ANNALES
UNIVERSITATIS MARIAE CURIE-SKŁODOWSKA
LUBLIN — POLONIA
VOL. XLV, 7 SECTIO AAA 1990

Instytut Fizyki UMCS

6354

Wiesław A. KAMIŃSKI

Generalization of the DCX Reaction Amplitudes for Transitions to an Arbitrary Final State

Uogólnienie amplitud reakcji DCX dla przejść do dowolnego stanu końcowego

1. INTRODUCTION

In a series of papers [1–4] (hereafter referred to as KF) it was shown that the double charge exchange reaction (DCX) with pions can be described within the proton-neutron quasiparticle random phase approximation (QRPA). Both transitions, to the ground state and to the double isobaric analogue state of the final $(A, Z+2)$ nucleus were studied and the expressions for the total amplitudes in these cases were found. The question arises as to whether such a result can be reached if we take into account the transition to any arbitrary excited state of the final nucleus. We shall investigate this problem in this paper.

2. KF MODEL REVISITED

In order to generalize the KF formulae, it is instructive to review briefly main goals of the previous results. We have considered the total transition amplitude $F(\mathbf{k}, \mathbf{k}')$ for the DCX reaction

$$\pi^+ \; + \; X(A,Z) \; \longrightarrow \; \pi^- \; + \; Y^*(A,Z+2) \tag{1}$$

defined in such a way, that the differential cross section is equal to

$$\frac{d\sigma}{d\Omega} = |\frac{1}{4\pi}F(\mathbf{k},\mathbf{k}')|^2. \tag{2}$$

Here $\mathbf{k}$ and $\mathbf{k}'$ are momenta of in- and outcoming pions, respectively. We applied to reaction (1) the sequential model in which an incident pion π^+ changes in a first step into a pion π^0 by changing a neutron into proton and producing the intermediate nucleus $(A, Z+1)$ from the target nucleus (A, Z). During this process, the intermediate nucleus is excited to all possible states. In our model, these excited states are described in the proton-particle neutron-hole QRPA formalism. We did not involve the effects of a nucleus penetration by the neutral pion explicitly because they are higher order terms in the linear response function. Next, a pion π^- is produced by changing another neutron into a second proton and leaves the final nucleus.

The total amplitude $F(\mathbf{k}, \mathbf{k}')$ is a sum over all excited states of the intermediate nucleus. We restrict our considerations to nucleon excitations only in the low pion energy domain. In principle there are no fundamental difficulties to include delta-isobar excitations, but because of the large delta-nucleon mass difference such terms are of no importance for pion kinetic energies lower than approximately 150 MeV. So, the full scattering amplitude is

$$F(\mathbf{k},\mathbf{k}') = \sum_{\{\alpha,J\}} [F_{\alpha J}(\mathbf{k},\mathbf{k}')] \quad , \tag{3}$$

where we sum over all intermediate states $\{\alpha, J\}$ as stated above. To describe the amplitude (3) in a more detailed way we introduced two phenomenological transition Hamiltonians reflected two different mechanisms for the DCX reaction. One of these Hamiltonians is generated from the non-relativistic reduction of the well-known pseudoscalar coupling Lagrangian. In this way one can obtain so-called p-wave effective pion-nucleus Hamiltonian (for details see [2])

$$h_p(\mathbf{q}) = -\sqrt{2}\, i\, \frac{f}{m_\pi} \sum_{pn} \sum_{JM} \mathcal{F}_{pn}^{JM}(\mathbf{q}) \mathcal{R}_{pn}^{JM} \ , \tag{4}$$

Another mechanism is connected with an exchange of a composite boson between nucleons. The $\pi\pi N$ interaction playing a crucial role in such a process gives us the phenomenological s-wave πN effective Hamiltonian in the Koltun-Reitan's form [2]

$$h_s(\mathbf{q}) = -4\pi \frac{\lambda_s}{m_\pi} \sqrt{2}\omega_q \sum_{pn} \hat{j}_p \delta_{pn} \mathcal{R}_{pn}^{00} \quad , \tag{5}$$

In the above equations the QRPA transition operator $\mathcal{R}_{pn}^{JM}$ is defined by the formula

$$\mathcal{R}_{pn}^{JM} = u_p v_n C^\dagger(pnJM) + v_p u_n \tilde{C}(pnJM) +$$

$$+ u_p u_n D(pnJM) - v_p v_n \tilde{D}^\dagger(pnJM) \quad . \tag{6}$$

Here $\omega_q = (\mathbf{q}^2 + m^2)^{\frac{1}{2}}$ is the incident or outgoing pion energy, $\hat{j} = \sqrt{2j+1}$ and u, v's are the occupation amplitudes appearing in the transformation from the particle operators to the quasiparticle operators. The pair-creation (annihilation) operators $C^\dagger(C)$ and additional $D^\dagger(D)$ operators are defined in the usual way (see e.g. [2, 5]). The function $\mathcal{F}_{pn}^{JM}(\mathbf{q})$ can be written as follows

$$\mathcal{F}_{pn}^{JM}(\mathbf{q}) = \sum_{m_p, m_n} (-1)^{j_n - m_n} (j_p m_p j_n - m_n | JM) \times$$

$$\times \left[\int d^3x \psi_p^*(\mathbf{x}, \xi) \boldsymbol{\sigma} \cdot \mathbf{q} e^{i\mathbf{q}\cdot\mathbf{x}} \psi_n(\mathbf{x}, \xi) \right] \quad . \tag{7}$$

The proton (neutron) space-spin wave functions $\psi_{p(n)}(\mathbf{x}, \xi)$ are calculated with some single particle potential (e.g. the harmonic oscillator or Woods--Saxon potential). After some algebra one can find a more handy expression

$$\mathcal{F}_{pn}^{JM}(\mathbf{q}) = \sqrt{4\pi}\sqrt{6} Y_{JM}^*(\Omega_q) G_{pn}^J(\mathbf{q}) \quad , \tag{8}$$

where Y_{JM} is the spherical harmonic depending on the solid angle Ω_q and $G_{pn}^J(q)$ is the nuclear form-factor, for which the final expression is different for different choices of the radial dependence of the nucleon wave functions. More details on the explicit form of G_{pn}^J can be found in [2].

Using perturbation theory we could express the transition amplitude $F_J(\mathbf{k}, \mathbf{k}')$ as follows:

$$F_J(\mathbf{k}, \mathbf{k}') = \sum_m \frac{\langle f, \pi^-(\mathbf{k}') | \hat{T} | mJM \rangle \langle mJM | \hat{T} | i, \pi^+(\mathbf{k}) \rangle}{E_i + \omega_k - E_m^J} \quad , \tag{9}$$

where $\hat{T}$ is the transition operator consisting of both s- and p- parts of the pion-nucleus Hamiltonian (eqs. (4) and (5)). $|i, \pi^+(\mathbf{k})\rangle$ is the ground state of the target (A, Z) nucleus with the incoming pion π^+ of momentum $\mathbf{k}$. $(E_i + \omega_k)$ is the initial energy of the nucleus-pion system and E_m^J's are the QRPA excitation energies of the intermediate nucleus. In eq. (9) we sum over all intermediate excited states. In point of fact, the question of this summation is more complicated because of two different mathematically but equivalent physically sets of the intermediate states calculated relative to the target nucleus $(A,\ Z)$ and the final nucleus $(A,\ Z+2)$, respectively. We do not intend to discuss this problem further in the paper.

In the approach developed earlier, the state $|f, \pi^-(\mathbf{k}')\rangle$ considered was either the ground state of the final nucleus or the double isobaric analogue state in the same nucleus. In principle there are no fundamental difficulties to treat the final state as an arbitrary excited state of the $(A,\ Z+2)$ nucleus. A need for such consistent formulae is urgent all the more, that the DCX data exist for the reaction $^{56}\mathrm{Fe}(\pi^+,\pi^-)^{56}\mathrm{Ni}$ in a full extension. Cross sections and angular distributions for the double isobaric analogue transition, non-analogue ground state transition and for some transitions to the low excited states $(0_2^+, 0_3^+, 2_1^+)$ were measured recently [6,7].

3. GENERALIZED DCX TRANSITION FORMULAE

The state $|f\rangle$ in the expression for the total amplitude (9) can be specified as the ground state (GS) of the final nucleus or one of the excited states among which the double isobaric analogue state (DIAS) is mostly discussed in literature. In KF we derived formulae for the DCX transition to the ground state [2, 4] and to the DIAS [1]. Below we shall show how to generalize such formulae in the case when the final nucleus is excited during process (1) to the one of its excited states. We are working within QRPA formalism to describe both sets of states: the excited states of the intermediate nucleus and the excited states of the final nucleus. Thus, we have to define two types of the angular-momentum-coupled phonon operators from the usual RPA ansatz [8]:
(1^0) for the excited states of the intermediate nucleus

$$\overline{Q}^{m}_{JM}{}^{\dagger} = \sum_{pn} \left[\overline{X}^{m}_{(pn)J} C^{\dagger}(pnJM) - \overline{Y}^{m}_{(pn)J} \tilde{C}(pnJM) \right] , \qquad (10)$$

(2^0) for the excited states of the final nucleus

$$\mathcal{Q}^{\nu}_{\mathcal{JM}}{}^{\dagger} = \sum_{pp'} \left[\overline{\mathcal{X}}^{\nu}_{(pp')\mathcal{J}} A^{\dagger}(pp'\mathcal{JM}) - \overline{\mathcal{Y}}^{\nu}_{(pp')\mathcal{JM}} \tilde{A}(pp'\mathcal{JM}) \right] +$$

$$+ \sum_{nn'} \left[\overline{\mathcal{X}}^{\nu}_{(nn')\mathcal{J}} B^{\dagger}(nn'\mathcal{JM}) - \overline{\mathcal{Y}}^{\nu}_{(nn')\mathcal{JM}} \tilde{B}(nn'\mathcal{JM}) \right] , \qquad (11)$$

where pair creation (annihilation) operators for the proton-neutron system $C^{\dagger}(C)$, proton-proton system $A^{\dagger}(A)$ and neutron-neutron system $B^{\dagger}(B)$ are defined in a usual way. $\overline{X}$'s and $\overline{Y}$'s are forward- and backwardgoing QRPA amplitudes of the mth J state of the odd-odd intermediate nucleus calculated relative to the final nucleus ground state with proton-particle neutron-hole excitations (the charge-changing mode). Similarly, $\overline{\mathcal{X}}_{(aa')}$ and

$\overline{\mathcal{Y}}_{(aa')}$ with (aa') standing for proton-proton or neutron-neutron indices are the QRPA amplitudes of the excited states in the final even-even nucleus calculated within the charge-non-changing mode. According to eq. (9) we need to calculate two matrix elements

$$\langle mJM|\hat{T}|i,GS;\pi^+(\mathbf{k})\rangle = \langle i,GS|Q^m_{JM}\hat{T}|i,GS;\pi^+(\mathbf{k})\rangle \tag{12}$$

and

$$\langle f_\nu,\mathcal{JM};\pi^-(\mathbf{k}')|\hat{T}|mJM\rangle = \langle f,GS;\pi^-(\mathbf{k}')|\mathcal{Q}^\nu_{\mathcal{JM}}\hat{T}\overline{Q}^{m}_{JM}{}^\dagger|f,GS\rangle \quad , \tag{13}$$

where JM is the angular momentum and its third component of the intermediate state and $\mathcal{JM}$ is the angular momentum and its third component for the excited state of the final nucleus. In real applications of the formalism we use two different phonon operators of type (10), because in the charge-changing mode one can calculate the excited states of the intermediate nucleus in two mathematically different but physically equivalent ways. From a formal point of view the only difference is that we should distinguish the RPA amplitudes X and Y's by adding a bar in the case we calculated them starting from the ground state of the final nucleus or leaving a bar in the case we calculated the amplitudes relative to the ground state of the initial nucleus. In this point a more detailed discussion of such difference is not needed and does not influence on generality of final results.

To find formulae for the matrix elements (12) and (13) we should remember that the QRPA phonon annihilation operators $\overline{Q}^m_{JM}$ and $\mathcal{Q}^\nu_{\mathcal{JM}}$ give zero operating on the RPA vacua. We also use the quasiboson approximation [5] and finally we are taking the BCS solutions as the RPA vacua. With the above approximations one finds after little algebra:
(1^0) for the s-wave transition operator h_s

$$\langle\overline{\mathrm{RPA}}|\left[\hat{\mathcal{R}}^{00}_{pn},C^\dagger(p'n'JM)\right]|\overline{\mathrm{RPA}}\rangle = v_p u_n \delta_{pp'}\delta_{nn'}\delta_{J0}\delta_{M0} \quad , \tag{14}$$

$$\langle\overline{\mathrm{RPA}}|\left[\hat{\mathcal{R}}^{00}_{pn},\tilde{C}(p'n'JM)\right]|\overline{\mathrm{RPA}}\rangle = (-1)^{J+M+1} u_p v_n \delta_{pp'}\delta_{nn'}\delta_{J0}\delta_{M0} \quad , \tag{15}$$

$$\begin{aligned}\langle\overline{\mathrm{RPA}}|\left[\left[A(pp'\mathcal{JM}),\hat{\mathcal{R}}^{00}_{p''n''}\right],C^\dagger(\bar{p}\bar{n}JM)\right]|\overline{\mathrm{RPA}}\rangle = \\ = u_{p''}u_{n''}(-1)^{j_p+j_{\bar{p}}+J+1}\hat{j}_{\bar{n}}\left(\delta_{pp''}\delta_{p'\bar{p}}\delta_{\bar{n}n''}\delta_{j_nj_p}+\right. \\ \left.+(-1)^J\delta_{p'p''}\delta_{p\bar{p}}\delta_{\bar{n}n''}\delta_{j_nj_{p'}}\right)\delta_{J\mathcal{J}}\delta_{M\mathcal{M}} \quad ,\end{aligned} \tag{16}$$

and

$$\langle\overline{\mathrm{RPA}}|\left[\left[\tilde{A}^\dagger(pp'\mathcal{J}\mathcal{M}),\hat{\mathcal{R}}^{00}_{p''n''}\right],\tilde{C}(\bar{p}\bar{n}JM)\right]|\overline{\mathrm{RPA}}\rangle =$$

$$= v_{p''}v_{n''}(-1)^{j_p+j_{\bar{p}}+J}\hat{j}_{n}\left(\delta_{pp''}\delta_{p'\bar{p}}\delta_{\bar{n}n''}\delta_{j_n j_p}+\right.$$

$$\left.+(-1)^{J}\delta_{p'p''}\delta_{p\bar{p}}\delta_{\bar{n}n''}\delta_{j_n j_{p'}}\right)\delta_{J\mathcal{J}}\delta_{M\mathcal{M}} \quad , \qquad (17)$$

Similar equations with the neutron-particle neutron-hole excitations can be written immediately by exchanging proton indices on neutron indices.

(2^0) for the p-wave QRPA transition operator $\hat{\mathcal{R}}^{J''M''}_{p''n''}$

$$\langle\overline{\mathrm{RPA}}|\left[\hat{\mathcal{R}}^{J''M''}_{p''n''},C^\dagger(pnJM)\right]|\overline{\mathrm{RPA}}\rangle =$$

$$= (-1)^{J+M}v_{p''}u_{n''}\delta_{pp''}\delta_{nn''}\delta_{JJ''}\delta_{MM''} \quad , \qquad (18)$$

$$\langle\overline{\mathrm{RPA}}|\left[\hat{\mathcal{R}}^{J''M''}_{p''n''},\tilde{C}(pnJM)\right]|\overline{\mathrm{RPA}}\rangle =$$

$$= (-1)^{J+M+1}u_{p''}v_{n''}\delta_{pp''}\delta_{nn''}\delta_{JJ''}\delta_{MM''} \quad , \qquad (19)$$

$$\langle\overline{\mathrm{RPA}}|\left[\left[A(pp'\mathcal{J}\mathcal{M}),\hat{\mathcal{R}}^{J''M''}_{p''n''}\right],C^\dagger(\bar{p}\bar{n}JM)\right]|\overline{\mathrm{RPA}}\rangle =$$

$$= u_{p''}u_{n''}(-1)^{j_p+j_n+J''}\hat{J}\hat{J}''(JMJ''M''|\mathcal{J}\mathcal{M})\left[\begin{Bmatrix} j_{\bar{n}} & j_{\bar{p}} & J \\ \mathcal{J} & J'' & j_p\end{Bmatrix}\delta_{pp''}\delta_{p'\bar{p}}\delta_{\bar{n}n''}+\right.$$

$$\left.+(-1)^{\mathcal{J}}\begin{Bmatrix} j_{\bar{n}} & j_{\bar{p}} & J \\ \mathcal{J} & J'' & j_{p'}\end{Bmatrix}\delta_{p'p''}\delta_{p\bar{p}}\delta_{\bar{n}n''}\right] \quad , \qquad (20)$$

and

$$\langle\overline{\mathrm{RPA}}|\left[\left[\tilde{A}^\dagger(pp'\mathcal{J}\mathcal{M}),\hat{\mathcal{R}}^{J''M''}_{p''n''}\right],\tilde{C}(\bar{p}\bar{n}JM)\right]|\overline{\mathrm{RPA}}\rangle =$$

$$= v_{p''}v_{n''}(-1)^{j_p+j_n+J''+1}\hat{J}\hat{J}''(JMJ''M''|\mathcal{J}\mathcal{M}\left[\begin{Bmatrix} j_{\bar{n}} & j_{\bar{p}} & J \\ \mathcal{J} & J'' & j_p\end{Bmatrix}\delta_{pp''}\delta_{p'\bar{p}}\delta_{\bar{n}n''}+\right.$$

$$\left.+(-1)^{\mathcal{J}}\begin{Bmatrix} j_{\bar{n}} & j_{\bar{p}} & J \\ \mathcal{J} & J'' & j_{p'}\end{Bmatrix}\delta_{p'p''}\delta_{p\bar{p}}\delta_{\bar{n}n''}\right] \quad , \qquad (21)$$

In expressions (14) – (20) the symbol [,] means the commutator and $\delta_{a,a'} = \delta_{n_a n_{a'}}\delta_{j_a j_{a'}}\,\delta_{l_a l_{a'}}$. Above we only quoted the terms with non-zero contributions within the quasiboson approximation.

With an aid of formulae (14) – (21) one finds the needed matrix element (13) containing two parts for h_s and h_p , respectively:

$$\langle f_\nu \mathcal{J}\mathcal{M};\pi^-(\mathbf{k}')|h_s|mJM\rangle = 4\pi\frac{\lambda_s}{m_\pi}\sqrt{2}\,\omega_k\delta_{J\mathcal{J}}\delta_{M\mathcal{M}}\times$$

$$\times\sum_{p\le p',\bar{p}\bar{n}}(-1)^{j_p+j_{\bar{n}}}\Big[\Big(\bar{\mathcal{X}}^\nu_{(pp')J}\bar{X}^m_{(\bar{p}\bar{n})J}u_p u_{\bar{n}} - \bar{\mathcal{Y}}^\nu_{(pp')J}\bar{Y}^m_{\bar{p}\bar{n})J}v_p v_{\bar{n}}\Big)\delta_{p'\bar{p}}\delta_{p\bar{n}}+$$

$$+(-1)^J\Big(\bar{\mathcal{X}}^\nu_{(pp')J}\bar{X}^m_{(\bar{p}\bar{n})J}u_{p'}u_{\bar{n}} - \bar{\mathcal{Y}}^\nu_{(pp')J}\bar{Y}^m_{\bar{p}\bar{n})J}v_{p'}v_{\bar{n}}\Big)\delta_{p\bar{p}}\delta_{p'\bar{n}}\Big]\quad, \tag{22}$$

$$\langle f_\nu \mathcal{J}\mathcal{M};\pi^-(\mathbf{k}')|h_p|mJM\rangle = i\frac{f}{m\pi}\sqrt{4\pi}\sqrt{12}\times$$

$$\times\sum_{p\le p',\bar{p}\bar{n}J''M''}\hat{J}\hat{J}''(JMJ''M''|\mathcal{J}\mathcal{M})(-1)^{j_p+j_{\bar{p}}+J+J''}Y^*_{J''}(\Omega_{k'})\Big[\begin{Bmatrix} j_{\bar{n}} & j_{\bar{p}} & J\\ \mathcal{J} & J'' & j_p\end{Bmatrix}\times$$

$$\times\delta_{p'\bar{p}}G^{J''}_{p\bar{n}}(k')\Big(\bar{\mathcal{X}}^\nu_{(pp')J}\bar{X}^m_{(\bar{p}\bar{n})J}\bar{u}_p\bar{u}_{\bar{n}} - \bar{\mathcal{Y}}^\nu_{(pp')J}\bar{Y}^m_{\bar{p}\bar{n})J}\bar{v}_p\bar{v}_{\bar{n}}\Big)+\begin{Bmatrix} j_{\bar{n}} & j_{\bar{p}} & J\\ \mathcal{J} & J'' & j_{p'}\end{Bmatrix}\times$$

$$\times(-1)^{\mathcal{J}}\delta_{p\bar{p}}G^{J''}_{p'\bar{n}}(k')\Big(\bar{\mathcal{X}}^\nu_{(pp')J}\bar{X}^m_{(\bar{p}\bar{n})J}\bar{u}_{p'}\bar{u}_{\bar{n}} - \bar{\mathcal{Y}}^\nu_{(pp')J}\bar{Y}^m_{\bar{p}\bar{n})J}\bar{v}_{p'}\bar{v}_{\bar{n}}\Big)\Big]\quad. \tag{23}$$

In all above expressions the quantities with bar correspond to the states generated from the daughter $(A, Z+2)$ nucleus. Both formulae (21) and (22) are quoted for the proton-particle proton-hole part of the total matrix elements only. The discarded neutron-neutron part of eqs. (22) and (23) may be easily constructed by exchanging p, p' indices upon n, n' indices.

Analogous equations can be written for the states generated relative to the initial (target) (A, Z) nucleus using the RPA amplitudes without bar. Thus, the matrix element (12) can be written as follows [2]:

$$\langle m'JM|h_s|i, G.S.;\ \pi^+(\mathbf{k})\rangle = \sqrt{2}\,\Big(4\pi\frac{\lambda_s}{m_\pi}\Big)\omega_k\delta_{J0}\delta_{M0}\times$$

$$\times\Big[\sum_{pn}\delta_{pn}\Big(X^{m'}_{(pn)J}u_p v_n\ +\ Y^{m'}_{(pn)J}v_p u_n\Big)\Big]\quad. \tag{24}$$

and

$$\langle m'J^\pi M|h_p|i, g.s.,\ \pi^+(\mathbf{k})\rangle = i\,\sqrt{12}\,\frac{f}{m_\pi}\,\sqrt{4\pi}\,(-1)^J\,Y^*_{JM}(\Omega_k)\times$$

$$\times\Big[\sum_{pn}G^J_{pn}(k)\Big(X^{m'}_{(pn)J}u_p v_n\ +\ Y^{m'}_{(pn)J}v_p u_n\Big)\Big]\quad. \tag{25}$$

Combining eqs. (22), (23), (24) and (25) with the expression for the total amplitude (9) one can find the final expressions for the s- and p- wave parts of the DCX amplitude in a case of the transition to an arbitrary excited state $|\mathcal{JM}\rangle$ of the final nucleus:

$$F^s_{\mathcal{J}}(\mathbf{k},\mathbf{k}') = -\delta_{J\mathcal{J}}\left(4\pi\frac{\lambda_s}{m_\pi}\right)^2 \omega_k\omega_{k'}\sum_{m,m'}\frac{\langle mJ|m'J\rangle}{E_i+\omega_k-\frac{E^J_m+E^J_{m'}}{2}}\times$$

$$\times\left\{\sqrt{2}\sum_{\bar{p}\bar{n},p\le p'}(-1)^{j_p+j_n}\left[\left(\overline{\mathcal{X}}^{\nu}_{(pp')\mathcal{J}}\overline{X}^{m}_{(\bar{p}\bar{n})J}\bar{v}_{\bar{p}}\bar{u}_n-\overline{\mathcal{Y}}^{\nu}_{(pp')\mathcal{J}}\overline{Y}^{m}_{(\bar{p}\bar{n})J}\bar{u}_p\bar{v}_{\bar{n}}\right)\delta_{p'\bar{p}}\delta_{p\bar{n}}+\right.\right.$$

$$\left.+(-1)^J\left(\overline{\mathcal{X}}^{\nu}_{(pp')\mathcal{J}}\overline{X}^{m}_{(\bar{p}\bar{n})J}\bar{u}_{\bar{p}'}\bar{u}_{\bar{n}}-\overline{\mathcal{Y}}^{\nu}_{(pp')\mathcal{J}}\overline{Y}^{m}_{(\bar{p}\bar{n})J}\bar{v}_{p'}\bar{v}_{\bar{n}}\right)\delta_{p\bar{p}}\delta_{p'\bar{n}}\right]\times$$

$$\left.\times\left[\sqrt{2}\,\delta_{J0}\sum_{pn}\hat{\jmath}\,\delta_{pn}\left(X^{m'}_{(pn)J}u_pv_n-Y^{m'}_{(pn)J}v_pu_n\right)\right]\right\}\quad, \tag{26}$$

$$F^p_{\mathcal{J}}(\mathbf{k},\mathbf{k}') = -\left(\frac{f}{m_\pi}\right)^2\sum_{m,m'}\frac{\langle mJ|m'J\rangle}{E_i+\omega_k-\frac{E^J_m+E^J_{m'}}{2}}\times$$

$$\times\sum_{J'',\mathcal{M}}\hat{J}\hat{J}''(-1)^{\mathcal{J}+\mathcal{M}}\{Y_{J''}(\Omega_{k'})\otimes Y_J(\Omega_k)\}_{\mathcal{JM}}\left\{\left[\sqrt{12}\sum_{p\le p',\bar{p}\bar{n}}(-1)^{j_p+j_p}\times\right.\right.$$

$$\times\left[\left\{\begin{matrix} j_{\bar{n}} & j_{\bar{p}} & J\\ \mathcal{J} & J'' & j_p\end{matrix}\right\}\delta_{p'\bar{p}}G^{J''}_{p\bar{n}}(k')\left(\overline{\mathcal{X}}^{\nu}_{(pp')J}\overline{X}^{m}_{(\bar{p}\bar{n})J}\bar{u}_p\bar{u}_{\bar{n}}-\overline{\mathcal{Y}}^{\nu}_{(pp')J}\overline{Y}^{m}_{\bar{p}\bar{n})J}\bar{v}_p\bar{v}_{\bar{n}}\right)+\right.$$

$$+\left\{\begin{matrix} j_{\bar{n}} & j_{\bar{p}} & J\\ \mathcal{J} & J'' & j_{p'}\end{matrix}\right\}(-1)^{\mathcal{J}}\delta_{p\bar{p}}G^{J''}_{p'\bar{n}}(k')\times$$

$$\left.\left.\times\left(\overline{\mathcal{X}}^{\nu}_{(pp')J}\overline{X}^{m}_{(\bar{p}\bar{n})J}\bar{u}_{p'}\bar{u}_{\bar{n}}-\overline{\mathcal{Y}}^{\nu}_{(pp')J}\overline{Y}^{m}_{(\bar{p}\bar{n})J}\bar{v}_{p'}\bar{v}_{\bar{n}}\right)\right]\times\right.$$

$$\left.\times\left[\sqrt{12}\sum_{pn}G^J_{(pn)J}(k)\left(X^{m'}_{(pn)J}u_pv_n-Y^{m'}_{(pn)J}v_pu_n\right)\right]\right\}\quad. \tag{27}$$

In eqs. (26) and (27) we involved a summation over both sets of the intermediate states $|mJM\rangle$ and $|m'JM\rangle$ generated independently from the parent (A,Z) nucleus and from the daughter $(A,Z+2)$ nucleus, respectively. They fulfil the orthogonality relation only approximately, so we are forced to introduce overlaps $\langle mJM|m'JM\rangle$ into the final expressions for the amplitude (9). In addition, the procedure introduces an uncertainty in the energy denominator and we decided to adopt an average value of the QRPA energies E^J_m and $E^J_{m'}$ instead of one of them. In general, this uncertainty is small however, compared to the mean value of the denominator.

4. SUMMARY AND CONCLUSIONS

In this paper we have generalized the formulae for the DCX transitions to any excited state of the final nucleus. As in the previous papers we used the proton-neutron, proton-proton and neutron-neutron QRPA to describe consistently both excitation modes (charge no — and — changing ones) appearing in the reaction model. The expressions obtained are in agreement with the previously derived result for the final double isobaric analogue [1] state if one assumes the angular momentum of the excited state in eqs. (24) and (25) equals only to zero.

The generalized result will be applied to the iron and nickel isotopes in which complete data for three types of transitions exist [6]. Because different possible routes influence on the DCX cross section [2,9,10] depending on the type of final state (GS, DIAS or another excited), one can expect to have in such calculations a very promising way to study various short-range correlation effects between nucleons in the nucleus. Such a project is being developed now and results will be published elsewhere.

This work was supported by the grant from the Polish Ministry of National Education.

REFERENCES

1. Kamiński W.A., Faessler Amand, *Phys. Lett.*, B 244 (1990), 255.
2. Kamiński W. A., Faessler Amand, *Nucl. Phys.*, A 529 (1991), 605.
3. Kamiński W. A., Faessler Amand, *J. Phys.* G, in press.
4. Kamiński W. A., *Annales UMCS*, AAA 43/44 (1990), 131.
5. Lane A. M., *Nuclear Theory*, Benjamin, New York–Amsterdam 1964.
6. H. Clement, private information.
7. Leitch M. J., in *Fundamental Symmetries and Nuclear Structure*, World Scientific, 1989, 163.
8. Ring P., Schuck P., *The Nuclear Many-Body Problem*, Springer Verlag, New York–Heidelberg–Bonn 1980.
9. Auerbach N., at al., *Phys. Rev.*, C 38 (1988), 1277.
10. Haider Q., Liu L. C., *Z. Phys.*, A 335 (1990) 437.
11. Clement H., PSI–LAMPF–Tübingen–Karlsruhe collaboration, private information.

STRESZCZENIE

W pracy uogólniono wyrażenia na przekrój czynny w reakcjach podwójnej wymiany ładunku z pionami. Rozważono przypadek, w którym jądro atomowe zostało wzbudzone w trakcie procesu do któregoś z jego stanów energetycznych. Przytoczono formuły na różniczkowe przekroje czynne jako funkcje kąta rozpraszania θ dla rozpraszania w fali s i w fali p.