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The Non-relativistic Quark Model for the M1 and E2 Electromagnetic Transition* in the Process $\Delta \to N\gamma$

Nierelatywistyczny model kwarkowy dla przejść elektromagnetycznych typu M1 i E2 w procesach $\Delta \to N\gamma$

There are many theoretical and experimental papers concerning the electric quadrupole (E2) and magnetic dipole (M1) transition amplitude in the process $\Delta \to N\gamma$ [1]. The ratio δ (E2/M1) may be a test for the models of hadron structure. Now it is the common opinion that the quark model is the best model describing structure of hadrons.

Our basic assumption is that baryons, consisting of three quarks, can be deformed. The possibility of a deformation of baryons, even in their ground state configurations has been already considered by other authors [2]. In the simple SU(6) quark model the E2 amplitude vanishes [3], but experiments indicate non zero δ(E2/M1) ratio [4]. Our model predicts M1 as well E2 transition in the process $\Delta \to N\gamma$.

In our previous work [5] we developed the generator coordinate method to deal with deformed three quarks system. The main problem was to construct an appropriate wave function describing deformed baryons within SU(6) symmetry. The expression for that function one can find in [5]. In present paper making use of generator function constructed in [5] we have calculated M1 and E2 amplitudes as a function of deformation to compare them with experimental data.

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The standard operators responsible for the M1 and E2 transitions can be written as:

$$\mathcal{M}(\mathrm{M1},\mu) = \sum_{k=1}^{3} \mu_{\mathrm{B}}(\mathrm{T}_3 + \frac{1}{6})^{(k)}(2s_\mu^{(k)} + 1_\mu^{(k)}) \nabla_{(k)} (r_{(k)}Y_{1\mu}(\theta_{(k)},\varphi_{(k)})), \quad (1)$$

$$\mathcal{M}(\mathrm{E},\mu) = \frac{5!!}{3q^2} \int \rho(\vec{r})\frac{\delta}{\delta r} - (rj_2(qr))Y_{2\mu}(\theta,\varphi)dV +$$

$$+ i\frac{5!!}{3cq^2}\int (\vec{r}\cdot\vec{j}(\vec{r}))j_2(qr)Y_{2\mu}(\theta,\varphi)\mathrm{d}V \quad (2)$$

where μ_{B} denotes the Bohr magneton, T_3 is the isospin third component operator, $s_\mu^{(k)}$ and l_μ^k are components of spin and angular momentum of k–th quark, q is the energy of emitted photon, $j_2(gr)$ denotes the spherical Bessel function, $\rho(\vec{r})$ and $j(\vec{r})$ are density and current of quarks in the baryon.

In the quark model based on the SU(3) flavour symmetry only second term in (2) contributes to the transition amplitude in $\Delta \to N\gamma$ process because the only operator which can change spin 3/2 of Δ to 1/2 of N is contained in the current density $\vec{j}(\vec{r})$ operator, which has the following form:

$$\vec{j}(\vec{r}) = \sum_k (T_3 + \frac{1}{6})^k \frac{1}{2}[\vec{v}_k\delta(\vec{r}-\vec{r}_k) + \delta(\vec{r}-\vec{r}_k)\vec{v}_k] + \frac{e\hbar}{2m}\sum_k \nabla\times\vec{s}^{(k)}\delta(\vec{r}-\vec{r}_k),$$

where $\vec{v}_k$ is a velocity of a k–th quark and e and m denote the elementary charge and the quark mass, respectively.

Consequently we can write approximately (for $qr < 1$) $\mathcal{M}(\mathrm{E2})$ operator, which gives a nonzero amplitude to the process $\Delta \to N\gamma$, in the form:

$$\mathcal{M}(\mathrm{E2},\mu) = -\frac{2\mu_{\mathrm{B}}}{3c\hbar^2}q\sum_{k=1}^{3} r_k^2 Y_{2\mu}(\theta_k,\varphi_k)(T_3 + \frac{1}{6})^{(k)}\vec{s}^{(k)}\ldots\vec{l}^{(k)}.$$

The reduced transition probability is given by the standard formula [6]

$$B(\mathrm{E(M)}\lambda; JTM_{\mathrm{T}} \to J'T'M') =$$

$$= \sum_{\mu M'} |\langle J'M'T'M'|\mathcal{M}(\mathrm{E(M)}\lambda,\mu)|JMTM_{\mathrm{T}}\rangle|^2 \quad (3)$$

where $|JMTM_{\mathrm{T}}\rangle$ are states (16) from the previous paper [5].

For the M1 and E2 transition rates one can write the expressions

$$T(\mathrm{M1}) = 1.76\cdot 10^{13}q^3[\mathrm{MeV}]\cdot \mathrm{B(M1)} \quad (4)$$

$$T(\mathrm{M2}) = 1.22 \cdot 10^9 q^5 [\mathrm{MeV}] B(\mathrm{E2}) \tag{5}$$

To remove the center of mass (CM) motion we have to express the transition matrix elements in relative and CM coordinates. The appropriate factorization of states is given in [5]. The operators can be factorized into the relative and CM parts making use of standard relations:

$$\vec{r} = \vec{\rho} + \vec{R}^{\mathrm{CM}}, \bar{\vec{l}} = \vec{l} + \vec{L}^{\mathrm{CM}}$$

and

$$r^2 Y_{2\mu}(\theta,\varphi) = \sqrt{\frac{15}{8\pi}} [\vec{r} \times \vec{r}]^2_{\mu}, \tag{6}$$

where we have denoted by: $\vec{r}$ — position of a quark in lab (the laboratory frame), $\vec{R}^{\mathrm{CM}}$ — position of CM in lab, $\bar{\vec{l}}$ — angular momentum in lab, $\vec{l}$ — relative angular momentum and $\vec{L}^{\mathrm{CM}}$ — angular momentum of CM.

Using of the equations (6) and applying the method that was used in the previous work to obtain matrix element (6) in [5], all required matrix elements between relative states we can express in laboratory and CM coordinates. One needs also to remember that the single particle state of k-th quark $|\phi_0^{(k)}(\lambda)\rangle$ and $|\mathrm{CM}\rangle$ are of Gaussian shape with the width equal to b and $\sqrt{\frac{2}{3}}b \equiv b_{\mathrm{CM}}$, respectively. After some involved but straightforward algebra we can obtain the required matrix elements within the deformed states:

$$\langle\lambda|[\vec{\rho} \times \vec{\rho}]^2_{\mu_1} R_Y\,|\lambda\rangle = (b^2 - b^2_{\mathrm{CM}})\,\langle\phi_0^{(3)}(\lambda)\,|R_Y^{(3)}|\phi_0^{(3)}(\lambda)\rangle \times$$

$$\times \langle\phi_0^{(3)}(\lambda)|[\frac{\vec{r}}{b} \times \frac{\vec{r}}{b}]^2_{\mu_1}(3)\vec{l}^{(3)}_{\mu} R_Y^{(3)}|\phi_0^{(3)}(\lambda)\rangle - (b^2 - b^2_{\mathrm{CM}})\langle\mathrm{CM}|L^{\mathrm{CM}}_{\mu} R_Y^{\mathrm{CM}}|\mathrm{CM}\rangle \times$$

$$\times \langle\phi_0^{(3)}(\lambda)|[\frac{\vec{r}}{b} \times \frac{\vec{r}}{b}]^2_{\mu_1}(3)\, R_Y^{(3)}|\phi_0^{(3)}(\lambda)\rangle \tag{7}$$

and

$$\langle\lambda|\mathcal{G}^+(\theta)R^+(\Omega)[T_3 + \frac{1}{6}]^{(k)} l_3^{(k)} R(\Omega')\mathcal{G}(\Omega')\,|\lambda\rangle =$$

$$= \frac{\langle\phi_0(\lambda)|\mathcal{G}^+ \overline{R}^+(\Omega)[T_3 + \frac{1}{6}]^{(k)}[l^{(k)} - \frac{1}{3}L^{\mathrm{CM}}]\overline{R}(\Omega')\mathcal{G}(\Omega')\,|\phi_0(\lambda)\rangle}{\langle\mathrm{CM}|R^{\mathrm{CM}}(\Omega^{-1}\Omega')|\mathrm{CM}\rangle} \tag{8}$$

In eq. (8) ~ denotes the rotational operator in laboratory frame which can be factorized into relative R and CM R^{CM} operators, the index Y means rotation around y axis, $\mathcal{G}$ stands for the isospin transformation, T_3 is the third component isospin operator and $\phi_0(\lambda)$ ($|\lambda\rangle$) is the internal generating function in lab (relative) coordinates given in [5] by eq. (12). Expressions

(7) and (8) are these matrix elements, which we have to put in eq. (3) in order to remove the center of mass motion.

We summarize the results obtained in our model in the figure, where we have plotted the ratio δ (E2/M1) of electric quadrupole to magnetic dipole transition amplitudes in the process $\Delta \to p + \gamma$ defined as follows:

$$\delta(\mathrm{E2/M1}) = \frac{\sqrt{3}}{10} q \frac{\langle J_2 \parallel \mathcal{M}(\mathrm{E2}) \parallel J_1 \rangle}{\langle J_2 \parallel \mathcal{M}(\mathrm{M2}) \parallel J_1 \rangle}, \tag{9}$$

where $\langle \parallel \parallel \rangle$ denotes the reduced matrix element. The deformation parameter (defined in the paper [5]) λ is positive for prolate shapes and negative for oblate shapes. The experimental value for δ (E2/M1) is $(-1.3 \pm 0.5)\%$ [4].

One can see from this figure, that this experimental value one can get for the elongation ratio of "z" to "x" axes of deformed baryons e^λ =1.2–1.3. The prolate shape of Δ and p is approximately consistent with that one obtained from minimum of energy with respect to deformation [5].

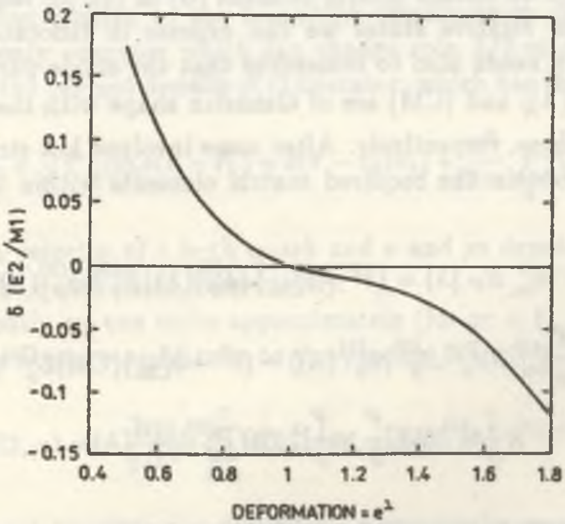

Fig. 1. In the figure is plotted the E2 to M1 transition amplitudes ratio in $\Delta \to N\gamma$ decay as a function of a common delta and nucleon deformation

We can add that magnetic dipol moments of delta and nucleon in our calculations also do not depend on deformation and are the same as in nondeformed quark model.

This preliminary result is only a hint that the hypothesis of deformation of baryon states should be treated more seriously and should be analysed carefully in more realistic models.

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STRESZCZENIE

W pracy znaleziono zależność pomiędzy deformacją rezonansu Δ i nukleonu a stosunkiem amplitud E2/M1 w reakcji $\Delta \rightarrow N\gamma$. Porównanie otrzymanych rezultatów z dostępnymi danymi eksperymentalnymi wskazuje na istnienie statycznej deformacji w obu barionach.

(7) and (8) are those ... [illegible] ... we have to put in eq. (2) in order to remove the center of mass motion.

[illegible] ... have plotted the ratio ... [illegible]

where ... [illegible] ... oblate shapes. The experimental value for ... [illegible]

One can see from this figure, that this experimental value can not ... for the elongation ratio of ... [illegible] ... = 1.3–1.5. The prolate shape of Δ and ... is approximately consistent with that ... [illegible]

Fig. 1. In the figure is plotted ... [illegible] ... as a function of ... [illegible] ... without ... [illegible]

We can add that ... [illegible] ... in any calculations also do not depend on ... [illegible] ... the same as in ... [illegible] model.

This preliminary result is only a hint that the hypothesis of deformation of baryon states should be treated more seriously and should be analyzed carefully in more realistic models.