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S. SZPIKOWSKI, M. TRAJDOS, R. TAMBOR

Four Particle Correlations in the Even Ca and Ti Isotopes

1. The Model

The existence of dynamical structures consisting of several nucleons on the nuclear surface is rather firmly established both experimentally and theoretically. We can refer to our earlier publications [1,2] where several arguments had been given for such clusters. Among those structures the most probable are two-particle (J=0) and four-particle alpha-like clusters. On the other hand, and especially from many works on the Interacting Boson Model, it follows that for low energy nuclear phenomena the building blocks of nuclei are not single nucleons but pairs of nucleons. It has been shown a long time ago [3] that two-body operators built from the creation and annihilation nucleon operators coupled to J = 0

are the generators of the orthogonal group SO(5). Hence we have constructed the Hamiltonian of the SO(5) structure in which the dominant role plays the pair of particles coupled to J=0. At first let us define the two-particle and four-particle structures for nucleons (both protons and neutrons) on the j-levels

$$P^{+}_{K}(j) = \frac{1}{2}(2j+1)^{1/2}\,[a^{+}_{j}a^{+}_{j}]^{J=0\,T=1}_{K}$$

$$Q^{+}_{\lambda\mu}(j_1j_2) = \frac{1}{4}\,[(2j_1+1)(2j_2+1)]^{1/2} \qquad (1)$$

$$\left\{[a^{+}_{j_1}a^{+}_{j_1}]^{J=0\;\;T=1}[a^{+}_{j_2}a^{+}_{j_2}]^{J=0\;\;T'=1}\right\}^{\lambda}_{\mu}$$

and $\qquad P_{K}(j) = [P^{+}_{K}(j)]^{+} \qquad Q_{\lambda\mu}(j_1j_2)) = [Q^{+}_{\lambda\mu}(j_1j_2)]^{+}$

The creation operators in the above formulas have been coupled by the Clebsch-Gordan coefficients in angular momentum and isospin spaces. The single particle isospin quantum number equal to 1/2 have been understood without being written. The four particle isospin quantum number λ is equal only to 0 or 2. The Hamiltonian of the SO(5) structure is now constructed in the form

$$H = \sum_{jm\tau}\varepsilon_j a^{+}_{jm\tau}a_{jm\tau} - G\sum_{j_1j_2k}P^{+}_{k}(j_1)P_{k}(j_2)$$

$$-\frac{1}{4}\sum_{j_1j_2j_3j_4\lambda\mu}\chi_{\lambda}\,Q^{+}_{\lambda\mu}(j_1j_2)\,Q_{\lambda\mu}(j_3j_4) \qquad (2)$$

where $\tau = 1/2$ for neutrons and $\tau = -1/2$ for protons. The first term in (2) is simple the single particle shell energy. The second term is the pairing part but taken among neutrons (k=1), protons (k=-1) and neutrons-protons (k=0) with the same strength G. The third term is a four-body interaction term which can be understood as an effective part of a real but unknown two-body interaction. The strength χ_{λ} has been made dependent on a scalar ($\lambda = 0$) or a tensor ($\lambda = 2$) character in the isospin space.

The present consideration is similar to that of our earlier paper [2] but it is more general as we allow the four-body clusters to be constructed from two pairs of nucleons not necessarily from the same j-level i.e. instead of considering only $Q^+(jj) \equiv Q^+(j)$ as in [2] we also consider $Q^+(j_1,j_2)$ with $j_1 \neq j_2$

The matrix elements of the Hamiltonian (2) have been calculated in the bases of irreducible representations of the orthogonal group SO(5) which are factorized by two numbers λ_1 and λ_2 (do not mix them with a tensor character of the operator $Q_{\lambda\mu}$) which have a group theory meaning as numbers of two fundamental representations used in the construction of a given irreducible representation (λ_1,λ_2) (4)

$$\lambda_1 = 2t \qquad \lambda_2 = j + \frac{1}{2} - \frac{v}{2} - t \tag{3}$$

where t is the reduced isotopic spin and v is the seniority number. In the case of an even number of nucleons the most important physical state vectors form the basis of the irreducible representations $(0,\lambda)$. For those representations the three physical quantum numbers provide a complete classification, namely the number of the total isospin T, its third component T_o and the particle number n. Then the state vectors read

$$|(0,\lambda)\ nTT_o\rangle \equiv |vnTT_o\rangle \tag{4}$$

The bases for the representations $(0,\lambda)$ have been constructed a long time ago [4]. However, for several non-degenerated j-levels a direct Kronecker product of several irreducible representations of the group SO(5) must be considered or, equivalently, a non-coupled in the SO(5) space basis can be used. In what follows we consider the schematic two-level model sd-$f_{7/2}$ with the core of the oxygen nucleus ^{16}O (Fig. 1).

For the first level, sd, instead of j+1/2 in (3) it should be taken $\sum_j (j+1/2) = 6$ and hence, the irreducible representation for that level is $(0,\lambda_2) = (0,6)$ if we take

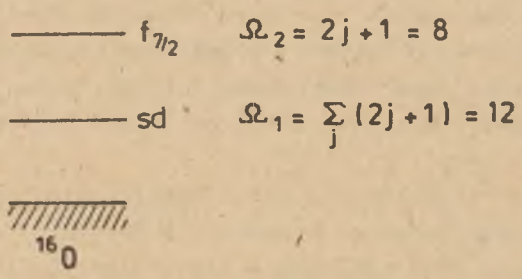

Fig. 1. The schematic two-level model sd-$f_{7/2}$ with the core of ^{16}O

$v = 0$ for the lowest energy states. The so called weak coupled basis is then used for the two-level model

$$|n_1 T_1;\ n_2 T_2;\ nTT_o\rangle \equiv$$

$$\equiv \sum_{T_{o1} T_{o2}} (T_1 T_{o1} T_2 T_{o2} | TT_o \lambda)\ |n_1 T_1 T_{o1}\rangle\ |n_2 T_2 T_{o2}\rangle \qquad (5)$$

where index "1" is for the sd-level and index "2" - for $f_{7/2}$.

The irreducible representations for both levels are not marked as they are fixed in what follows, i.e. $(0,\lambda_2) = (0,6)$ for the sd-shell and $(0,\lambda_2) = (0,4)$ for the $f_{7/2}$ shell. The symbol $(|)$ in (5) means, as usually, the Clebsch-Gordan coupling coefficient. The Hamiltonian (2) has been constructed from the SO(5) generators and hence it does not mix the state vectors of the bases of other irreducible representations, i.e. the states with $v_1 \neq 0$ or $v_2 \neq 0$, with our basis (5).

The matrix elements of the Hamiltonian (2) had been given in [1,2] except of that part in which the four body Q operators are formed from the pairs in different Ω levels. Hence, in the appendix we give the missing matrix elements of the four-body interaction expressed by the reduced matrix elements of the $P^+(\Omega)$ and $Q^+(\Omega\ \Omega)$ operators which were already calculated [1,2].

2. The Applications

At first we have solved the eigenenergy problem of the Hamiltonian (2) for the Ca and Ti isotopes. Because the pairing interaction is already involved in the four-body part of (2), we put G = 0 without making the model less applicable. Let us write the Hamiltonian (2) in the form

$$H = \varepsilon_1 \hat{n} + \Delta\varepsilon \left\{ n_2 - \frac{1}{4} \sum_{j_1 j_2 j_3 j_4 \lambda\mu} \frac{\chi}{\Delta\varepsilon} Q^+_{\lambda\mu}(j_1 j_2)\, Q_{\lambda\mu}(j_3 j_4) \right\} \qquad (6)$$

where $\hat{n}$ is the total number operator, n_2 is the particle operator on the $j = 7/2$ level and $\Delta\varepsilon = \varepsilon_2 - \varepsilon_1$. The first term in the Hamiltonian (6) does not enter the excited energy calculated relatively to the ground state energy which we take on the level $E_o = 0$. Hence the Hamiltonian has three free parameters, namely $\Delta\varepsilon$, $\chi_o/\Delta\varepsilon$, $\chi_2/\Delta\varepsilon$. After detailed inspection of the several calculated results we were able to put $\chi_o/\Delta\varepsilon = \chi_2/\Delta\varepsilon = 0.0012$. The second parameter, $\Delta\varepsilon = 2.3$ MeV except ^{42}Ti and ^{44}Ti for which $\Delta\varepsilon = 1.7$ MeV and $\Delta\varepsilon = 1.4$ MeV respectively. In our previous calculations [2] in which the space for four-body interactions had been restricted its effective strength was larger, namely $\chi_o/\Delta\varepsilon = \chi_2/\Delta\varepsilon = 0.0018$. However in our old calculations we did not obtained even fairly good agreement with experimental energy data for, especially, the ^{44}Ti nucleus. In Fig. 2 we show the comparison of the experimental data [5-7] with the previous and present calculations for Ti isotopes. In Fig. 3 we show the similar comparison for the Ca isotopes. In this case the old and new calculations are of the same quality.

The solution of the eigenvalue problem is also important for the construction of the state vectors of a given energy E

$$|nTT_oE\rangle = \sum_{n_1 n_2 T_1 T_2} C_{nET}(n_1T_1; n_2T_2)\ |n_1T_1; n_2T_2; nTT_o\rangle \qquad (7)$$

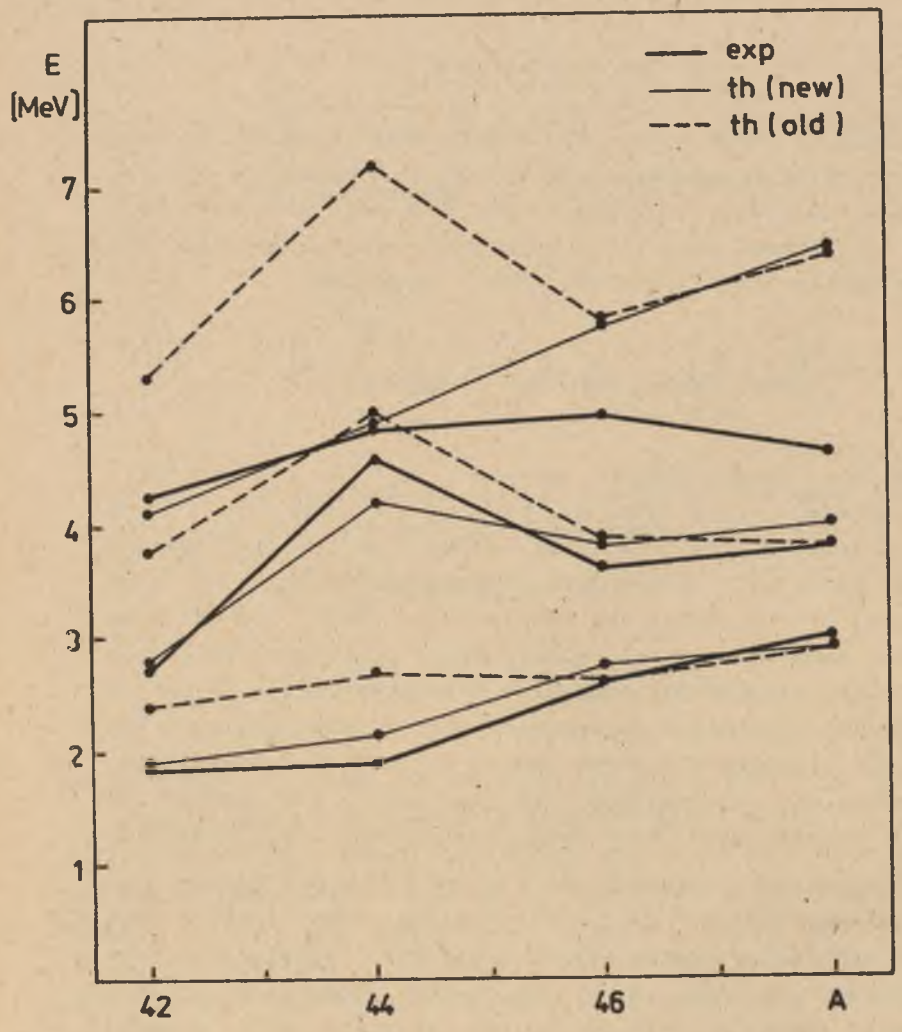

Fig. 2. The first excited 0^+ levels for Ti isotopes taken relatively to the ground state energy (E_o=0). The present calculations ("new") are compared with the previous ones ("old") [2] and with experimental data [5-7].

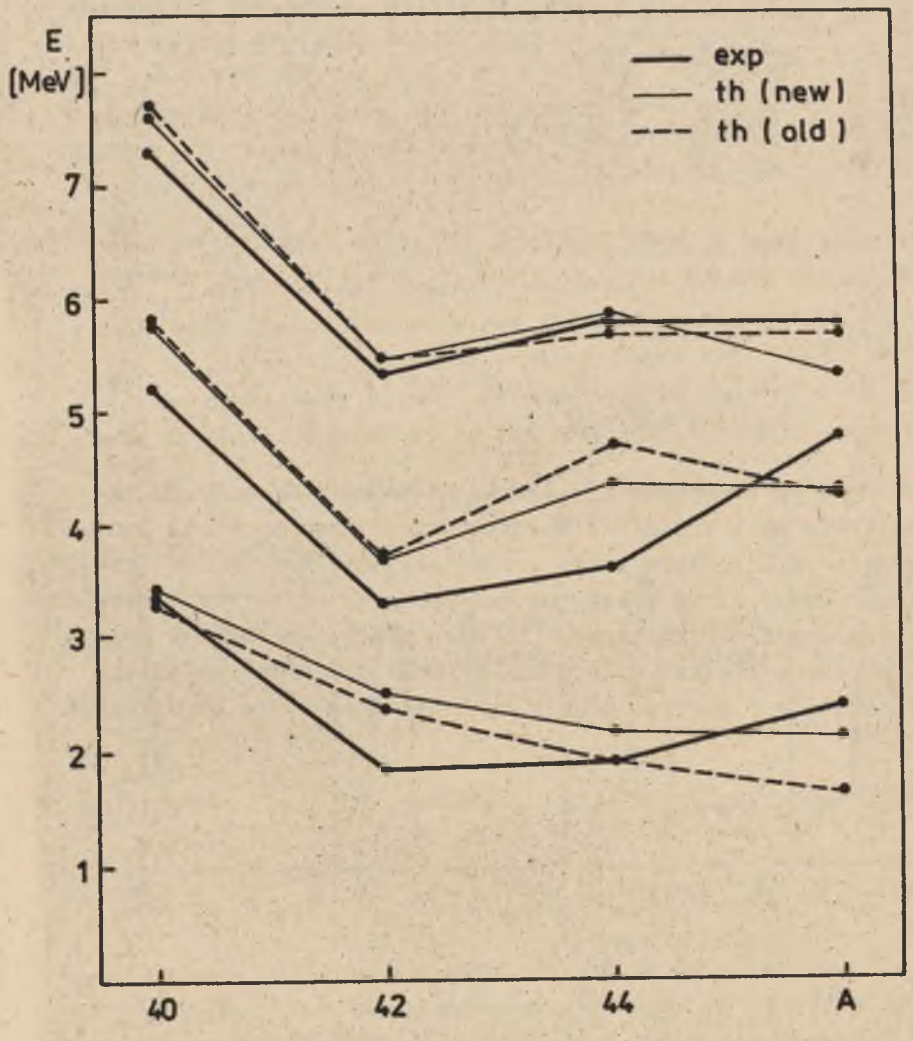

Fig. 3. The same as for Fig. 2 but for Ca isotopes.

where C are the calculated expansion coefficients of a state vector in the basis (5). We use now (7) to discuss the

α-cluster probabilities on the nucleus surface. Consider at first the probability $K(N_2)$ of finding in the state of a given energy E the number N_2 of pairs on the $f_{7/2}$ shell. That is

$$K(N_2) = \sum_{T_1 T_2} |C_{nET}(n_1 T_1;\ n_2 T_2)|^2 \tag{8}$$

where $N_2 = \frac{1}{2} n_2$ for $v_2 = 0$. Similarly we write the probability $K(T_2)$ to couple the nucleons on the $f_{7/2}$ shell to the isospin T_2

$$K(T_2) = \sum_{n_1 n_2 T_2} |C_{nET}(n_1 T_1;\ n_2 T_2)|^2 \tag{9}$$

Suppose there is no α-cluster (with, of course, $T_\alpha = 0$) on the $f_{7/2}$ shell. In such a case at any instant of time $N_2 = T_2$ and hence $K(N_2) = K(T_2)$. This means a complete overlapping of the two curves. However, if these two curves have different shape or they are shifted with respect to each other, we can say that α-structures are present with some probability. To calculate that probability P one can simply take the mean value of $\bar{N}_2$ and $\bar{T}_2$

$$\bar{N}_2 = \sum_{N_2} N_2 K(N_2) \qquad \bar{T}_2 = \sum_{T_2} T_2 K(T_2) \tag{10}$$

and then

$$P = \frac{\bar{N}_2 - \bar{T}_2}{\bar{N}_2} \ 100\% \tag{11}$$

We illustrate the problem for two isotopes ^{40}Ca and ^{44}Ti which differ by one like α-cluster, Fig. 4 and Fig. 5. For ^{40}Ca (Fig.4) there is no particles on the $f_{7/2}$ shell if there is no interactions. Then for the ground state even with the four-body interaction there is rather a low probability of finding a pair of nucleons on the $f_{7/2}$ shell (the first part of Fig. 4). The

next three parts of Fig. 4 show the situation in the three excited 0^+ states of ^{40}Ca. It is very interesting to compare the second and third excited states. For the second excited state we get the high probability of α-clusters equal to 83% but the third excited state having less, on average, pairs of nucleons on the $f_{7/2}$ shell shows the very low α-cluster probability equal to 12%. It explains why lower number of excited nucleons from the sd to $f_{7/2}$ shell forms a structure higher in energy.

For the ^{44}Ti isotope which has one more like α-cluster on the $f_{7/2}$ shell we can consider the probability of α-clustering for as well the ground state as for the three excited 0^+ states (Fig. 5). In this case we get respectively 99%; 68%; 89% and 61%. The comparison of the second and third excited levels shows the same similarity as for the ^{42}Ca isotope.

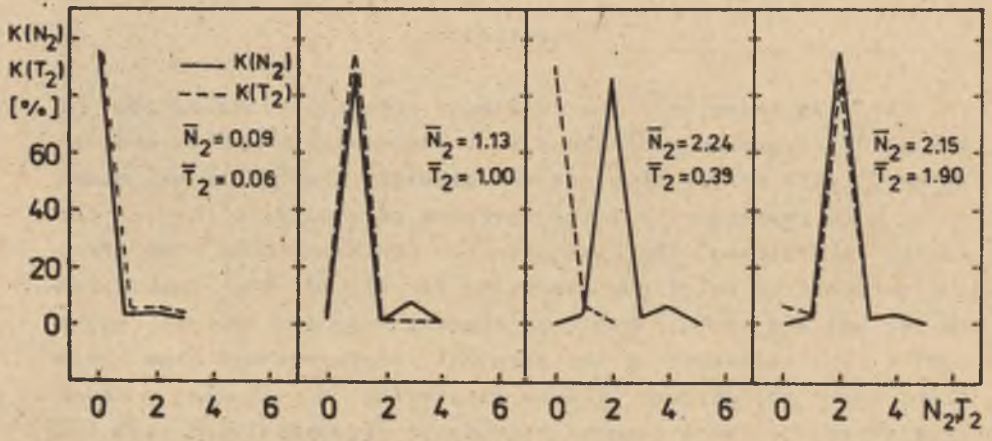

Fig. 4. The probability $K(N_2)$ of finding N_2 pairs on the $f_{7/2}$ shell for the ground and excited 0^+ levels for the nucleus ^{40}Ca versus N_2 and also the probability $K(T_2)$ of the total isospin of nucleons on the $f_{7/2}$ shell versus T_2. There are also given, in each case, the mean values of $\bar{N}_2$ and $\bar{T}_2$.

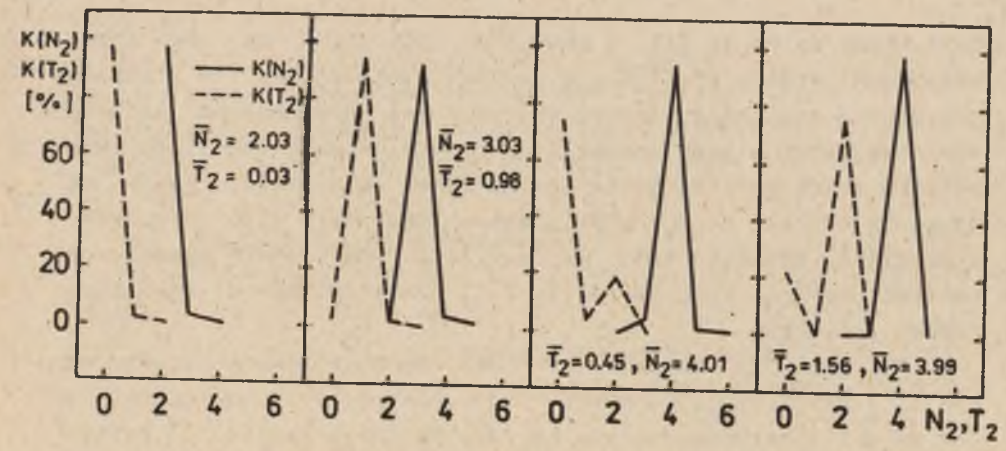

Fig. 5. The same as for Fig. 4 but for the nucleus ^{44}Ti

3.Conclusions

We have shown that the four-body effective interaction of the SO(5) structure contains also enough pairing energy and can be used, with single particle energies for the two level model sd-$f_{7/2}$ to reproduce well the energies of excited 0^+ levels for Ca and Ti isotopes. The Hamiltonian contains only two free parameters from which one is fixed for all of the considered nuclei and the another has been changed only for two of eight nuclei. The structure of the physical state-vectors has been then used to discuss the α-clustering of nuclei under consideration. The α-cluster calculated probabilities are on line with physical reasoning.

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Appendix

Matrix elements of the four body part of the Hamiltonian (6) for $\Omega_1 \neq \Omega_2$

I.

$$\langle \nu_1, n_1 - 2, T_1'; \nu_2, n_2 + 2, T_2'; nTT_0 |$$

$$\Big| \sum_{\mu} Q^{+}_{\lambda\mu}(\Omega_1\Omega_2) Q_{\lambda\mu}(\Omega_1\Omega_1) \Big| \nu_1 n_1 T_1; \nu_2 n_2 T_2; nTT_0 \rangle$$

$$= \sum_{L} (-1)^{T_1+T_1'+T_2+T_2'+T+L+\lambda} \sqrt{2\lambda+1}\,\sqrt{2T+1} \begin{Bmatrix} T_1 & T_2 & T \\ T_2' & T_1' & 1 \end{Bmatrix} \begin{Bmatrix} T_1 & T_1' & 1 \\ 1 & \lambda & L \end{Bmatrix}$$

$$\times \langle \nu_1, n_1 - 2, T_1' \| P^{+}(\Omega_1) \| \nu_1, n_1 - 4, L \rangle$$

$$\times \langle \nu_2, n_2 + 2, T_2' \| P^{+}(\Omega_2) \| \nu_2 n_2 T_2 \rangle \langle \nu_1 n_1 T_1 \| Q^{+}_{\lambda}(\Omega_1\Omega_1) \| \nu_1, n_1 - 4, L \rangle$$

II.

$$\langle \nu_1, n_1 - 2, T_1'; \nu_2, n_2 + 2, T_2' \; nTT_0 |$$

$$\Big| \sum_{\mu} Q^{+}_{\lambda\mu}(\Omega_2\Omega_2) Q_{\lambda\mu}(\Omega_1\Omega_2) \Big| \nu_1 n_1 T_1; \nu_2 n_2 T_2; nTT_0 \rangle$$

$$= \sum_{L} (-1)^{T_1+T_1'+2T_2'+T+L} \sqrt{2\lambda+1}\,\sqrt{2T+1} \begin{Bmatrix} T_1 & T_2 & T \\ T_2' & T_1' & 1 \end{Bmatrix} \begin{Bmatrix} T_2 & T_2' & 1 \\ \lambda & 1 & L \end{Bmatrix}$$

$$\times \langle \nu_1, n_1, T_1 \| P^{+}(\Omega_1) \| \nu_1, n_1 - 2, T_1' \rangle \langle \nu_2 n_2 T_2 \| P^{+}(\Omega_2) \| \nu_2, n_2 - 2, L \rangle$$

$$\times \langle \nu_2, n_2 + 2, T_2' \| Q^{+}_{\lambda}(\Omega_2\Omega_2) \| \nu_2, n_2 - 2, L \rangle$$

III.

$$\langle v_1 n_1 T_1' ; v_2 n_2 T_2' ; nTT_o |$$

$$\Big| \sum Q^+_{\lambda\mu}(\Omega_1\Omega_2) Q_{\lambda\mu}(\Omega_1\Omega_2) \Big| v_1 n_1 T_1 ; v_2 n_2 T_2 ; nTT_o \rangle$$

$$= \sum_{\nu L'} (-1)^{T+T_1+T_1'+2T_2'+L+L'} \sqrt{2T+1}\; a_{\lambda\mu} \begin{Bmatrix} T_1 & T_2 & T \\ T_2' & T_1' & \nu \end{Bmatrix} \begin{Bmatrix} T_1 & T_1' & \nu \\ 1 & 1 & L' \end{Bmatrix} \begin{Bmatrix} T_2 & T_2' & \nu \\ 1 & 1 & L' \end{Bmatrix}$$

$$\times \langle v_1, n_1, T_1' \,\|\, P^+(\Omega_1) \,\|\, v_1, n_1 - 2, L \rangle \langle v_1 n_1 T_1 \,\|\, P^+(\Omega_1) \| v_1, n_1 - 2, L \rangle$$

$$\times \langle v_2, n_2, T_2' \,\|\, P^+(\Omega_2) \,\| v_2, n_2 - 2, L' \rangle \langle v_2 n_2 T_2 \| \, P^+(\Omega_2) \| v_2, n_2 - 2, L' \rangle$$

where

$$a_{00} = {}^1/_3 \qquad a_{20} = {}^5/_3$$

$$a_{01} = -1 \qquad a_{21} = {}^5/_2$$

$$a_{02} = {}^5/_3 \qquad a_{22} = {}^5/_6$$

The λ index in the above formulas means either a scalar ($\lambda = 0$) or a tensor ($\lambda = 2$) character of the $Q_{\lambda\mu}$ operator and should not be mixed with the (0λ) irreducible representation of the group SO(5).