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Some Geometrical Constructions with (0,2)–Tensor Fields on Higher Order Cotangent Bundles

ABSTRACT. We study some geometrical properties of the r-th order cotangent bundle, which are closely connected with liftings of $(0,2)$-tensor fields to this bundle.

1. Introduction. The r-th order cotangent bundle is defined as the space $T^{r*}M = J^r(M,\mathbb{R})_0$ of all r-jets of smooth functions $\varphi : M \to \mathbb{R}$ with the target $0 \in \mathbb{R}$. Every local diffeomorphism $f : M \to N$ is extended to a vector bundle morphism $T^{r*}f : T^{r*}M \to T^{r*}N$, $j^r_x\varphi \mapsto j^r_{f(x)}(\varphi \circ f^{-1})$, where f^{-1} is constructed locally, [3]. Then T^{r*} is a functor on the category $\mathcal{M}f_m$ of all m-dimensional manifolds and their local diffeomorphisms. Using the general concept of the bundle of geometric objects, $T^{r*}M$ is a natural bundle on $\mathcal{M}f_m$. Obviously, $T^{r*}M$ is a vector bundle over M and for $r=1$ we obtain the classical cotangent bundle T^*M. In what follows a tensor field of the type (r,s) will mean a smooth section of the vector bundle $T^{(r,s)}M = \overset{r}{\otimes} TM \otimes \overset{s}{\otimes} T^*M$ and $T^{(r,s)}$ will denote the corresponding vector bundle functor.

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In [2] we have studied the problem, how a tensor field of the type (r,s) on M can induce a tensor field of the same type on T^*M. We have studied this problem for $(r,s)=(0,1)$, $(r,s)=(0,2)$ and $(r,s)=(1,1)$. Such geometrical constructions are called liftings. Using a more general point of view, [3], [7], the liftings from [2] are in fact natural differential operators $T^{(r,s)} \rightsquigarrow T^{(r,s)}T^*$. In some particular cases it is possible to classify all natural operators of a certain type (in other words to describe the full list of all geometrical constructions in question), see e.g. [1], [2], [3], [4], [5] and [6].

The aim of this paper is to classify all natural operators $T^{(0,2)} \rightsquigarrow T^{(0,2)}T^{r*}$ for $r \geq 2$ and to study some related geometrical properties of higher order cotangent bundles. In particular, we will study linear homomorphisms $TT^{r*}M \to T^*T^{r*}M$, natural $(0,2)$–tensor fields on $T^{r*}M$ and natural $(0,3)$–tensor fields on $T^{3*}M$. We also show that unlike the classical cotangent bundle T^*M, the higher order cotangent bundle $T^{r*}M$ has no canonical symplectic structure for $r \geq 2$. In particular, we prove that the only closed 2-form on $T^{r*}M$ is the pull-back of the canonical symplectic form from T^*M. We remark that $T^{r*}M$ is the classical example of a non product preserving functor. On the other hand, every product preserving functor can also be defined as the Weil functor T^A of A–velocities (cf. [3]) and Mikulski has in [6] classified all natural operators $T^{(0,2)} \rightsquigarrow T^{0,2)}T^A$ for any Weil functor T^A. All manifolds and maps are assumed infinitely differentiable.

2. Liftings of $(0,2)$–tensor fields to higher order cotangent bundles. The aim of this section is to show how an arbitrary $(0,2)$–tensor field on M can induce a $(0,2)$–tensor field on $T^{r*}M$ for $r \geq 2$, i.e. to classify all natural operators $T^{(0,2)} \rightsquigarrow T^{(0,2)}T^{r*}$.

Let $q_M : T^{r*}M \to M$ be the vector bundle projection and let $q_M^{r,s} : T^{r*}M \to T^{s*}M$ be the projection which is defined for $r > s$ by $j_x^r\varphi \mapsto j_x^s\varphi$. The canonical coordinates on $T^{r*}M$ will be denoted by $(x^i, u_i, \dots, u_{i_1\dots i_r})$. Let G_m^r be the group of all invertible r–jets from $\mathbb{R}^m$ to $\mathbb{R}^m$ with the source and the target zero. Then the canonical coordinates on G_m^r are denoted by $(a_j^i, a_{jk}^i, \dots, a_{j_1\dots j_r}^i)$, while the coordinates of the inverse element will be denoted by a tilde. Roughly speaking, $a_j^i = \frac{\partial \overline{x}^i}{\partial x^j}, \dots, a_{j_1\dots j_r}^i = \frac{\partial \overline{x}^i}{\partial x^{j_1}\dots\partial x^{j_r}}$ express the partial derivatives of the coordinate changes $\mathbb{R}^m \to \mathbb{R}^m$ given by $\overline{x}^i = \overline{x}^i(x^j)$. Using the coordinates of G_m^r, one can easily express the transformation laws of $(u_i, u_{ij}, u_{ijk}, \dots, u_{i_1\dots i_r})$ by

$$\begin{aligned} \overline{u}_i &= \widetilde{a}_i^k u_k, \\ \overline{u}_{ij} &= \widetilde{a}_i^k \widetilde{a}_\ell^j u_{k\ell} + \widetilde{a}_{ij}^k u_k, \end{aligned} \tag{1}$$

$$\overline{u}_{ijk} = \widetilde{a}_i^\ell \widetilde{a}_j^m \widetilde{a}_k^n u_{\ell mn} + \widetilde{a}_{ij}^\ell \widetilde{a}_k^m u_{\ell m} + \widetilde{a}_{ik}^\ell \widetilde{a}_j^m u_{\ell m} + \widetilde{a}_i^\ell \widetilde{a}_{jk}^m u_{\ell m} + \widetilde{a}_{ijk}^\ell u_\ell,$$

$$\dots$$

$$\overline{u}_{i_1 \dots i_r} = \widetilde{a}_{i_1}^{j_1} \dots \widetilde{a}_{i_r}^{j_r} u_{j_1 \dots j_r} + \dots + \widetilde{a}_{i_1 \dots i_r}^k u_k.$$

In other words, the formulae (1) express the action of G_m^r on the standard fibre of $(T^{r*}\mathbb{R}^m)_0$.

The canonical symplectic 2–form $\Lambda_M = du_i \wedge dx^i$ on T^*M is natural with respect to the following definition. Consider a natural bundle F over m–manifolds.

Definition. A natural $(0,r)$–tensor field on F is a system of $(0,r)$–tensor fields $\omega_M : FM \to T^{(0,r)}FM$ for every m–manifold M satisfying $T^{(0,r)}Ff \circ \omega_M = \omega_N \circ Ff$ for all $f : M \to N$ from $\mathcal{M}f_m$.

We have

Lemma. *Let*

(2) $\alpha = du_i \otimes dx^i - u_{ij} dx^i \otimes dx^j$ *and* $\beta = dx^i \otimes du_i - u_{ij} dx^i \otimes dx^j$.

Then α and β are natural $(0,2)$–tensor fields on $T^{r*}M$ for $r \geq 2$.

Proof. Using (1) we easily prove that $\overline{\alpha} = \alpha$ and $\overline{\beta} = \beta$, i.e. that α and β are defined geometrically (independently of the coordinate changes). □

Let $g = g_{ij} dx^i \otimes dx^j$ be a $(0,2)$–tensor field on M. Then the relation $\langle g, X \otimes Y \rangle = \langle g', Y \otimes X \rangle$ defines another $(0,2)$– tensor field $g' = g_{ji} dx^i \otimes dx^j$ on M. Further, let $\lambda_M = u_i dx^i$ be the classical Liouville 1–form on T^*M. We prove

Proposition 1. *All natural operators $T^{(0,2)} \rightsquigarrow T^{(0,2)}T^{r*}$ transforming $(0,2)$–tensor fields on M into $(0,2)$–tensor fields on $T^{r*}M$ for $r \geq 2$ are of the form*

(3) $$g \mapsto c_1 q_M^* g + c_2 q_M^* g' + c_3 (q_M^{r,1})^* (\lambda_M \otimes \lambda_M) + c_4 \alpha + c_5 \beta,$$

where $*$ means the pull-back and $c_1, \dots, c_5$ are arbitrary real numbers.

Proof. By [3], it suffices to find all G_m^r–equivariant maps

$$(J^r T^{(0,2)})_0 \mathbb{R}^m \oplus (T^{r*})_0 \mathbb{R}^m \to (T^{(0,2)} T^{r*})_0 \mathbb{R}^m$$

between standard fibres, which correspond to the r-th order natural operators in question. The canonical coordinates on the standard fibre $(T^{r*})_0\mathbb{R}^m$

are $(u_i, u_{ij}, \dots, u_{i_1\dots i_r})$ with the action of G^r_m given by (1). Further, we will denote by $(g_{ij}, g_{ij,k}, \dots, g_{ij,k_1\dots k_r})$ the canonical coordinates on the standard fibre $(J^rT^{(0,2)})_0\mathbb{R}^m$. Finally, the coordinate expression of a (0,2)–tensor field on $T^{r*}M$

$$\begin{aligned} G = {} & A_{ij}dx^i \otimes dx^j + B^i_j du_i \otimes dx^j + C^j_i dx^i \otimes du_j + D^{jk}_i dx^i \otimes du_{jk} \\ & + D^{jk\ell}_i dx^i \otimes du_{jk\ell} + \dots + D^{j_1\dots j_r}_i dx^i \otimes du_{j_1\dots j_r} + E^{jk}_i du_{jk} \otimes dx^i \\ & + E^{jk\ell}_i du_{jk\ell} \otimes dx^i + \dots + E^{j_1\dots j_r}_i du_{j_1\dots j_r} \otimes dx^i + F^{ij} du_i \otimes du_j \\ & + F^{ijk} du_i \otimes du_{jk} + \dots + F^{i_1\dots i_s j_1\dots j_r} du_{i_1\dots i_s} \otimes du_{j_1\dots j_r} + \dots \end{aligned}$$

define the canonical coordinates $(A_{ij}, \dots, F^{i_1\dots i_s j_1\dots j_r}, \dots)$ on the standard fibre $(T^{(0,2)}T^{r*})_0\mathbb{R}^m$. Consider first the maps

$$D^{jk}_i = D^{jk}_i(g_{ij}, g_{ij,k}, \dots, g_{ij,k_1\dots k_r}, u_i, u_{ij}, \dots, u_{i_1\dots i_r}).$$

Then the homotheties $a^i_j = k\delta^i_j$ yield

$$kD^{jk}_i = D^{jk}_i\Big(\frac{1}{k^2}g_{ij}, \frac{1}{k^3}g_{ij,k}, \dots, \frac{1}{k^{r+2}}g_{ij,k_1\dots k_r}, \frac{1}{k}u_i, \frac{1}{k^2}u_{ij}, \dots, \frac{1}{k^r}u_{i_1\dots i_r}\Big).$$

Multiplying both sides of this equation by $\frac{1}{k}$ and then setting $\frac{1}{k} \to 0$ we obtain that $D^{jk}_i = 0$. Quite analogously we prove that all $D's = 0$, $E's = 0$ and $F's = 0$. Moreover, by homotheties, each r-th order natural operator is reduced to the zero order one (the coordinates of G do not depend on $(g_{ij,k}, \dots, g_{ij,k_1\dots k_r})$). Now it suffices to find the form of

$$\begin{aligned} A_{ij} &= A_{ij}(g_{ij}, u_i, \dots, u_{i_1\dots i_r}), \\ B^i_j &= B^i_j(g_{ij}, u_i, \dots, u_{i_1\dots i_r}), \\ C^j_i &= C^j_i(g_{ij}, u_i, \dots, u_{i_1\dots i_r}). \end{aligned}$$

Since $D's = 0$, $E's = 0$ and $F's = 0$, in the transformation laws of A_{ij}, B^i_j and C^j_i the terms with $D's$, $E's$ and $F's$ may be omitted. One evaluates easily the following transformation laws

$$\begin{aligned} \overline{A}_{ij} &= \tilde{a}^k_i \tilde{a}^\ell_j A_{k\ell} - \tilde{a}^m_{pj} a^p_\ell u_m C^\ell_i - \tilde{a}^m_{pi} a^p_\ell u_m B^\ell_j, \\ \overline{B}^i_j &= \tilde{a}^\ell_j a^i_k B^k_\ell, \\ \overline{C}^j_i &= \tilde{a}^k_i a^j_\ell C^\ell_k. \end{aligned}$$

Using homotheties again we find that

$$\begin{aligned} A_{ij} &= c_1 g_{ij} + c_2 g_{ji} + c_3 u_{ij} + c_4 u_i u_j, \\ C^j_i &= c_5 \delta^j_i, \\ B^i_j &= c_6 \delta^i_j. \end{aligned}$$

Moreover, by equivariance we prove that $c_3 = -c_5 - c_6$, which corresponds to the coordinate form of (3). Finally, by [3], every natural operator in question has a finite order. □

Remark 1. Notice that the difference $\alpha - \beta$ is exactly the pull-back of the canonical symplectic form Λ_M on T^*M, so that the list (3) can be rewritten as

$$(3^*) \quad g \mapsto c_1 q_M^* g + c_2 q_M^* g' + c_3 (q_M^{r,1})^*(\lambda_M \otimes \lambda_M) + c_4 (q_M^{r,1})^* \Lambda_M + c_5 \beta.$$

Remark 2. By [2], all natural operators $T^{(0,2)} \rightsquigarrow T^{(0,2)}T^*$ transforming $(0,2)$–tensor fields to the cotangent bundle are linearly generated by the following 4–parameter family

$$g \mapsto c_1 q_M^* g + c_2 q_M^* g' + c_3 \lambda_M \otimes \lambda_M + c_4 \Lambda_M,$$

while for $r \geq 2$ we have 5–parameter family (3^*) with an extra $(0,2)$–tensor field β (or α).

By (3^*), the only closed 2–form on $T^{r*}M$ for $r \geq 2$ is $(q_M^{r,1})^* \Lambda_M$ and we have

Corollary 1. *There is no canonical symplectic structure on $T^{r*}M$ for $r \geq 2$.*

Corollary 2. *There is no linear canonical isomorphism $TT^{r*}M \to T^*T^{r*}M$ over the identity of $T^{r*}M$ for $r \geq 2$.*

On the other hand, in the case $r = 1$ we have the well known natural equivalence $TT^*M \to T^*T^*M$ which is induced by the canonical symplectic structure of the cotangent bundle.

Corollary 3. *The only natural $(0,2)$–tensor fields on $T^{r*}M$ for $r \geq 2$ are $(q_M^{r,1})^*(\lambda_M \otimes \lambda_M)$, $(q_M^{r,1})^* \Lambda_M$ and α (or β).*

3. Natural tensor fields. By [2], the only natural $(0,1)$–tensor field on T^*M is the classical Liouville form λ_M. It is not difficult to prove, that the pull-back $(q_M^{r,1})^* \lambda_M$ is the only natural $(0,1)$–tensor field on $T^{r*}M$ for all $r \geq 2$. Using tensor product and the exterior differential, we have two natural $(0,2)$–tensor fields $\lambda_M \otimes \lambda_M$ and $d\lambda_M = \Lambda_M$ on T^*M. By [2], all natural $(0,2)$–tensor fields on T^*M form a 2–parameter family linearly generated by $\lambda_M \otimes \lambda_M$ and Λ_M. For $r \geq 2$ we have an additional $(0,2)$–tensor field α (or β) on $T^{r*}M$ and by Corollary 3 the family of all natural

(0, 2)–tensor fields on $T^{r*}M$ is linearly generated by three tensor fields for all $r \geq 2$.

Further, natural (0, 3)–tensor fields on $T^{r*}M$ can be constructed by means of tensor products of λ_M, α and β. We have a question: Is there a natural (0, 3)–tensor field on $T^{3*}M$, which does not arise from $\lambda_M \otimes \alpha$, $\lambda_M \otimes \beta$, $\alpha \otimes \lambda_M$, $\beta \otimes \lambda_M$ and $\lambda_M \otimes \lambda_M \otimes \lambda_M$? This question is a particular case of a more general problem of finding all natural $(0, r)$–tensor fields on $T^{r*}M$. Put

$$\tag{4} \begin{aligned} \gamma_1 &= u_k dx^i \otimes dx^k \otimes du_i - u_k u_{ij} dx^i \otimes dx^k \otimes dx^j, \\ \gamma_2 &= u_k du_i \otimes dx^k \otimes dx^i - u_k u_{ij} dx^i \otimes dx^k \otimes dx^j. \end{aligned}$$

We have

Proposition 2. *All natural (0, 3)–tensor fields on $T^{3*}M$ are linearly generated by $(q_M^{3,1})^*(\lambda_M \otimes \lambda_M \otimes \lambda_M)$, $(q_M^{3,1})^*\lambda_M \otimes \alpha$, $(q_M^{3,1})^*\lambda_M \otimes \beta$, $\alpha \otimes (q_M^{3,1})^*\lambda_M$, $\beta \otimes (q_M^{3,1})^*\lambda_M$, γ_1 and γ_2.*

Proof. The proof is quite similar to that of Proposition 1, so that we sketch the principal steps only. The coordinate expression of a (0, 3)–tensor field on $T^{3*}M$ is of the form

$$\begin{aligned} G = {} & A_{ijk} dx^i \otimes dx^j \otimes dx^k + B^k_{ij} dx^i \otimes dx^j \otimes du_k \\ & + C^k_{ij} dx^i \otimes du_k \otimes dx^j + D^k_{ij} du_k \otimes dx^i \otimes dx^j \\ & + E^{k\ell}_{ij} dx^i \otimes dx^j \otimes du_{k\ell} + F^{k\ell}_{ij} dx^i \otimes du_{k\ell} \otimes dx^j \\ & + G^{k\ell}_{ij} du_{k\ell} \otimes dx^i \otimes dx^j + \dots \end{aligned}$$

where all the coefficients are functions of $(u_i, u_{ij}, \dots, u_{i_1 \dots i_r})$. By homotheties, all the coefficients except $A_{ijk}, \dots, G^{k\ell}_{ij}$ are zero. Using this fact, we compute the transformation laws in the form

$$\begin{aligned} \overline{A}_{ijk} = {} & A_{ijk} + a^r_{nk} u_r B^n_{ij} + a^r_{nj} u_r C^n_{ik} + a^r_{ni} u_r D^n_{jk} \\ & + (a^r_{npk} u_r + a^r_{nk} u_{rp} + a^s_{pk} u_{ns}) E^{np}_{ij} \\ & + (a^r_{npj} u_r + a^r_{nj} u_{rp} + a^s_{pj} u_{ns}) F^{np}_{ik} + (a^r_{npi} u_r + a^r_{ni} u_{rp} + a^s_{pi} u_{ns}) G^{np}_{jk}, \\ \overline{B}^k_{ij} = {} & B^k_{ij} + a^k_{rs} E^{rs}_{ij}, \\ \overline{C}^k_{ij} = {} & C^k_{ij} + a^k_{rs} F^{rs}_{ij}, \\ \overline{D}^k_{ij} = {} & D^k_{ij} + a^k_{rs} G^{rs}_{ij}, \end{aligned}$$

while the remaining coordinates $E^{k\ell}_{ij}$, $F^{k\ell}_{ij}$ and $G^{k\ell}_{ij}$ have tensorial transformation laws. The rest of the proof is then an easy exercise with the homotheties and equivariances analogously to the proof of Proposition 1. □

4. Canonical homomorphisms. We have proved that there is no linear canonical isomorphism $TT^{r*}M \to T^*T^{r*}M$ for $r \geq 2$. Each $(0,2)$–tensor field $g = g_{ij}dx^i \otimes dx^j$ on M can be identified with a linear homomorphism $g_L : TM \to T^*M, (x^i, y^i) \mapsto (x^i, u_i = g_{ij}y^j)$. If M is a symplectic manifold and g is the corresponding symplectic form, then g_L is an isomorphism.

Analogously, a $(0,2)$–tensor field G on $T^{r*}M$ induces a linear homomorphism $G_L : TT^{r*}M \to T^*T^{r*}M$ over the identity of $T^{r*}M$. Denoting by $(x^i, u_i, \dots, u_{i_1 \dots i_r}, X^i, U_i, \dots, U_{i_1 \dots i_r})$ the canonical coordinates on $TT^{r*}M$ and $(x^i, u_i, \dots, u_{i_1 \dots i_r}, \alpha_i dx^i + \beta^i du_i + \dots + \beta^{i_1 \dots i_r} du_{i_1 \dots i_r})$ the canonical coordinates on $T^*T^{r*}M$, the $(0,2)$–tensor fields α and β induce the homomorphisms $\alpha_L, \beta_L : TT^{r*}M \to T^*T^{r*}M$. The equations of α_L are

$$\alpha_i = -u_{ij}X^j, \quad \beta^i = X^i, \quad \beta^{ij} = 0, \dots, \beta^{i_1 \dots i_r} = 0 \tag{5}$$

and the equations of β_L are

$$\alpha_i = -u_{ij}X^j + U_i, \quad \beta^i = 0, \dots, \beta^{i_1 \dots i_r} = 0. \tag{6}$$

At the end we prove the stronger form of Corollary 2 for $r = 2$.

Proposition 3. *There is no canonical isomorphism $TT^{2*}M \to T^*T^{2*}M$ over the identity of $T^{2*}M$.*

Proof. The action of G^3_m on the standard fibre $T^*T^{2*}M$ is

$$\overline{\beta}^{ij} = a^i_k a^j_\ell \beta^{k\ell}, \quad \overline{\beta}^i = a^i_j \beta^j + a^i_{jk}\beta^{jk}$$

while we will not need the equations of $\overline{\alpha}_i$.

By equivariance, $\beta^{ij} = \beta^{ij}(u_i, u_{ij}, X^i, U_i, U_{ij})$ do not depend of U_{ij}. Further, introduce new variables $p_i, q_j \in \mathbb{R}^{m*}$ and consider the function $f(u_i, u_{ij}, X^i, U_i, p_i, q_i) = \beta^{ij}p_i q_j$. Then f is G^1_m–invariant. By the tensor evaluation theorem from [3],

$$f = f(u_i X^i, u_{ij}X^i X^j, U_i X^i, p_i X^i, q_i X^i).$$

Replace $u_i X^i$, $U_i X^i$ and $u_{ij}X^i X^j$ by $I_1 = u_i X^i, I_2 = U_i X^i - u_{ij}X^i X^j$ and $u_{ij}X^i X^j$, so that $f = f(I_1, I_2, p_i X^i, q_i X^i, u_{ij}X^i X^j)$. One evaluates easily that I_1, I_2, $p_i X^i$ and $q_i X^i$ are G^2_m–invariant expressions. Then the equivariance yield that f does not depend of the fifth variable, so that $f = f(I_1, I_2, p_i X^i, q_i X^i)$. Differentiating with respect to p_i and then setting $p_i = 0$ we have $\beta^{ij}q_j = \tilde{f}(I_1, I_2, q_i X^i)X^i$. Analogously, differentiating this with respect to q_i and then setting $q_i = 0$ we prove that $\beta^{ij} = \psi(I_1, I_2)X^i X^j$

where ψ is an arbitrary function of two variables. Further, using a similar procedure as that for β^{ij} we deduce that $\beta^i = \varphi(I_1, I_2)X^i$. By equivariance,

$$\varphi(I_1, I_2)X^i + a^i_{jk}\psi(I_1, I_2)X^jX^k = \varphi(I_1, I_2)X^i$$

which reads $\psi = 0$. Up till now, we have proved $\beta^i = \varphi X^i$, $\beta^{ij} = 0$, which can not be equations of an isomorphism. □

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