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Dynamics of a Field-Sail Spaceship

ABSTRACT - The equation of motion of a spaceship powered by a field sail driven by an external particle beam is carried out. Although dynamics is independent of the particular field and beam, field-sail is meant to be a magnetic sail pushed by a plasma launched from an orbital device. Relativistic dynamics is considered. Major purposes of this paper are: (1) to compare the field-sail mode with the pure-rocket mode, (2) to demonstrate that the field-sail mode is equivalent, from a dynamics viewpoint, to either a photon-sail propulsion mode or a ram-braking mode. Numerical results are discussed.

1 INTRODUCTION

In the course of the last two decades an impressively high number of propulsion system concepts have been proposed and analysed not only for interplanetary missions, but also for interstellar flight. One could group most basic concepts into three main categories or propulsion modes from a conceptual viewpoint, namely, without considering the enormous technological differences between the proposed propulsion devices: (a) the pure-rocket mode, (b) the photon-sail mode, (c) the ramjet mode. A fourth mode, that one may call a field-sail mode (although the effects of interest develop in a volume, not on a surface), would consist of a spaceship generating a low-magnitude extended magnetic field around its body, for instance by means of superconducting loops; charged particles entering the field from a "side" change their momentum so that a net momentum is ultimately transferred to the ship which the loops are anchored to. Depending on the direction of the relative velocity between the ship and the external particle beam, the space vehicle can acquire or loss energy. In Ref.-1 is is suggested that the Alfven Engine might be suitable for decelerating a starship instead of a rocket: a number of very thin conducting loops, spaced apart and arranged in a sort of sail, may generate a drag with the interstellar plasma through the produced magnetic field. In Ref.-2 an independent quantitative analysis of this problem is issued by considering one large superconducting wire about the ship body; the ship is pushed by the solar wind particles interacting with the loop's magnetic field. A significant saving of propellant may be achieved by means of a magnetic sail, especially used as a decelerating device: the ship's kinetic energy would be transferred to the particles of the interstellar medium in large volumes about the moving ship body.

It is possible to envisage a ship endowed with two or more propulsion modes working either simultaneously or sequentially. In ref.-3 a unified picture of the powered spaceship motion is presented. One of the problems to be dealt with is to characterise the fundamental propulsion modes. In other words, one has to model those propulsion modes which are conceptually different from a dynamics point of view; two propulsion modes are basically different when the vehicle's mass and vector velocity histories (over a finite interval of the ship proper time) cannot be made equal for both modes.¹ For it is important to investigate whether the field sail mode, in particular, may be made dynamically equivalent to some other mode. This is also of interest when a numerical code is implemented for computing the trajectory profiles coming from one or more propulsion modes.

In this paper we are interested in the dynamics of a field sail and we do not enter technological problems but those relevant to the motion equation. We first carry out the ship motion equation by means of the following general picture: a plasma beam (for instance, a high-intensity high-energy hydrogen plasma) is emitted from a *controllable* source orbiting a planet; it enters the "interaction volume" of the field sail after travelling a certain distance; the effect of the interaction between this plasma and the ship's magnetic field is a momentum change of the vehicle.

¹ For example, the velocity and mass profiles of a pure-rocket can in no way be made equal to those ones of a pure solar-sail ship, especially when a three-dimensional trajectory is considered.

2 BASIC ASSUMPTIONS AND PRELIMINARY CALCULATIONS

Ignoring details of the plasma source technology, let us focus our attention on the interaction between plasma and sail in terms of dynamical output. For we picture a beam of collimated particles entering a three-dimensional interaction box representing the effective field volume. After a non-zero time a fraction of the incoming particles is reflected by the field at a certain angle, while the remaining fraction is transmitted through the box at a generally different angle (Fig. 1). We make the following assumptions:

- (a) no particles are produced, destroyed and/or captured by the vehicle during the interaction between beam and field;
- (b) although the ship velocity in the Galactic Frame (GF), say, V is three-dimensional, however the plasma beam velocity in GF (Vb) is parallel to V; therefore, the beam velocity in the Ship Frame (SF) ² is ν ; $-(\nu, -\nu)\nu(1-\nu, \nu)$ ³ (the superscript 's' denotes a ship frame quantity hereafter);
- (c) both reflection and transmission of the beam particles develop in a cylindrical symmetry about the ship velocity V;
- (d) transmitted and reflected particles have the same value of speed;
- (e) the reflected beam does not interact with the incoming beam;
- (f) in SF the incoming particles enter the interaction box and lag a time τ_i the interaction time before exiting;
- (h) the beam is assumed to be all focused on the sail by some space-born focusing device which does not change the beam energy.

We can consider the surface of the interaction box as a *control surface*. In SF, from a generic time τ and over $d\tau$, the four-momentum entering the control surface amounts to:

(1)
$$\Delta P_b^{s,\xi} = V_b^{s,\xi} m_b \Delta N_b$$

² Both GF and SF are defined in the usual way Relativity adopts; therefore, we do not repeat such definitions here.

³ Throughout this paper we use normalised units, namely, the speed of light is set to unity, the time unit is the sidereal year and the ship's initial mass is set to one. All other units of interest follow. We use MKSA units on explicit specification.



where the superscript ξ ranges from 1 to 4, the first three components referring to as the space-like part 4. ΔN_s is the number of incoming plasma particles corresponding to the SF and GF time intervals:

(2)
$$\Delta \tau = \Delta t / \gamma = \Delta t_{em} \sqrt{1 - V^2 V_b} / (V_b - V)$$

where ydenotes the Lorentz factor related to the ship speed V. The equality on the left in (2) is obvious. The equality on the right requires a short proof. In fact, the number of particles emitted from the plasma source during the (arbitrary) GF time $\exists t_{sh}$ is expressed, according to assumption (g), by:

(3)
$$\Delta N_b = \int_{V_{b,min}}^{V_{b,max}} F_b dV_b \Delta t_{em}$$

where the integration interval extends to the speed limits which the particles are emitted with. Let D(t) be the distance between the sail and the plasma source at time t when the fastest particles, out of those ones emitted at t_{em} , reach the sail; then, all particles given by Eq.-3 strike the field sail in a GF time interval utdilated, with respect to u_{em} , by both the relative speed of the beam with respect to the ship and the emission speed spread. By means of the world lines of the beam source, travelling beam and ship it is easy to carry out

(4)
$$\Delta t = \frac{(1 - V_{b, \min} / V_{b, \max}) D(t) + V_{b, \min} - It_{em}}{V_{b, \min} - V}$$

Without entering technological considerations that would go well beyond the purposes of this paper, we further assume that both the plasma source is sufficiently monochromatic and the acceleration ship time sufficiently short that the space term of Eq.-4 is negligible. Then, the rightmost equality (2) follows. In addition, the function Fb reduces to the number flow rate:

$$(3a) \qquad \Delta N_b = F_b(V_b) \ \Delta t_{em}$$

With regard to the exiting particles, over the interval $\exists \tau$ starting at time $\tau - \tau$ the following four-momentum exits the control volume:

(5)
$$\Delta P_o^{s,\xi} = m_b \Delta N_b [(1-\lambda)V_l^{s,\xi} + \lambda V_r^{s,\xi}]$$

where the subscripts 'e', 't' and 'r' refer to as exiting, transmitted and reflected particles, respectively (see Fig.-1). The sail reflection coefficient has been denoted by λ

⁴ We assume a metrics with the following signature [-1 -1 -1 +1]; thus, the invariant magnitude of any four-velocity is +1.

3 THE FIELD SAIL MOTION EQUATION

As a consequence of the beam-sail interaction, the whole ship will experience a four-momentum change, we denote by $aP^{t,t}$, over dr. The four-momentum conservation law can be then written by recalling Eq.-1 and assumptions (e,h) ⁵:

(6)
$$-V_b^{s,\xi} + \lambda V_r^{s,\xi} + (1-\lambda)V_t^{s,\xi} + \Delta P^{s,\xi}/(m_b \Delta N_b) = 0$$

Combining Eqs. 2 and 3a, one gets:

(7)
$$m_b \Delta N_b = (m_b F_b) \frac{V_b - V}{V_b \sqrt{1 - V^2}} \Delta \tau$$

$$\equiv \phi_b^s \Delta \tau$$

Thus, the coefficient of the proper time in Eq.-7 is the beam mass flow rate as observed in SF. Note how it decreases as V increases.

By inserting Eq.-7 into Eq.-6, noting that $\Delta P^{3,4}=0$ by assumption (a) and recalling assumption (d), we can make the *time-like* part of Eq.-6 explicit as follows:

(8)
$$\gamma_e^s|_{\tau+\tau_i+\chi} - \gamma_b^s|_{\tau+\chi} = 0$$
 $\chi = [0, \Delta \tau]$

that is, in the current environments a *finite* interaction time does not alter the magnitude of the interacting-particle velocity. In practice, the value of τ could take on few milliseconds, at most, even for a very large interaction box. Equation 8 tells us that no energy is ultimately absorbed in SF.

With regard to the space-like part of Eq.-6, we use assumption (c) also. Introducing the three-dimensional unit vector $\underline{u} - \underline{v}_{1}^{\prime}/\nu_{1}^{2}$ and the reflection and transmission angles (Fig.-1), we carry out the equation of the axial three-momentum balance as follows:

⁵ considering any interaction over a finite time interval, one can always write down the energy-momentum conservation law by comparing quantities at the beginning and the end of the interaction interval, ignoring the details of the interaction fields.

(9)
$$\Delta \underline{P}^* = - \Phi_b^s \Delta \tau [(\lambda \cos \alpha_r +$$

 $(1-\lambda)\cos\alpha_i P_{e,m}^s - P_{b,m}^s]$

where:

(9a)
$$P_{e,m}^s = \gamma_e^s V_e^s u = P_{b,m}^s$$

(9b) $\underline{P}_{b,m}^{s} = \gamma_{b}^{s} V_{b}^{s} \underline{u}$

stand for the SF axial momenta per mass unit of the exiting and incoming particles, respectively. Equation 9 represents the momentum balance in the non-inertial ship-frame. The quantity $\Delta E^{r}/M \, dr$, where M is the ship's rest mass, is sensed onboard as a three-dimensional inertial acceleration. Equation 9 is to be transformed to GF in order to get the ship motion equation. Such a transformation, applied indeed to the ship four-momentum, is performed by the well-known four-dimensional *boost*. say, *(*)*, of Special Relativity as follows:

(10)
$$\Delta \begin{bmatrix} (\gamma \underline{V} M) \\ (\gamma M) \end{bmatrix} = \Delta P^{\xi} =$$
$$\Gamma^{\xi}{}_{\beta} \Delta P^{s,\beta} = \begin{bmatrix} \Xi & (\gamma \underline{V}) \\ (\gamma \underline{V}^{T}) & \gamma \end{bmatrix} \begin{bmatrix} (\Delta \underline{P}^{s}) \\ 0 \end{bmatrix}$$

where the *boost* has been partitioned ⁶. By inserting Eqs.-9 into Eq.-10 and expanding the leftmost term, one can make the space-like and time-like parts explicit. A remark is in order. Although assumption (b) entails that the ship motion is rectilinear, however a three-dimensional calculation has been made in order to keep the ship velocity at any orientation in the galactic inertial frame. That forces us to consider *first* the time-like part of the motion equation and *then* substitute it into the space-like component. The former gives the behaviour of the velocity magnitude, whereas the latter one furnishes the behaviour of the three-dimensional velocity. We are going to report the final results:

(11)
$$dV = \frac{\mu \Phi_b \gamma_b}{M V_b} (V_b - V)^2 d\tau$$

⁶ The entries of the boost can be found in any good text about Relativity; therefore, we do not repeat them here. We only cite the following property: $\exists \underline{\nu} = y\underline{\nu}$ that we use in the SF-to-GF transformation.

(12)
$$d\underline{V} = \frac{\mu \Phi_b \gamma_b}{M V_b} (V_b - V)^2 \frac{V}{V} d\tau$$

where we have denoted the plasma source mass flow rate by *, and defined the coefficient μ as follows:

$$\mu = 1 - [\lambda \cos \alpha_r + (1 - \lambda) \cos \alpha_t]$$
$$\pi/2 \le \alpha_r \le \pi \quad 0 \le \alpha_t \le \pi/2$$

it may be identified as the *effective* coefficient of the field sail for changing the ship momentum. Which values of the transmission and reflection angles translate the spread of the exiting particles depends on the sail's field structure and the (time-varying) relative momentum of the incoming beam.

Equation 11 is easily integrated to give:

(13)
$$V(\tau) = V_b - \left(\rho \frac{\gamma_b}{V_b} \tau + \frac{1}{V_b - V_0}\right)^2$$
$$\rho = \mu \phi_b / M$$

Equation 13 represents the motion equation of a field sail spaceship. It could be carried out more rapidly by equalling the proper acceleration to the relativistic dynamic pressure on the sail. However, we have preferred to show all the way starting from the underlying assumptions down to the motion equation through the balance of four-momenta. The current proof is *strict*.

What one has to specify to get the speed profile are *two* parameters of the spaceship (mass and effective sail coefficient) and *two* parameters of the beam source (either the rest-mass flow rate and the beam particle Lorentz factor or the total-mass flow rate and the beam speed 7). Note that the speed values depend on the mass-flow-rate on ship-mass ratio, namely, on how much mass the source releases *relatively* to the mass (of the ship) to be accelerated. The spaceship's speed gain is always asymptotically limited by v_{s} - v_{o} : the speed gain $v - v_{o}$ decreases as v_{o} increases (standing the other quantities).

⁷ Remember that $v_{11} = \sqrt{1 - 1/r_{11}^2}$ by definition.

4 COMPARISON BETWEEN FIELD-SAIL MODE AND ROCKET-MODE

It is interesting to compare the performances of the field-sail propulsion mode and the pure-rocket mode. We are going to define the terms of comparison:

- 1: both ships start with the same speed;
- 2: the field-sail mode is characterised by a beam source and a sail ship with the parameters described above; in particular, the ship mass is thought to be composed of a net payload and the field-sail system: where the thrusting lasts r in SF;
- 3: the rocket-mode is accomplished with an effective exhaust speed v_{\star} ; the ship mass rate equals $w_{\star} w_{\star}$; the initial rocket mass amounts to $M_0 M_{\star} M_{\star} M_{\star}$; the last term accounts for the mass of the propulsion system, whereas the first two ones fix the net payload and the propellant to be exhausted away, respectively.

The above points aim at comparing a pure-rocket mode with a sail-mode conceived by envisaging of stripping the rocket-ship of both its rocket engines and propellant, but adding a field-generating sail system weighing as well as the engines. The original rocket propulsion system is then transformed into a plasma source system (with increased mass, if it is needed) orbiting a planet; it launches an equal amount of propellant at the same original ejection speed for an equal time interval (in SF). Such a "transformation" does not change the mass of the delivered net payload. Then, at least to within our current framework, the question about the propulsion performance comparison reduces to: which thrusting mode exhibits the highest final speed ?

According to points 1 through 3 the rocket-mode and field-sail mode final speeds are expressed, respectively, as follows:

(14)
$$V_{R} = \tanh \left[\tanh^{-1} V_{0} + V_{b} \ln \frac{M_{0}}{M_{0} - \gamma_{b} \phi_{b} \tau} \right]$$

(15)
$$V_{FS} = V_b - \left[\mu \frac{\gamma_b \phi_b}{\left(M_b + M_w\right) V_b} \tau + \frac{1}{V_b - V_0} \right]^2$$

where the initial rocket mass is given in point 3. Figure 2 shows the final speeds as function of the SF time and for different values of the initial speed. The following parameters have been considered:

(16) $M_0 = 32$ $M_L = 10$ $M_W = M_L$ $\tau_f = 10$ $\mu = 1.8$ $\gamma_b = 1.2$ $\phi_b \tau = \Lambda_L$



G. Vulpetti

382

The above values are meant to emphasise a large payload fraction (> 30%) and an acceleration to achieve more than 20 percent the speed of light after 10 years. The plasma beam Lorentz factor corresponds to a particle kinetic energy of about 190 MeV for a hydrogen plasma. The related speed, about 0.55, is compatible with the effective exhaust speeds of currently envisaged beam-core matter-antimatter annihilation rocket engines [4], according to point 3. The first feature, which is apparent in Fig.-2, of the field-sail dynamics is to exhibit a speed curvature opposite to the rocket's; a crossing point is eventually achieved as the propulsion time increases. Such point is shifted to lower speed values for higher initial speeds of the ship. Thus, the plasma-driven field-sail mode appears to be favourable with respect to the high-performance pure-rocket for accelerating the first stage of a multi-staged starship. Figure 2 shows that there is an *optimum* acceleration time for which, namely, the field sail provides the maximum gain of speed with respect to the rocket's. Setting V0=0 and ignoring the hyperbolic tangent operator in the rocket equation it is possible to carry out an approximate value of the acceleration time:

(17)
$$\tau_{accel} = \sqrt{M_{L} + M_{W}} \frac{\sqrt{4\mu M_{0} + 5(M_{L} + M_{W})} - 3\sqrt{M_{L} + M_{W}}}{2\mu \Phi_{b} \gamma_{b}}$$

Inserting the values (16) into Eq. 17, the acceleration time takes on 4.93 yr. In contrast, the numerical solution of the actual difference of speed between rocket and sail modes gives 4.47 yr. (That is not excessively significant because the maximum is somewhat broad). The corresponding final speed amounts to 0.18. Such a starship would therefore exhibit a really high payload fraction, a not too long propulsion time and a significant final speed. Were the deceleration time equal to the acceleration's, then a flight to the Barnard star would last 36.7 yr in SF (about 37.3 yr in GF).

5 EQUIVALENCE BETWEEN PHOTON MODE AND FIELD SAIL MODE

Let us consider Eq.-13 again. If Vb approaches 1, then where W is recognised to express a photon power constant in GF. In addition, $\alpha_i \rightarrow 0$ $\alpha \rightarrow \pi$ because the field-sail transforms into a material sail to reflect photons; thus, $\mu \rightarrow 2\lambda$. Then, Eq.-13 results in

(18)
$$V(\tau) = 1 - \left(\frac{2\lambda W}{M}\tau + \frac{1}{1 - V_0}\right)^{-1}$$

Equation 18 describes the relativistic speed profile of a photon-sail ship pushed by a (collimated) beam originated from a GF source of constant power W, as one can easily verify by realising that the power, say, L in SF equals W(1-V)/(1+V). According to the current sail model, the amount $(1-\lambda)W$ is transmitted through. There is a number of realistic options regarding the power not directly reflected by a photon sail [3]; however, this would bring us beyond the purposes of the current paper. To within the current framework, we can make the field-sail and photon sail speed profiles equal to one another by controlling the source of light according to the following equation:

(19)
$$W' = \frac{\mu}{2\lambda} \frac{M\Phi_{b}\gamma_{b}(V_{b} - V_{0})^{2}}{(1 - V_{0})[MV_{b}(1 - V_{0}) + \mu\Phi_{b}\gamma_{b}(V_{b} - V_{0})(1 - V_{b})\tau]}$$

Equation 19 can be read in two ways:

- 1--if one is interested in getting equal speeds at some particular instant, say, Y then Eq. 19 provides the *constant* power W(Y) of the light source. The speed profiles for times less than Y are not equal for the two modes;
- 2--if one wants to obtain the two speed profiles identical at any instant, then the photon power has to be *modulated* according to Eq. 19.

In the particular case of V0=0, that has been noted in sect. 4 to be the most efficient for a sail mode, one gets the following significant equation:

(20)
$$W = \frac{\mu}{2\lambda} \frac{M \phi_b V_b \gamma_b}{\left[M + \mu \phi_b \gamma_b (1 - V_b)\tau\right]}$$
$$= \frac{\mu}{2\lambda} \frac{P_b^*}{\left[1 + \mu (E_b^* - P_b^*)\tau/M\right]}$$

where the superscript * denotes differentiation with respect to GF time. Thus, the source power of the *equivalent* photon mode depends, in particular, on both the momentum and energy rates of the plasma source. Equation 20 also expresses that the photon-sail mode is more efficient than the field-sail mode. In fact, the photon power in GF, although starting at the same level of the plasma source intensity in GF, is to be decreased with time in order to get the same speed of the field sail ship ⁸. Equation 20 can be cast into a different form by means of Eq.-13 considered at some desired speed Vf:

(20a)
$$W = \frac{\mu}{2\lambda} \phi_b \gamma_b (V_b - V_f) / (1 - V_f)$$

One should not be mistaken when Vf approaches Vb; in fact, the thrusting time diverges in that case.

To summarise this section, Eq.-19 clearly establishes the equivalence between a field-sail mode and a photon-sail mode; in addition, the latter one represents the upper limit of the former from a ship dynamics viewpoint.

⁸ This feature may be viewed as the counterpart of the pure rocket one: standing the same mass-into-energy conversion rate, the photon rocket exhibits the highest exhaust speed and thrust out of all possible rocket jets.

6 EQUIVALENCE BETWEEN RAM MODE AND FIELD SAIL MODE

In refs. [1-2] it is pointed out that a magnetic sail may find a high performance utilisation as decelerating stage instead of a rocket stage. In fact, a field-sail might be activated to interact with a plasma beam for accelerating the starship, switched off during a long coasting and, finally, activated again to reflect the interstellar plasma particles in order to decelerate. If we denote the *effective* sail surface by S⁹ and the interstellar medium density by ρ_m , the sail sees a flux of mass coming from ahead equal to $\gamma V \rho_m S^{10}$. One then writes down the four-momentum conservation in SF and, finally, transforms it into GF. One thus obtains the following equation:

(21)
$$V(\tau) = \left[\mu \rho_m S \tau / M + 1 / V_0\right]^{-1}, \quad V_0 \ge V$$

Equation 21 can be directly got from Eq.-13 by the following substitutions (in the order):

 $V_{b} \rightarrow -V_{b}, \quad V_{b} \rightarrow 0$ $\lim_{V_{b} \rightarrow 0} \Phi_{b} \gamma_{b} / V_{b} = S_{b} \rho_{b}$ $S_{b} \rho_{b} \rightarrow \rho_{m} S$

where ρ_{s} . s, denote the density and the emission surface of the plasma source, respectively. The above steps transform a sail accelerated by particle beam into a sail decelerated by drag.

In Ref.-3 the general ramjet-mode equation has been carried out under assumptions regarding onboard interaction, loss and trapping of particles. By removing the jet component, retaining the particle reflection terms and considering the one-dimensional case, one obtains Eq.-21. Because the vehicle mass is constant, the equivalence between ram braking mode and field sail deceleration mode is proved. Figure 3 shows Eq.-21 for a deceleration starting from V0=0.2 and different S/M ratios. The deceleration is very slow because of the very low density of the interstellar plasma. Even in the case of ten protons per cubic centimetres a high value such as S/M = 50 Km^2/tonne would require about 35 years to reduce the final speed to one hundreth of the initial one. In general, the SF time necessary to brake the ship from V0 to k V0 (k < 1) is expressed by:

⁹ The effective area may include field extensions depending on the particle energy; therefore, it may be larger than the geometric area of the field-generating loop. See Ref.-2 for examples.

¹⁰ In sects. 3-5 plasma beam and photon beam were supposed to be collimated toward the magnetic and photon sails. respectively. For this reason the effective area of such sails do not enter the field-sail and photon-sail motion equations explicitly. In the ramjet case, the effective area explicitly appears because the incoming flux is by effect of the ship motion. It is plain that such formal differences are of no matter for the starship dynamics: only the received flux intervenes in the momentum exchange, of course. In contrast. from a technology viewpoint, the plasma collimation device will depend on the sail size.

(22)
$$\tau_{brake}(k) = \frac{1-k}{A \, k \, V_0}$$

where A 11 denotes the coefficient of the SF time in Eq.-21.



¹¹ If we express the medium density and S/M in the same units of Fig.-3, than we must multiply by 63.2 to get the SF time of Eq.-22 in years.

7 ADDITIONAL NUMERICAL RESULTS

In the previous sections we considered the field-sail motion relatively to one phase of flight, namely, either acceleration or deceleration. We shortly investigate how the field sail mode behaves in a two-boundary flight and with another propulsion mode too. For we examine a flight to Proxima Centauri.

A probe is accelerated by means of a magnetic-sail stage up to a certain speed to be optimised; then, the plasma source and the superconducting loops are deactivated. The ship coasts for a number of years; this phase gets most of the path to the target. Then, the superconducting coils are re-energised. The interaction between sail and interstellar plasma is such that the ensuing phase is actually a slow deceleration phase. This phase, which can be longer in time than the true coasting, is devoted mainly to lose speed as much as possible. In fact, because of its inefficiency at low speeds, the sail is jettisoned at a certain velocity and a retiorocket is switched on for inserting the payload into the target star system. The problem is to find the dynamical profile which minimises the flight time in SF, while the final mass is at a prefixed value. The trajectory is a straight line in the three-dimensional space. The control parameters are assumed to be the source plasma flow rate and the time intervals of the coasting and thrustings but the last one. In fact, in order to keep the payload high, we fix the time of the phase which consumes ship mass. Two equality constraints play a major role; the flight path and speed at destination. Although the path of a field-sail can be expressed in closed form, however the related expression, interfaced with the other equations of the current problem, gives rise to solving equations so complex to require a separate paper for presenting and discussing them appropriately. Rather, we solve our problem (a flight to Proxima) by means of the general code SMAC (Starship Mission Analysis Code). This code is written in FORTRAN-77 and currently runs in protected mode on a full 32-bit i80386-based work-station endowed with the co-processor Weitek 3167.

We do not consider the gravitational fields of the departure and arrival star systems, here. They are important for the initial and final transition phases. Although SMAC can take such fields into account, nevertheless that would bring us far from the current aim here. Some of major features of SMAC are described in Ref.-3.

In order to solve the current problem elegantly, we make use of the theory developed in sects. 2-3. We also assume that the field-sail mass come largely from the superconducting loops. For we model the mass of the field-sail consisting of (few) n coils, all of radius R, as:

(23)
$$M_{FS} = (1+h)n 10^7 \frac{R^2 B_{max}}{J/d}$$
 (MKSA)

where J/d denotes the current density on mass density ratio [A m/Kg], Bmax being the maximum magnetic induction at the coil centre line. The factor h accounts for the field generator and the physical connection of the coils to the ship body. Note that the field-sail mass scales as R, as Bmax is proportional to 1/R, standing the other parameters.

With regard to modelling the rocket system mass, we force the sophisticated model of SMAC to a simplified mass breakdown. We write:

(24)
$$M_R = \alpha W_k + M_a + (1+q) M_i$$

where Ma represents the active mass (which releases energy), Mi is the inert mass which receives part of that energy, Wk denotes the jet kinetic power, q is the tankage factor. By α we denote the overall specific mass of the rocket system. Because the rocket is to be activated after tens of years after departure, it is hard to think of antimatter rocket propulsion. We consider a very advanced Nuclear Propulsion system. In general, both jet speed and kinetic power of the rocket can be expressed in terms of only three key parameters which have a basic physical meaning [3]. We do not repeat them here for the sake of simplicity; in contrast, we will report only the engineering quantities of Eq.-24.

The ultimate performances of nuclear electric propulsion have been examined in [5,6]. Those engines do not fit the current example essentially because of their specific mass, too high with respect to a light-mass fast-mission such as the current one (see Tab. 1). Possibly, advanced Fusion propulsion [2] or direct Fission propulsion [7] might offer a specific mass of 1 Kg/MW like that assumed here. However, the important point to grasp in our example is the strong "interaction" between the field sail and rocket technologies. Such a conflicting situation may be relaxed if the orbiter mission is transformed into a fly-by one.

Figure 4 shows the profiles of path and speed as function of the ship proper time. Figure 5 shows thrust and kinetic beam power versus the beam speed as measured in SF for the envisaged flight to Proxima Centauri. Table 1 reports engineering and dynamical quantities. Table 1 and Fig.4-5 are self-explanatory. We note only that the magnetic sail is able to deliver a final mass fraction as high as 25 percent. That would imply a fly-by flight at 1500 Km/s, namely, 23 days of inertial flight to cover 20 AU around the star target. A small rocket system would be necessary for the final guidance. If we want an injection to the star system, the mission increases in performance, of course; however, the payload mass fraction falls down to 6 percent, because of a large rocket system. In any case a big problem is to keep the plasma beam on the sail for over 2600 AU, whereupon the need of considering sources of intensity significantly higher than 1 gramme per second, compatibly with the high acceleration resulting to the ship.

TABLE 1 Field-Sail plus Rocket Minimum-Time Prefixed-Final-Mass Mission to Proxima Centauri: ship configuration. The field sail performance has been conservative (effective-area=geometric-area) for the acceleration phase because no simulation has been made for particles of 50 MeV impinging a magnetic sail. (In Ref.-2 a large effective area has been found out for particles in the 1.3-2.9 KeV range, typical of the interplanetary conditions). The same configuration has been kept for the deceleration phase where the current ship sees the incoming interstellar plasma decaying in energy from 14 MeV down to 12 KeV.

** Initial Ship Mass **	200	[tonne]	
FULL ACCELERATION STAGE			
hydrogen plasma source:			
mass flow rate	0.010	[Kg/s]	
particle kinetic energy	50	[MeV]	
magnetic sail:			
current density/mass density	1.É7	[A m/Kg]	
loop radius	100	[Km]	
max magnetic induction	1.E-5	[T]	
field-generator+cables mass	50	[tonne]	
sail reflection coefficient	1.8		
sail stage mass	150	[tonne]	
acceleration time (SF)	143	[day]	
acceleration time (GF)	144	[day]	
path	2662	[AU]	
COASTING			
coasting time (SF)	20.02	[year]	
coasting time (GF)	20.31	[year]	
cruise speed	0.170	[C]	
The second se			
COARSE DECELERATION STAGE			
interstellar mean density	1	[p/cm^3]	1
final speed	1515	[Km/s]	
deceleration time (SF)	42.93	[yr]	
deceleration time (GF)	42.95	[yr]	
FINE DECELERATION STAGE	100 C		
jettisoned mass	150	[tonne]	
** Probe Mass **	50	[tonne]	
retrorocket.			
overall specific mass	1	[Kg/MW]	
propulsion system mass	6.2	[tonne]	
propellant (active+inert) mass	28.9	[tonne]	
tanks mass	2.9	[tonne]	
true exhaust speed	2000	[Km/s]	
thrust	5000	[N]	
mass utilisation efficiency	0.90		
energy efficiency	0.80	Se 12- 11	
thrusting time (SF,GF), flight time (GF)	117 - 64	[day] [yr]	
Final Orbit about Proxima:			
semimajor axis - eccentricity	0.15 - 0.38	[AU]	
** Net Payload **	12	[tonne]	



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390



391

8 FINAL CONSIDERATIONS

In this paper the dynamics of a field sail has been analysed. The relativistic equation of motion has been carried out under a number of assumptions realistic enough, at least as far as one can imagine today about a field sail. Substantially, the only approximation made here regarded the size of the interaction box, which has been assumed of constant size, namely, independent of the particle energy as seen in the ship frame. This is of no matter for a sail accelerated by a controllable beam which, in principle, could be focused on the effective sail area as the ship increases its speed. In contrast, if the sail were used for decelerating by interstellar drag which one cannot fully control, a time-varying effective area may result in a significant change of the ship speed profile. This depends on the details of the interaction between particles and field. This interaction could and should be simulated in a particle energy range from, say, 1 KeV to 100 MeV in order to cover an interval of speeds appropriate to an interstellar mission.

Other major aspects dealt with in this paper have been the comparisons of the field sail dynamics with the rocket, photon sail and ram-braking dynamics. It has been found that no equivalence between the speed profiles of the field sail mode and the rocket mode is possible. In contract, it is possible to make the field sail dynamics equivalent to either an accelerating light sail's or a decelerating ram's. This last sentence can be reversed. All that, together with what is well-known in space dynamics, proves that one can control the evolution of the spaceship velocity by means of three fundamental propulsion modes, in spite of their vast range of technology realisations. Such practical designs, though, will determine the selection of the space vehicle for a certain mission.

A field sail mode, in its possible realisation as magnetic sail, may represent a valid alternative to or parallel the rocket mode in interstellar missions, as hinted in this paper by a numerical example. It also may be considered with success in some mission precursor to the first interstellar flight.

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