0 : \mathrm{G} \to \mathrm{G}$ satisfies (2.17) and (2.18) then, similarly, to the second part of the proof of the Theorem 2.3 we can define the left invariant integrable almost product structure on G. Now we consider the left invariant almost paracontact structure $\Sigma = (\phi, \xi, \eta)$ on G. Since $(L_a)_*\xi = 0$ and $(L_a)^*\eta = \eta$, then the linear map $\eta \otimes \xi$ is left invariant i.e.:

$$(L_a)^* \circ (\eta \otimes \xi) = (\eta \otimes \xi) \circ (L_a)^* .$$

Let $F_1 = \phi - \eta \otimes \xi$, $F_2 = \phi + \eta \otimes \xi$, then F_1 and F_2 are the tensors of the left invariant almost product structures on G [1].

Definition 2.3 [1]. The left invariant almost paracontact structure $\Sigma = (\phi, \xi, \eta)$ on G is said to be weak-normal, if $[F_1, F_1] = 0$ and $[F_2, F_2] = 0$.

In virtue of the Theorem 1.1 Σ is weak-normal if

$$\psi(\phi X, \phi Y) = 0 \text{ and } (\phi \circ \psi)(X, \xi) = 0 .$$

Now let $X, Y \in \mathrm{G}$. Then:

$$\psi(\phi X, \phi Y) = \psi(\phi_0 X, \phi_0 Y) =$$
$$= \phi_0 [\phi_0 X, \phi_0 Y] - [\phi_0^2 X, \phi_0 Y] - [\phi_0 X, \phi_0^2 Y] + \phi_0 [\phi_0^2 X, \phi_0^2 Y],$$

$$(\phi \circ \psi)(X, \xi) = \phi_0^2 [X, Y] - [\phi_0 X, Y]$$

where ϕ_0 denotes the restriction of ϕ to G.

Theorem 2.5. *The left invariant almost paracontact structure $\Sigma = (\phi, \xi, \eta)$ on a Lie group G is weak-normal if and only if:*

$$\phi_0 [\phi_0 X, \phi_0 Y] - [\phi_0^2 X, \phi_0 Y] - [\phi_0 X, \phi_0^2 Y] + \phi_0 [\phi_0^2 X, \phi_0^2 Y] = 0$$

$$\phi_0^2 [X, Y] = [\phi_0 X, Y]$$

for any $X, Y \in$ G.

Now we have:

Theorem 2.6. *Let $(q, [\ ])$ be a Lie algebra, such that $q' \neq q$, where q' denotes the linear space spanned by $[X, Y]$ for all $X, Y \in q$. Then, there exists a paracontact structure on q.*

Proof. Let q_1 be such subspace of q, that $q = q' \oplus q_1$. Take an arbitrary $0 \neq \xi_0 \in q_1$. We can define a form η_0 such, that:

$$\eta_0(\xi_0) = 1, \bigwedge_{x \in q'} \eta_0(X) = 0.$$

Putting $\phi_0 = Id_q - \eta_0 \otimes \xi_0$, we have: $\phi_0^2 = Id - \eta_0 \otimes \xi_0$ and

$$\phi_0 [X, Y] - [\phi_0 X, Y] - [X, \phi_0 Y] + \phi_0 [\phi_0 X, \phi_0 Y] =$$
$$= [X, Y] + \eta_0 [X, Y]\xi_0 - [X, Y] + \eta_0(X)[\xi_0, Y] -$$
$$- [X, Y] + \eta_0(Y)[X, \xi_0] + [X, Y] - \eta_0(X)[\xi_0, Y] - \eta_0(Y)[X, \xi_0] - \eta_0 [\phi_0 X, \phi_0 Y] = 0.$$

Corollary 2.7. *Every nilpotent (and solvable as well, since nilpotency implies solvability) Lie algebra admits a paracontact structure.*

Let GL(n) denotes the Lie algebra of the linear group $GL(n)$. Since $tr[X, Y] = 0$ for all $X, Y \in$ GL(n), then GL$'(n) \neq$ GL(n). Hence we have:

Corollary 2.8. *The linear group $GL(n)$ admits a left invariant normal almost paracontact structure.*

Example 2.1. Now we give an example of a left invariant paracontact structure on $GL(n)$.

Let $X = (X_j^i)_{ij=1,\ldots,n}$ and $\xi_0 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$. Put $\eta_0(X) = n^{-1}\, tr\, X = n^{-1}(X_1^1 + \ldots + X_n^n)$

and $\phi_0 = Id_{GL(n)} - \eta_0 \otimes \xi_0$. Then $\eta_0(\xi_0) = 1$ and $\phi_0 [X, Y] - [\phi_0 X, Y] - [X, \phi_0 Y] + \phi_0 [\phi_0 X, \phi_0 Y] = 0$. Hence (ϕ_0, ξ_0, η_0) is a paracontact structure on $GL(n)$.

Example 2.2. In this example we present the left invariant almost paracontact weak-normal structure on $GL(2)$, that is not normal. To this end, let g^i_j denote the natural coordinates in $GL(2)$. We consider the following forms: $\eta = \tilde{g}^2_1\, dg^1_1 + \tilde{g}^2_2\, dg^2_1$ and

$$\alpha = -\tilde{g}^1_1\, dg^1_1 - \tilde{g}^1_2\, dg^2_1 + \tilde{g}^2_1\, dg^1_2 + \tilde{g}^2_2\, dg^2_2\,,$$

where g^i_j denote entries of an inverse matrix to a matrix (g^i_j). It is easy to verify, that η and α are left invariant and $d\eta = \eta \wedge \alpha$. Putting $\xi = g^1_2 \dfrac{\partial}{\partial g^1_1} + g^2_2 \dfrac{\partial}{\partial g^2_1}$ and $\phi = Id_{GL(2)} - \eta \otimes \xi$ we obtain a weak-normal [1] left invariant almost paracontact structure on $GL(2)$.

Example 2.3. Here we show a paracontact structure on $GL(n)$ that is not weak-normal. Let $\eta_0(X) = n^{-1}\, tr\, X$, $\xi_0 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ and $\phi_0(X) = X^T - n^{-1}\, tr\, X \cdot \xi_0$. The structure (ϕ_0, ξ_0, η_0) is not weak-normal.

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STRESZCZENIE

W tej pracy zajmujemy się badaniem związków między normalnością i słabą normalnością [1] prawie para-kontaktowych struktur na grupie Lie'go. Okazuje się, że problemy te sprowadzają się do rozpatrywania zagadnień czysto algebraicznych w algebrze Lie'go.

Ponadto podajemy przykłady: lewo-niezmienniczej para-kontaktowej struktury na $GL(n)$, lewo-niezmienniczej prawie para-kontaktowej słabo-normalnej struktury na $GL(2)$, która nie jest normalna oraz para-kontaktowej struktury na $GL(n)$, która nie jest słabo-normalna.

РЕЗЮМЕ

В этой работе мы изучаем зависимости между нормальными и слабо-нормальными почти параконтактными структурами на группе Ли.

Кроме того дасм примеры: лево-инвариантной параконтактной структуры на $GL(n)$ лево-инвариантной почти параконтактной слабо-нормальной структуры на $GL(2)$, которая не нормальная и параконтактная структуры на $GL(n)$, и не является слабо-нормальной.