ANNALES UNIVERSITATIS MARIAE CURIE-SKŁODOWSKA
LUBLIN-POLONIA

VOL. XLI, 14 SECTIO A 1987

Instytut Kształcenia Nauczycieli
Piotrków Trybunalski

T. POREDA

On the Univalent Holomorphic Maps of the Unit Polydics in C^n Which Have the Parametric Representation I – the Geometrical Properties

O odwzorowaniach jednokrotnych policylindra jednostkowego w C^n mających przedstawienie parametryczne I – własności geometryczne

In this paper we consider univalent holomorphic maps of the unit polydisc in C^n into C^n, having the parametric representation. It is shown that this class of functions have basic geometrical properties analogous to those of the class of univalent functions of one variable.

Let C^n denote the space of n complex variables $z = (z_1, \ldots, z_n)$, $z_j \in C$, $j = 1, 2, \ldots, n$. For $(z_1, \ldots, z_n) = z \in C^n$, define $\|z\| = \max_{1 \le j \le n} |z_j|$. Let $P^n(r) = \{z \in C^n; \|z\| < r\}$ and $P^n = P^n(1)$. We shall denote by I the identity map on C^n. The class of holomorphic maps of a domain Ω (contained in C^n) into C^n is denoted by $H(\Omega)$.

Let $M(P^n(r))$ be the class of maps $h : P^n(r) \to C^n$ which are holomorphic and satisfy the following conditions: $h(0) = 0$, $Dh(0) = I$ and $\operatorname{re}(h_j(z)/z_j) \ge 0$ when $\|z\| = |z_j| > 0$ $(1 \le j \le n)$, where $h = (h_1, \ldots, h_n)$ (see [6], [11]).

A mapping $v \in H(P^n)$ is called a Schwarz function if $v(0) = 0$ and $\|v(z)\| \le \|z\|$ for $z \in P^n$.

We shall say that the function f from $[s, \infty)$ (where $s \ge 0$) into C^n is almost absolutely continuous on $[s, \infty)$ if it is absolutely continuous on every bounded closed interval contained in $[s, \infty)$.

By $\delta P^n(r)$ we denote the boundary of the polydisc $P^n(r)$.

Lemma 1. *If $h \in M(P^n)$, then*

$$\|h(z) - z\| \le \frac{2\|z\|^2}{1 - \|z\|} \qquad \text{for } z \in P^n .$$

Proof. Let $h \in M(P^n)$. Then by the definition of the class $M(P^n)$, we have that $h(0)$, $Dh(0) = I$ and $\operatorname{re}(h_k(z)/z_k) \ge 0$ when $z \in P^n$ and $\|z\| = |z_k| > 0$ $(1 \le k \le n)$, where $h = (h_1, \ldots, h_n)$. Denote by

$$E_k{}^n = \{z \in P^n; \|z\| \le |z_k| , \text{ where } z = (z_1, \ldots, z_n)\}$$

for $k = 1, 2, \ldots, n$.

Fix k , $1 \leq k \leq n$. Let $F_k(z) = \frac{h_k(z)}{z_k}$ for $z \in E_k{}^n - \{0\}$. It is easy to see that $\operatorname{re} F_k(z) \geq 0$ for $z \in E_k{}^n - \{0\}$. Now, we define a function H_k in the following way:

$$H_k(t_1, \ldots, t_n) = F_k(t_1 t_k, \ldots, t_{k-1} t_k, t_k, t_{k+1} t_k, \ldots, t_n t_k)$$

for all $t = (t_1, \ldots, t_n) \in P^n$ such that $t_k \neq 0$. Since h_k is a holomorphic function on P^n and $Dh(0) = I$, therefore we can represent it in the form of the absolutely convergent power series

$$h_k(z) = z_k + \sum_{\substack{|\nu|>1 \\ \nu \in \mathbf{N}^n}} \alpha_\nu{}^{(k)} z^\nu \qquad \text{for } z \in P^n$$

(compare [2], chapter IX). Using this representation, we obtain that

$$H_k(t) = 1 + \sum_{\substack{|\nu|>1 \\ \nu \in \mathbf{N}^n}} \alpha_\nu{}^{(k)} \cdot t_1{}^{\nu_1} \cdot \ldots \cdot t_{k-1}{}^{\nu_{k-1}} \cdot t_{k+1}{}^{\nu_{k+1}} \cdot \ldots \cdot t_n{}^{\nu_n} \cdot t_k{}^{|\nu|-1} ,$$

(where $\nu = (\nu_1, \ldots, \nu_n)$, $|\nu| = \nu_1 + \cdots + \nu_n$), for all $t = (t_1, \ldots, t_n) \in P^n$ such that $t_k \neq 0$.

Let us extend the function H_k to the entire polydisc P^n by putting, for $t = (t_1, \ldots, t_{k-1}, 0, t_{k+1}, \ldots, t_n) \in P^n$, $H_k(t) = 1$. It is obvious that H_k is holomorphic on P^n and satisfies the following conditions: $H_k(0) = 1$, $\operatorname{re} H_k(t) \geq 0$ for $t \in P^n$. Taking the function H_k as a function of one complex variable t_k (with other variables fixed) we obtain by Herglotz formula (see [9], theorem 2.4) following inequality

$$|H_k(t_1, \ldots, t_n) - 1| \leq \frac{2|t_k|}{1 - |t_k|} \qquad \text{for } (t_1, \ldots, t_n) \in P^n .$$

Now, let $z = (z_1, \ldots, z_n)$ be any fixed point of $E_k{}^n - \{0\}$. Put $t_i^0 = \frac{z_i}{z_k}$ for $i \neq k$, $1 \leq i \leq n$, and $t_k^0 = z_k$. It is obvious that $t^0 = (t_1^0, \ldots, t_n^0) \in P^n$ and, since $H_k(t^0) = F_k(z)$, therefore

$$|F_k(z) - 1| \leq \frac{2|z_k|}{1 - |z_k|} .$$

By the free choice of z, we obtain that this inequality takes place for all $z \in E_k{}^n - \{0\}$. This implies that

$$|h_k(z) - z_k| \leq \frac{2|z_k|^2}{1 - |z_k|} \qquad \text{for } z \in E_k{}^n - \{0\} .$$

Further, observe that $(r e^{i\varphi_1}, \ldots, r e^{i\varphi_n}) \in E_k{}^n - \{0\}$ for any $r \in (0, 1)$ and $\varphi_m \in [0, 2\pi]$, $m = 1, \ldots, n$. Hence, we have

$$|h_k(r e^{i\varphi_1}, \ldots, r e^{i\varphi_n}) - r e^{i\varphi_k}| \leq \frac{2r^2}{1 - r}$$

for any $r \in (0,1)$ and $\varphi_m \in [0,2\pi]$, $m = 1,2,\ldots,n$.

Considering the form of the Bergman–Silov boundary for the polydisc P^n, we obtain

$$|h_k(z) - z_k| \leq \frac{2\|z\|^2}{1-\|z\|} \qquad \text{for } z \in P^n .$$

From the arbitrariness of k $(1 \leq k \leq n)$ we have

$$\|h(z) - z\| \leq \frac{2\|z\|^2}{1-\|z\|} \qquad \text{for } z \in P^n .$$

From the above lemma immediately arises

Corollary 1. *If $h \in M(P^n)$, then*

$$\|h(z)\| \leq \|z\| \,\frac{1+\|z\|}{1-\|z\|} \qquad \textit{for } z \in P^n .$$

Lemma 2. *Let $h = h(z,t)$ be a function from $P^n \times [0,\infty)$ into $\mathbb{C}^n$ such that*
(i) for every $t \in [0,\infty)$, $h(\cdot\,,t) \in M(P^n)$,
(ii) for every $z \in P^n$, $h(z,\cdot)$ is a measurable function on $[0,\infty)$.
Then for any $s \geq 0$ and $z \in P^n$ the equation

$$(1) \qquad \frac{\partial v}{\partial t} = -h(v,t) \qquad \textit{for a.e. } t \geq s , \qquad v(s) = z$$

posesses exactly one almost absolutely continuous solution $v = v(z,s,\cdot)$ on interval $[s,\infty)$. Moreover, for any $t \geq s$, the function $v(\cdot,s,t)$ is a univalent Schwarz function on P^n and $Dv(0,s,t) = e^{s-t}I$.

Proof. After introduction of semi–inner product in space $\mathbb{C}^n$ (for definition of semi–inner product see [6]) and after using lemma 1.3 from [7] and corollary 1, the proof of this lemma runs similarly as that of the theorem 2.1 from [8].

With the assumption of lemma the following corollary is true.

Corollary 2. *If $v = v(z,s,t)$ for $z \in P^n$, $0 \leq s \leq t < \infty$ satisfies equation (1) then the following inequalities take place*

$$(2) \qquad \begin{cases} \dfrac{e^t\|v(z,s,t)\|}{(1-\|v(z,s,t)\|)^2} \leq \dfrac{e^s\|z\|}{(1-\|z\|)^2} \\[2ex] \dfrac{e^s\|z\|}{(1+\|z\|)^2} \leq \dfrac{e^t\|v(z,s,t)\|}{(1+\|v(z,s,t)\|)^2} \end{cases}$$

for $z \in P^n$ and $0 \leq s \leq t < \infty$.

Using lemma 4 from [6] the proof of this corollary runs similarly to that of lemma 2.2 from [8].

Lemma 3. *Let $h = h(z,t)$ be a function from $P^n \times [0,\infty)$ into $\mathbf{C}^n$, which satisfies assumptions* (i)–(ii) *from lemma* 2. *Then there exists a limit*

(3)
$$\lim_{t\to\infty} e^t v(z,s,t) = f(z,s)\,, \qquad \text{for } z \in P^n\,,\ s \geq 0\,,$$

where $v = v(z,s,t)$, for $z \in P^n$ and $0 \leq s \leq t$, is a solution of equation (1) *such that for any $z \in P^n$ and $s \geq 0$ the function $v(z,s,\cdot)$ is almost absolutely continuous on $[s,\infty)$ and for any $s \geq 0$ the function $f(\cdot,s)$ is holomorphic and univalent on P^n, and $Df(0,s) = e^s I$.*

Proof. The fact that for any $s \geq 0$ the function $f(\cdot,s)$ is holomorphic on P^n can be proved similarly as in the theorem 2 from [10].

From lemma 2 it follows that $Df(0,s) = e^s I$ for $s \geq 0$.

Since for any $t \geq s$ ($s \geq 0$) the function $v(\cdot,s,t)$ is univalent and holomorphic on P^n and $Df(0,s) = e^s I$, therefore the map $f(\cdot,s)$ is biholomorphic as the limit of biholomorphisms (compare [5], theorem 20.2, p. 333).

Definition 1. We say that $f \in S(P^n)$ if and only if $f \in H(P^n)$, $f(0) = 0$, $Df(0) = I$ and f is univalent on P^n.

Definition 2. We say that $f \in S^0(P^n)$ if and only if there exists a function $h = h(z,t)$ from $P^n \times [0,\infty)$ into $\mathbf{C}^n$ which satisfies conditions

(i) for every $t \in [0,\infty)$, $h(\cdot,t) \in M(P^n)$

(ii) for every $z \in P^n$, $h(z,\cdot)$ is a measurable function on P^n such that

$$\lim_{t\to\infty} e^t v(z,t) = f(z) \qquad \text{for } z \in P^n$$

where $v = v(z,t)$ (for $z \in P^n$, $t \geq 0$) is such a solution of the equation

$$\frac{\partial v}{\partial t} = -h(v,t) \qquad \text{for a.e. } t \in [0,\infty), \quad v(z,0) = z$$

that for every $z \in P^n$, $v(z,\cdot)$ is an almost absolutely continuous function on $[0,\infty)$.

Remark 1. The correctness of definition 2 follows from lemma 3.

Remark 2. The class $S^0(P^n)$ will be called the class of functions which have the parametric representation.

Remark 3. It is obvious that $S^0(P^n) \subset S(P^n)$.

Remark 4. On account of theorem 6.1 and 6.3 from [9] for $n = 1$ we have

$$S^0(P^1) = S(P^1)\,.$$

The example, which is in the latter part of this paper, shows that for $n \geq 2$, the class $S^0(P^n)$ is a proper subclass of the class $S(P^n)$.

Theorem 1. *If $f \in S^0(P^n)$ then*

(4)
$$\frac{\|z\|}{(1+\|z\|)^2} \leq \|f(z)\| \leq \frac{\|z\|}{(1-\|z\|)^2} \qquad \text{for } z \in P^n\,.$$

Proof. If $f \in S^0(P^n)$, then there exists a map $h = h(z,t)$ from $P^n \times [0,\infty)$ into $\mathbf{C}^n$ satisfying conditions (i)–(ii) of definition 2. Hence $f(z) = \lim_{t\to\infty} e^t v(z,t)$, for $z \in P^n$, where $v = v(z,t)$, for $z \in P^n$ and $t \geq 0$, is a solution of the equation

$$\frac{\partial v}{\partial t}(z,t) = -h\big(v(z,t),t\big) \qquad \text{for a.e. } \ t \in [0,\infty)\ , \quad v(z,0) = z\ .$$

By corollary 2 we have the following inequalities

$$\text{(5)} \qquad \begin{cases} \dfrac{e^t\|v(z,t)\|}{(1-\|v(z,t)\|)^2} \leq \dfrac{\|z\|}{(1-\|z\|)^2} \\[2ex] \dfrac{e^t\|v(z,t)\|}{(1+\|v(z,t)\|)^2} \geq \dfrac{\|z\|}{(1+\|z\|)^2}\ , \end{cases}$$

for $z \in P^n$ and $t \geq 0$. Since $\|v(z,t)\| \leq 1$ for $z \in P^n$ and $t \geq 0$, therefore from above inequalities we obtain that $\lim_{t\to\infty} \|v(z,t)\| = 0$. Taking this fact and inequalities (5) into account we get that

$$\frac{\|z\|}{(1+\|z\|)^2} \leq \|f(z)\| \leq \frac{\|z\|}{(1-\|z\|)^2}\ .$$

Now we shall prove a theorem which, with regard to remark 4, is a generalization of Koebe theorem (compare [3], theorem 2.3).

Theorem 2. *If $f \in S^0(P^n)$, then $P^n(\frac{1}{4}) \subset f(P^n)$.*

Proof. Let $f \in S^0(P^n)$. Then from theorem 1 it follows that

$$\text{(6)} \qquad \liminf_{\|z\|\to 1} \|f(z)\| \geq \frac{1}{4}\ .$$

Let η be a fixed number from $(0,\frac{1}{4})$. By (6) we get that there exists $\rho \in (0,1)$ such that for $w \in f\big(\delta P^n(\rho)\big)$, $\|w\| > \eta$. Since $\delta P^n(\rho)$ cuts $\mathbf{C}^n$, therefore also $f\big(\delta P^n(\rho)\big)$ cuts $\mathbf{C}^n$ in two disjoint parts – one which is bounded and the other which is not bounded, and $f\big(\delta P^n(\rho)\big)$ is the boundary of these parts (see [4]). As $f\big(P^n(\rho)\big)$ is a connected set with the boundary $f\big(\delta P^n(\rho)\big)$, so far any w such that $\|w\| = \eta$ the segment $[0,w]$ does not cut $f\big(\delta P^n(\rho)\big)$. The point $0 \in f\big(P^n(\rho)\big)$, hence $P^n(\eta) \subset$ $\subset f(P^n)$ for any $\eta \in (0,\frac{1}{4})$. As a consequence we obtain that $P^n(\frac{1}{4}) \subset f(P^n)$.

The next theorem will be preceded by following lemmas.

Lemma 4. *Let $f = f(z,s)$ for $z \in P^n$ and $s \geq 0$ be a map defined as that in lemma 3. Then for any $z \in P^n$, $f(z,\cdot)$ is an almost absolutely continuous function on $[0,\infty)$. Moreover,*

$$\text{(7)} \qquad f(z,s) = f\big(v(z,s,\tau),\tau\big) \qquad \textit{for} \quad z \in P^n\ , \quad \tau \geq s \geq 0\ ,$$

where v fulfils the assumptions of lemma 3.

Proof. Equality (7) can be proved similarly to that in theorem 3 from [10].

Now, let z_0 be a fixed point of polydisc P^n, and s_1, s_2 – be any positive numbers. We can assume that $s_1 \leq s_2$ (in the contrary case the proof runs likewise). By the definition of the function v we have

$$v(z_0, s_1, s_2) - z_0 = -\int_{s_1}^{s_2} h(v(z_0, s_1, t), t)\, dt\ .$$

This and corollary 1 imply that

$$\|v(z_0, s_1, s_2) - z_0\| \leq |s_2 - s_1|\ \|z_0\|\ \frac{1 + \|z_0\|}{1 - \|z_0\|}\ . \tag{8}$$

Since $\|f(z_0, s)\| \leq \dfrac{e^s \|z_0\|}{(1 - \|z_0\|)^2}$ for $s \geq 0$, therefore by the Cauchy formula and by the mean–value theorem it is not difficult to show that for every $T > 0$ and $r \in (0,1)$ there exists $L > 0$ such that

$$\|f(z_1, s) - f(z_2, s)\| \leq L\, \|z_1 - z_2\| \tag{9}$$

for any $z_1, z_2 \in P^n(r)$ and $s \in [0, T]$.

Next, notice that from (7), (8) and (9) it follows

$$\|f(z_0, s_1) - f(z_0, s_2)\| \leq L\, \|z_0\|\ \frac{1 + \|z_0\|}{1 - \|z_0\|}\ |s_1 - s_2|$$

for any $s_1, s_2 \in [0, T]$.

From the above inequality it appears at once that for any fixed $z \in P^n$ the function $f(z, \cdot)$ is absolutely continuous on $[0, T]$, where T is any positive number. Hence for any $z \in P^n$, $f(z, \cdot)$ is an almost absolutely continuous function on $[0, \infty)$.

Lemma 5. *If $h \in M(P^n)$, then*

$$\left\|\frac{1}{2!} D^2 h(0)(z, z)\right\| \leq 2 \qquad \textit{for } z \in P^n\ . \tag{10}$$

Proof. Let z be any fixed point of P^n. Let us define a function H_z in the following way :

$$H_z(\lambda) = h(\lambda z) - \lambda z \qquad \text{for } |\lambda| < 1\ .$$

Such defined function is holomorphic in unit ball and

$$H_z''(0) = D^2 h(0)(z, z)\ . \tag{11}$$

By theorem 5.2 from [1] it follows that

$$H_z''(0) = \frac{2!}{2\pi i} \int_{C_r} \frac{H_z(\lambda)}{\lambda^3}\, d\lambda\ , \tag{12}$$

where C_r $(0 < r < 1)$ is positively directed circle with center 0 and radius r. From lemma 1 we have

$$\|H_z(\lambda)\| \leq \frac{2|\lambda|^2\|z\|^2}{1-|\lambda|\,\|z\|} \qquad \text{for } |\lambda| < 1 .$$

Taking this inequality and equality (12) into account we get that

$$\|H_z''(0)\| \leq 2! \frac{2\|z\|^2}{1-r\|z\|} \qquad \text{for } r \in (0,1) .$$

This immediately implies that $\|\frac{1}{2!}H_z''(0)\| \leq 2$.

Hence from (11) and by the free choice of z we get inequality (10).

Theorem 3. *If $f_0 \in S^0(P^n)$, then*

$$\|\frac{1}{2!} D^2 f_0(0)(z,z)\| \leq 2 \qquad \textit{for } \|z\| \leq 1 . \tag{13}$$

Proof. By the definition of the class $S^0(P^n)$ it follows that there exists a function h from $P^n \times [0,\infty)$ into $\mathbb{C}^n$ which fulfils assumption (i)–(ii) from lemma 2 and such that

$$f_0(z) = \lim_{t\to\infty} e^t v_0(z,t) \qquad \text{for } z \in P^n ,$$

where v_0 is a solution of the equation

$$\frac{\partial v_0}{\partial t}(z,t) = -h(v_0(z,t),t) \qquad \text{for a.e. } \; t \in [0,\infty) \; , \; v_0(z,0) = z .$$

Let us observe that in accordance with lemma 2 for any $s \geq 0$ and $z \in P^n$ the equation

$$\frac{\partial v}{\partial t} = -h(v,t) \qquad \text{for a.e. } \; t \in [0,\infty) \; , \; v(s) = z$$

possesses exactly one almost absolutely continuous solution $v = v(z,s,t)$ on interval $[s,\infty)$. Next, let the function $f = f(z,s)$ for $(z,s) \in P^n \times [0,\infty)$ be defined as that in lemma 3. By lemma 4 the function f is differentiable with respect to the variable s for almost all $s \in [0,\infty)$. Differentiating equality (7) and considering that $v(z,s,s) = z$, we get

$$\frac{\partial f}{\partial s}(z,s) = Df(z,s) \circ h(z,s) \tag{14}$$

for $z \in P^n$ and a.e. $s \geq 0$.

Let T be any positive number. Then we can write equality (14) in the form

$$f(z,T) - f(z,0) = \int_0^T Df(z,s) \circ h(z,s)\, ds \qquad \text{for } z \in P^n . \tag{15}$$

Now, let us introduce two functions $G_{z_0}(\lambda) = f(\lambda z_0, T) - f(\lambda z_0, 0)$ and $H_{z_0}(\lambda) = \int_0^T Df(\lambda z_0, s) \circ h(\lambda z_0, s)\, ds$ for $|\lambda| < 1$, where z_0 is a fixed point of polydisc P^n. Such defined functions are holomorphic and map unit ball into C^n ; besides considering (15) $G_{z_0} = H_{z_0}$. Hence by lemma 3, corollary 1 and the theorem about the differentiation of integrals dependent on parameter we obtain

$$H_z''(0) = \int_0^T [2D^2 f(0,s)(z_0, z_0) + e^s D^2 h(0,s)(z_0, z_0)]\, ds .$$

Hence at once we get

$$D^2 f(0,T)(z_0, z_0) - D^2 f(0,0)(z_0, z_0) = \int_0^T [2D^2 f(0,s)(z_0, z_0) + e^s D^2 h(0,s)(z_0, z_0)]\, ds.$$

By simple transformations this equality takes form

$$(16) \qquad e^{-2T} D^2 f(0,T)(z_0, z_0) - D^2 f(0,0)(z_0, z_0) = \int_0^T e^{-s} D^2 h(0,s)(z_0, z_0)\, ds .$$

In virtue of corollary 2 and lemma 3 we have the inequality

$$\|f(z,T)\| \le \frac{e^T \|z\|}{(1-\|z\|)^2} \qquad \text{for } z \in P^n ,$$

hence using the Cauchy formula it is not difficult to show that $\lim_{T\to\infty} e^{-2T} D^2 f(0,T)(z_0, z_0) = 0$. Next, making use of the inequality

$$\left\|\frac{1}{2!} D^2 h(0,s)(z_0, z_0)\right\| \le 2 \qquad \text{for } s \ge 0$$

(compare lemma 5) and considering the fact that $f(z,0) = f_0(z)$ for $z \in P^n$ and equality (16) we obtain that

$$\left\|\frac{1}{2!} D^2 f_0(0)(z_0, z_0)\right\| \le 2 .$$

By the free choice of z_0 it follows inequality (13).

Example. Let $n \ge 2$ and $f : P^n \to C^n$ be defined by formula

$$f(z) = (z_1 + 3z_2^2, z_2, \ldots, z_n) \qquad \text{for } z = (z_1, \ldots, z_n) \in P^n .$$

It is easy to see that $f \in S(P^n)$. We shall show that $f \notin S^0(P^n)$. Let us observe that $\left\|\frac{1}{2!} D^2 f(0)(z_0, z_0)\right\| = 3$ for $z_0 = (0,1,0,\ldots,0)$, hence the function f does not satisfy the necessary condition, so it does not belong to $S^0(P^n)$. Hence for $n \ge 2$ the class $S(P^n)$ is essentially wider than the class $S^0(P^n)$.

REFERENCES

[1] Alexiewicz, A., *Analiza funkcjonalna*, PWN, Warszawa 1969.

[2] Dieudonné, J., *Foundations of modern analysis*, Academic Press, New York, London 1960.

[3] Duren, P.L., *Univalent Functions*, Springer–Verlag, New York 1983.

[4] Engelking, R., Sieklucki, K., *Geometria i topologia*, part II, PWN, Warszawa 1980.

[5] Fuks, B.A., *Special chapters in the theory of analytic functions of several complex variables*, (Russian), Fizmatgiz, Moscow 1963.

[6] Gurganus, K.R., Φ *–like holomorphic functions in* $\mathbb{C}^n$ *and Banach spaces*, Trans. Amer. Math. Soc. 205 (1975), 389–406.

[7] Kato, T., *Nonlinear semigroups and evolution equations*, J. Math. Soc. Japan 19 (1967), 508–520.

[8] Pfaltzgraff, J.A., *Subordination chains and univalence of holomorphic mappings in* $\mathbb{C}^n$, Math. Ann. 210 (1974), 55–68.

[9] Pommerenke, J.Ch., *Univalent Functions*, Vandenhoeck and Ruprecht, Göttingen 1975.

[10] Poreda, T., *On the univalent subordination chains of holomorphic mappings in Banach spaces*, Comment. Math. Prace Mat., (to appear).

[11] Suffridge, T.J., *Starlike and convex maps in Banach spaces*, Pacific J. Math. 46 (1973).

STRESZCZENIE

W pracy tej wyróżnione zostały jednokrotne odwzorowania holomorficzne policylindra jednostkowego P^n w $\mathbb{C}^n$ mające przedstawienie parametryczne. Okazuje się, że ta klasa funkcji ma podstawowe własności geometryczne analogiczne jak klasa funkcji jednokrotnych jednej zmiennej zespolonej.

SUMMARY

The author considers univalent holomorphic mappings of the unit polydisc in $\mathbb{C}^n$ into $\mathbb{C}^n$ which have the parametric representation. He points out an analogy between these mappings and the univalent functions of one complex variable.