ANNALES
UNIVERSITATIS MARIAE CURIE-SKŁODOWSKA
LUBLIN — POLONIA
VOL. XXIX, 6 SECTIO A 1975

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Hadamard Products of Convex Schlicht Functions

Iloczyny Hadamarda funkcji wypukłych jednolistnych

Произведения Адамара выпуклых однолистных функций

1. Introduction

The disk in **C** with radius 1 and center at the origin is denoted by Δ. The set $\mathscr{A}$ of all functions holomorphic on Δ is a linear space with pointwise operations. The compact open topology on $\mathscr{A}$ makes $\mathscr{A}$ a Montel space, with the usual metric structure. Let S be the set of f in $\mathscr{A}$ which are univalent on Δ and C the subset of f in S which have convex ranges. Each f in C induces a continuous linear operator Λ on $\mathscr{A}$ by convolution

$$\Lambda(g)(z) = f^* g(z).$$

If $f(z) = \sum_{n=0}^{\infty} a_n z^n$ and $g(z) = \sum_{n=0}^{\infty} b_n z^n$ then

$$\Lambda(g)(z) = \sum_{n=0}^{\infty} a_n b_n z^n = \frac{1}{2\pi i} \int_{|\zeta|=1+|z|/2} f(\zeta) g\left(\frac{z}{\zeta}\right) \frac{d\zeta}{\zeta}.$$

There are two important results which came to the fore in discussing such operators. First, the result of T. J. Suffridge [9]. He shows that if f and g and in C then $f^* g(z)$ is again a univalent function. The second and perhaps more striking result is that indeed under the same hypothesis $f^* g(z)$ is a convex function. This last result is due to St. Ruscheweyh and Sheil-Small [8]. In [7] Sheil-Small has defined a very general class of operators which generalize the simple convolution operators defined above.

In the latter part of this paper we shall need the definitions of the Hardy spaces H^p and H^∞ and also the class of functions of bounded mean oscillation (BMO). A function f in $\mathscr{A}$ is in the space H^p $(1 \leqslant p < \infty)$ if

$$\sup_{0<r<1} \left(\frac{1}{2\pi} \int |f(re^{i\theta})|^p d\theta \right)^{1/p} = M < \infty$$

and $f \in H^\infty$ if

$$\sup_{|z|<1} |f(z)| = M < \infty.$$

H^p is a Banach space. The dual of H^1 can be identified in a natural way with a space of functions. More precisely let $f(e^{ix})$ be a function in L^2 of the unit circle. The function f has bounded mean oscillation (or f is in BMO) if for each interval I we have

$$\frac{1}{|I|} \int_I \left| f(e^{ix}) - \frac{1}{|I|} \int_I f(e^{it})\, dt \right| dx \leqslant M$$

where M is a fixed constant depending on f and $|I|$ is the length of I. This set of functions modulo the constants is a Banach space when the obvious norm is introduced. The pairing which establishes the duality is as follows. Let $g \in BMO$ and $f \in H^1$. We choose a sequence $\{f_n\}$ in H^2 with $f_n \to f$ in H^1 and define

$$\Lambda(f) = \lim_{n\to\infty} \frac{1}{2\pi} \int_{-\pi}^{\pi} f_n(e^{ix}) g(e^{ix}) dx.$$

We have

$$\left| \frac{1}{2\pi} \int_{-\pi}^{\pi} f_n(e^{ix}) g(e^{ix}) dx \right| \leqslant \|\Lambda\| \cdot \|f_n\|_1.$$

For a reference on BMO see C. Fefferman and E. Stein [2] or the notes of J. Garnett [3].

We have written two essential sections in this work. The first is a study of the operators induced by composition with the functions $K_a(z) = \left(\frac{1+z}{1-z}\right)^a$. The second is a collection of assorted cases describing when convolutions map H^1 into H^∞.

1. Operators induced by $K_a(z)$.

The univalent mapping $\left(\frac{1+z}{1-z}\right)$ maps Δ onto the right half plane in C. If $a \in (0, 1)$ then $K_a(z) = \left(\frac{1+z}{1-z}\right)^a$ are univalent mappings of Δ onto a cone, symmetric with respect to the positive real axis. The cone has its vertex at $z = 0$ and aperture opening $a\pi$. We compute the Taylor coefficients of $K_a(z) = \sum_{k=0}^{\infty} g_k(a) z^k$. The differential equation

$$K'_a(z) = \frac{2a}{(1-z^2)} K_a(z) \tag{1.1}$$

allows the recursion relation

$$\begin{aligned} g_0(a) &= 1 \\ g_1(a) &= 2a \end{aligned} \tag{1.2}$$

$$g_{k+1}(a) = \frac{1}{k+1}[2ag_k(a) + (k-1)g_{k-1}(a)]$$

for $k = 1, 2, 3, \ldots$ to be established.

Theorem 1. *For each $a \in (0,1)$ the linear operator Λ_a induced on $\mathscr{A}$ by convolution with K_a is a one to one, onto, continuous linear operator on $\mathscr{A}$.*

Proof. The convolution is obviously linear and continuous. Also since $g_0(a) > 0$ and $g_1(a) > 0$ if $a \in (0,1)$ the relation (1.2) shows that $g_k(a) > 0$ $k = 1, 2, 3, \ldots$ and every $a \in (0,1)$. This means the operator induced by K_a is one to one. Hence, we examine the behavior of the polynomials $g_k(a)$ for (i) $a = \frac{1}{2}$, (ii) $a \in \left(\frac{1}{2}, 1\right)$, and (iii) $a \in \left(0, \frac{1}{2}\right)$.

Assuming $a = \frac{1}{2}$ we find $g_0\left(\frac{1}{2}\right) = g_1\left(\frac{1}{2}\right) = 1$. Also $g_2\left(\frac{1}{2}\right) = g_3\left(\frac{1}{2}\right) = \frac{1}{2}$. In general a simple finite induction argument shows that

$$g_{2k}\left(\frac{1}{2}\right) = g_{2k+1}\left(\frac{1}{2}\right) = \frac{(2k-1)!}{2^{2k-1}k!(k-1)!}.$$

Recall that Sterling's inequalities can be written (for large n)

$$\frac{\sqrt{2\pi}\, n^{n+1/2}}{e^n} < n! < \frac{\sqrt{2\pi}\, n^{n+1/2}\left(1 + \frac{1}{4n}\right)}{e^n}.$$

This of course implies that

$$g_{2k}\left(\frac{1}{2}\right) \geqslant \frac{1}{\sqrt{2\pi}}\left(\frac{(2k-1)^2}{2k^2-4k}\right)^k \left(\frac{k-1}{2k^2-k}\right)^{1/2} \frac{1}{2^{2k-1}\left(1+\frac{1}{4(k-1)}\right)\left(1+\frac{1}{4k}\right)}.$$

Taking the $2k$ th root and then the limit inferior we find

$$\liminf_{k\to\infty}\left(g_{2k}\left(\frac{1}{2}\right)\right)^{1/2k} \geqslant \liminf_{k\to\infty}\left(\frac{k-1}{2k^2-2k}\right)^{1/k} = 1.$$

Hence,

$$\lim_{k\to\infty} g_{2k}\left(\frac{1}{2}\right)^{1/2k} = 1.$$

A similar computation for the coefficients g_{2k+1} (1/2) shows that

$$\lim_{k\to\infty} g_k\left(\frac{1}{2}\right)^{1/k} = 1.$$

But assuming that $h(z) = \Sigma a_k z^k$ is in $\mathscr{A}$ we can choose the function

$$f(z) = \sum_{k=0}^{\infty} \frac{1}{g_k\left(\frac{1}{2}\right)} a_k z^k.$$

Clearly f is in $\mathscr{A}$ and $\Lambda(f) = K_{1/2}^{*} f = h$.

We consider now the behavior of the polynomials $g_k(a)$ for $a \epsilon \left(\frac{1}{2}, 1\right)$. Assume a is fixed and we observe that $g_1(a) > g_2(a)$. We will show that in general $g_{k-1}(a) > g_k(a)$. Fix $k \geqslant 1$ and use (1.2) to write

$$\begin{aligned}
&(k+1)\big(g_k(a) - g_{k+1}(a)\big) \\
&= (k+1-2a)\big(g_k(a) - g_{k-1}(a)\big) + \\
&\quad + 2(1-a) g_{k-1}(a) \\
&= (2a-1)\big(g_{k-1}(a) - g_k(a)\big) + 2(1-a) g_{k-1}(a) + \\
&\quad + k\left(\frac{1}{k}\,[2a g_{k-1}(a) - (k-2) g_{k-2}(a)] - g_{k-1}(a)\right) \\
&= (2a-1)\big(g_{k-1}(a) - g_k(a)\big) + (k-2)\big(g_{k-2}(a) - g_{k-1}(a)\big).
\end{aligned} \tag{1.3}$$

The induction argument then establishes that $g_{k-1}(a) > g_k(a)$. We will now show that $\lim_{k\to\infty} g_k(a)^{1/k} = 1$. Again referring to (1.2) we find

$$(k+1) g_k(a) - 2a g_k(a) > (k-1) g_{k-1}(a),$$

or

$$g_k(a) > \frac{(k-1)}{(k+1-2a)} g_{k-1}(a).$$

A finite repetition of this result establishes the inequality

$$g_k(a) > \frac{(k-1)!(4a)}{(k+1)!\left(1-\frac{2a}{3}\right)\left(1-\frac{2a}{4}\right)\dots\left(1-\frac{2a}{k+1}\right)}$$

$$= \frac{4a}{k(k+1)\prod_{j=3}^{k+1}\left(1-\frac{2a}{j}\right)}$$

We consider

$$\lim_{k\to\infty}\left(\sum_{j=3}^{k+1}\left(1-\frac{2a}{j}\right)\right)^{1/k}.$$

It is clear that

$$0 > \log\left(1-\frac{2a}{j+1}\right) > \log\left(1-\frac{2a}{j}\right).$$

The integral test shows

$$\int_3^{k+1}\left|\log\left(1-\frac{2a}{x}\right)\right| dx = \int_3^{k+1}[\log x - \log(x-2a)]dx$$

$$= \log(1+k) - \log(1+k-2a) + o(k).$$

Hence,

$$\lim_{k\to\infty}\frac{1}{k}\sum_{j=3}^{k+1}\log\left(1-\frac{2a}{j}\right) = \lim_{k\to\infty}\log\left(\frac{k+1}{k+1-2a}\right) = 0.$$

Thus

$$\liminf_{k\to\infty} g_k(a)^{1/k} \geqslant \liminf_k \frac{(4a)^{1/k}}{(k^2+k)^{1/k}\left(\prod_{j=3}^{k+1}\left(1-\frac{2a}{j}\right)\right)^{1/k}} = 1$$

and $\lim g_k(a)^{1/k} = 1$. As in the case $a = \frac{1}{2}$ we find that K_a induces an onto operator.

We consider finally the case $a \in \left(0, \frac{1}{2}\right)$. The functions $g_k(a)$ are not point wise decreasing so a somewhat different approach is necessary to treat this case. Again fix $a \in \left(0, \frac{1}{2}\right)$ and add the trivial equality

$$g_{k-1}(a) - g_k(a) = g_{k-1}(a) - g_k(a)$$

to (1.3) written for k and $(k-1)$ to obtain

$$(k+1)\big(g_{k-1}(a)-g_{k-1}(a)\big)$$
$$= 2a\big(g_{k-2}(a)-g_{k}\cdot(a)\big)+(k-3)\cdot\big(g_{k-3}(a)-g_{k-1}(a)\big). \qquad (1.4)$$

It is true that for $a\epsilon\left(0,\frac{1}{2}\right)$

$$g_1(a)-g_2(a)>0$$

and

$$g_2(a)-g_3(a)<0.$$

We can apply (1.3) and finite induction to conclude that

$$g_{2j+1}(a)-g_{2j+2}(a)>0$$

and

$$g_{2j+1}(a)-g_{2j}(a)<0$$

for all $j=1,2,3,\ldots$. Now applying (1.4) we establish

$$g_{2k-2}(a)>g_{2k}(a)$$

and

$$g_{2k-1}(a)>g_{2k+1}(a),$$

for all $k=2,3,4,\ldots$. We proceed with the coefficient estimate. An application of equation (1.4) shows

$$\big(g_{2j}(a)-g_{2j+2}(a)\big)>\frac{(2j-2)}{2j+2}\big(g_{2j-2}(a)-g_{2j}(a)\big).$$

A repetition of this inequality yields

$$g_{2j}(a)>g_{2j+2}(a)+\frac{2(j-1)!}{(j+1)!}\big(g_2(a)-g_4(a)\big)$$
$$=\frac{1}{(2j+2)}\,\{2ag_{2j+1}(a)+2jg_{2j}(a)\}+\frac{2(j-1)!}{(j+1)!}\big(g_2(a)-g_4(a)\big).$$

Letting $a(a)\equiv\big(g_2(a)-g_4(a)\big)>0$ we can write

$$g_{2j}(a)>\frac{2}{j}\,a(a).$$

Taking $2j$-th roots yields

$$\lim_{j\to\infty} g_{2j}^{1/2j}(a)=1.$$

Similarly one proves

$$g_{2j-1}(a)>\frac{3}{2(2j-1)}\,b(a)$$

where $b(a) > 0$. This proves that

$$\lim_{k\to\infty} g_k(a)^{1/k} = 1$$

for each fixed $a \in \left(0, \frac{1}{2}\right)$ and so K_a induces an onto operator. This completes the proof of Theorem 1.

As an easy corollary to this theorem we obtain information about the fixed points of the operators Λ_a induced by the composition with K_a.

Corollary 1. *There are a countable number of* $\{a_j\}_{j=1}^{\infty}$, *with* $a_1 = \frac{1}{2} < a_2 < \ldots$ *and* $\lim_{n\to\infty} a_n = 1$ *such that* Λ_a *has non-trivial* (e.g. $f(z) \not\equiv$ constant) *fixed points. If* a_j *is such a number there exists an integer* n_j *such that all fixed points of* Λ_{a_j} *are of the form* $a + bz^{n_j}$.

Before stating Theorem 2 we need a lemma. A result of Cargo [1, p. 472] implies that K_a is in H^p for all $p < \frac{1}{a}$. An application of the Hausdorff-Young theorem then shows that if $f(z) = \sum_{n=0}^{\infty} a_n z^n$ is in H^p, $1 \leqslant p \leqslant 2$, and if p' is the index conjugate to p then the sequence of Taylor coefficients $\{a_n\}_{n=0}^{\infty}$ is in the sequence space $l^{p'}$. It is clear then that for a fixed in $(0, 1)$ $\lim_{k\to\infty} g_k(a) = 0$. We find the following result more useful for coefficients.

Lemma 1. *For each* a *in* $(0,1)$ *and* $k = 1, 2, 3, \ldots$ *we have* $g_k(a) \leqslant 2\left(\frac{1}{k}\right)^{1-a}$.

Proof. The cases $k = 1$ and 2 are easily checked. Assuming the result is valid for k it suffices to prove

$$2ag_k(a) + (k-1)g_{k-1}(a) < 2(k+1)^a.$$

We apply the induction statement to conclude that this inequality above is valid if

$$2a\left(\frac{1}{k}\right)^{1-a} + (k-1)^a < (k+1)^a.$$

We know

$$\left(1+\frac{1}{k}\right)^a = \sum_{n=0}^{\infty} \binom{a}{n} \left(\frac{1}{k}\right)^n$$

and hence the last inequality can be written as

$$2ak^a + kk^a\left(\sum_{n=0}^{\infty}(-1)^n\binom{a}{n}\left(\frac{1}{k}\right)^n\right) < kk\left(\sum_{n=0}^{\infty}\binom{a}{n}\left(\frac{1}{k}\right)^n\right).$$

The transposition of terms reduces this to an obviously valid inequality

$$2ak^a < kk^a\left(\frac{2a}{k} + \frac{2(a)(a-1)(a-2)}{3!}\frac{1}{k^3} + \dots\right)$$

where all the terms in the series are positive.

Theorem 2. *Let a be given in $(0,1)$ and suppose $\beta \in (0,1)$ is such that $a+\beta < 1$, then $K_a^* K_\beta(z) = h(z)$ is in H^∞.*

Proof. An application of Lemma 1 to the coefficients of $h(z)$ yields the proof.

We wish to make a few remarks on the coefficients $g_k(a)$. The last theorem is reasonably sharp in that if $a = \beta = \frac{1}{2}$ one sees that the coefficients $g_k^2\left(\frac{1}{2}\right)$ of $K_{1/2}^* K_{1/2}(z)$ are of order of magnitude $\frac{1}{k}$ so that $K_{1/2}^* K_{1/2}$ is in H^2 but not in H^∞. In fact the function is unbounded on the positive axis as $x \to 1-$.

In the proof of Lemma 1 we used the expansion

$$(1+z)^a = \sum_{n=0}^{\infty}\binom{a}{n} z^n,$$

where

$$\binom{a}{n} = \frac{(a)(a-1)(a-2)\dots(a-n+1)}{n!}.$$

Thus one deduces

$$\left(\frac{1+z}{1-z}\right)^a = \left(\sum_{n=0}^{\infty}\binom{a}{n} z^n\right)\left(\sum_{n=0}^{\infty}(-1)^n\binom{-a}{n} z^n\right)$$

$$= \sum_{n=0}^{\infty}\left(\sum_{k=0}^{n}(-1)^{n-k}\binom{a}{k}\binom{-a}{n-k}\right) z^n$$

so that

$$g_n(a) = \left(\sum_{k=0}^{n}(-1)^{n-k}\binom{a}{k}\binom{-a}{n-k}\right).$$

There are rather comprehensive reference works and tables on products of similar forms (see [6] and [4]) but we have been unable to use any arithmetic or combinatorial simplifications to obtain information about the $g_k(a)$ or products of the $g_k(a)$.

2. Mappings of H^1 into H^∞.

The set of convex mappings can be divided into several groups. The function $k(z) = (1-z)^{-1}$ mapping Δ into the right half plane is the identity under convolution. Other convex mappings onto half planes are just rotations and translations of k and their behavior under convolution is determined by the behavior of k. Also the integral representation shows that the mapping induced by a function $f(z)$ in H^∞ maps H^1 into H^∞. The function $f(z) = \frac{1}{2}\log\left(\frac{1+z}{1-z}\right)$ is a convex mapping of Δ onto an infinite strip. A well known result of Hardy and Littlewood [5, p.] will show that the operator

$$\Lambda(g)(z) = f^* g(z)$$

maps H^1 into H^∞. We have the following extension of this result.

Theorem 3. *Let h be a function in $\mathscr{A}$ and assume that h is subordinate to $f(z) = \frac{1}{2}\log\left(\frac{1+z}{1-z}\right)$. Then the operator $\Lambda(g)(z) = g * h(z)$ is a mapping of H^1 into H^∞.*

Proof. The convex mappings are continuous from Δ into the Riemann sphere. The condition that h be subordinate to f means there exists a univalent mapping $\eta(z)$ from Δ into Δ with η a Schwarz function and

$$h(z) = f\circ\eta(z).$$

It is sufficient to prove that

$$\log(1+\eta)^* g$$

is in H^∞ for all $g \in H^1$. First, we may choose $g_r(z) = g(rz)$ and observe g_r tends in H^1 to g and $\|g_r\|_1 \leqslant \|g\|_1$. Also $g_r \in H^2$. A criterion developed by C. Fefferman and E. Stein [2] states that a function H is in BMO if and only if

$$H = u + \tilde{v}$$

where u and v are in L^∞ and v is the Fourier transform of $\tilde{v}$. We write

$$\log\big(1+\eta(z)\big) = \log|1+\eta(z)| + i\arg\big(1+\eta(z)\big).$$

The function $\arg(1+\eta(z))$ is in L^∞ and its harmonic conjugate is its

Fourier conjugate in this case so that $\log(1+\eta(z))$ is in BMO. Now if $z = re^{i\theta}$, we may pass to the limit in the integral representation to obtain

$$P_\varrho(z) = \log(1+\eta)^* g_\varrho(z) = \frac{1}{2\pi}\int_0^{2\pi} \log(1+\eta(e^{it}))g_\varrho(re^{i(\theta-t)})\,dt.$$

We have, treating $\log(1+\eta)$ as a continuous linear functional on H^1,

$$|P_\varrho(z)| \leqslant M\|g_\varrho\|_1 \leqslant M\|g\|_1$$

where M is a positive constant independent of z and ϱ. But

$$P_\varrho(z) = \log(1+\eta)^* g_\varrho(z) \to \log(1+\eta)^* g(z)$$

as $\varrho\to 1$. Hence,

$$\|\log(1+\eta)^* g\|_\infty \leqslant M\|g\|_1.$$

This is sufficient to show that $h^* g$ is in H^∞ if $g \in H^1$.

3. Some open questions.

There are integral representations of convex functions. For example one can easily find an increasing function $u_a(t)$ such that

$$K_a(z) = \int_0^z \exp\left[-\frac{1}{\pi}\int_0^{2\pi} \log(1-we^{-it})\,du_a(t)\right]dw,$$

where $u_a(2\pi)-u_a(0) = 2\pi$. It is an easy consequence of the form of the first three functions $g_k(a)$, $k = 0, 1, 2$ that $h_{a\beta} = K_a{}^* K_\beta$ is never equal to a K_r $(a, \beta \in (0,1))$. Although we can determine the Fourier coefficients of the measure corresponding to $h_{a,\beta}$ we have not found a "simple" increasing function $l(t)$ with $du = dl$. It is a problem then to determine this function $l(t)$ for these cases.

A result of the St. Ruscheweyh and Sheil-Small paper is that if φ and ψ are convex with $f < \psi$ then $\varphi^* f$ is subordinate to $\psi^*\varphi$. Consider an analogous question. Suppose $a \in (0,1)$ is fixed and g is convex function with range $g \subseteq \operatorname{range} K$. What conditions on range g will imply that convolution of H^1 with g will be in H^∞? For example it is easy to see that

$$\operatorname{dist}_C(\operatorname{range} g, \operatorname{boundary} K_a) = \delta > 0$$

is not sufficient.

It would be interesting to find the precise values of a such that $g_k(a) = 1$.

We observe that the operators Λ_a on $\mathscr{A}$ induced by composition with K_a have non-void spectrum. In particular the spectrum of Λ_a is the countable set $\{g_k(a)\}_{k=0}^\infty$. Note that our proof of Theorem 1 shows that the spectrum $\sigma(\Lambda_a)$ is not compact. We can ask then, if the convex function $f(z) = \sum_{n=0}^{\infty} a_n z^n$ induces the operator Λ on $\mathscr{A}$ is the spectrum of Λ the set $\{a_n\}_{n=0}^{\infty}$?

Finally, I would like to thank Professors John Pfaltzgraff and Ladnor Geissinger for their helpful comments on parts of this material.

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STRESZCZENIE

Niech A oznacza zbiór wszystkich funkcji holomorficznych w kole jednostkowym Δ, zaś S, C podzbiory zbioru A funkcji odpowiednio jednolistnych i wypukłych.

Dla dowolnie ustalonego elementu $f \in C$ określamy na zbiorze A operator liniowy Λ:

$$\Lambda(g)(z) = f^* g(z).$$

Jeśli $f(z) = \sum_{0}^{\infty} a_n z^n$, $g(z) = \sum_{0}^{\infty} b_n z^n$, wówczas

$$\Lambda(g)(z) = \sum_{0}^{\infty} a_n b_n z^n.$$

W pierwszej części pracy autor zajmuje się własnościami operatora Λ gdy $f(z) = \big((1+z)/(1-z)\big)^{a}$, $0 < a < 1$.

W drugiej części autor podaje warunki, przy których operator Λ przeprowadza klasę Hardy'ego H^1 w H^{∞}.

РЕЗЮМЕ

Пусть A обозначает множество всех голоморфических функций в единичном круге Δ, зато S, C подмножество множества A соответственно однолистных и выпуклых функций.

Для произвольно установленного элемента $f \in C$ определяем на множестве A линейный оператор Λ:

$$\Lambda(g)(z) = f^{*} g(z).$$

Если $f(z) = \sum^{\infty} a_n z^n$, $g(z) = \sum_{n=0}^{\infty} b_n z^n$, тогда

$$\Lambda(g)(z) = \sum_{n=0}^{\infty} a_n b_n z^n$$

В первой части работы автор занимается свойствами оператора A, когда $f(z) = \big((1+z)/(1-z)\big)^{a}$, $0 < a < 1$

Во второй части автор представляет условия, в которых оператор Λ проводит класс Гарди H^1 в H^{∞}.