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Cyclotron Resonance in Parabolic Quantum Wells in Tilted Magnetic Field

1. Introduction

Experimentally, the application of the additional parallel component of the magnetic field to the sample provides insight into very nontrivial behaviour of the quasi-two-dimensional (Q2D) system and enables extraction of additional information on its properties. The method has successfully been used to study the combined and coupled intersubband cyclotron resonances [1] in Q2D system, where the coupling of modes is expected to occur. It was shown to give the new insight into the spin structure of states in the fractional quantum Hall effect [2-4] customarily treated as purely two-dimensional phenomenon in which case the parallel field should have no effect.

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The material engineering has reached now the state of art when the quantum wells can be produced with a given shape of confining potential. In a recent work [5] the quantum wells with parabolic confining potential $V(z) \propto z^2$ in the growth direction (by varying quadratically the composition x in $Al_xGa_{1-x}As$ material from 0 for z = 0 to 0.19 at the well edges) have been studied. The precision of material processing when combined with very precise optical experiments gives very detailed information the handling of which requires better theoretical treatment not only of the many-body but also one-particle properties.

Recently the problem of a single electron moving in a quantum well confined by the parabolic potential V(z) and with magnetic field $\vec{B}$ applied at angles $\theta \neq 0$ with respect to the z direction has been solved for isotropic material analytically [6,7]. The exact solution gives the possibility of correct interpretation of the data outside perturbative regimes of small angles θ or weak fields B.

In particular, the origin of a gap in optical spectra of quantum well studied in tilted magnetic field has been clarified [8]. The line shape, however, seen in [5] and also in [9] remains still unexplained. The satellite structure observed on both lines in the coupled resonance in tilted field configuration remains visible at $\theta = 0$ and may be due to spin dependent many-body interactions mixing the spin states in a given energy level resulting from hybridization of subband and Landau levels. In this work we shall neglect all the possible many-body complications and will discuss the one-particle property of the parabolic quantum well in tilted magnetic field. Special attention will be focused on the anisotropy of the system, and the polarization effects.

The organization of the paper is as follows. In Section 2 we shall present our anisotropic model Hamiltonian and sketch the solution of corresponding eigenproblem. Some comments on the degeneracy spectrum and the effects connected with anisotropy of the mass tensor will be made. The calculation of matrix elements entering the formula for oscillator strengths

is presented in Section 3. This is the main part of our work. The discussion of results as well as the proposals of new experiments and limited comparison with available experimental data is given in last Section.

2. Model and (exact) solution

A single electron Hamiltonian in a confining potential V(z) and magnetic field $\vec{B}$ described by vector potential $\vec{A}$ in effective mass approximation is given by

$$H = \sum_i P_i^2/2m_i + V(z) , \qquad (1)$$

where m_i are effective masses in three principal directions, by assumption coinciding with (x,y,z) coordinates. $P_i = (p_i + eA_i)$, $p_i = -i\hbar \frac{\partial}{\partial x_i}$, $V(z) = m_z\Omega^2 z^2/2$, where Ω has been taken [7] in the form $\hbar\Omega = E_{01}$ (0 → 1 intersubband energy). We take $\vec{B} = (0,B_y,B_z) = B\,(0, \sin\theta, \cos\theta)$ and represent it by the potential $\vec{A}$ in a Landau-like gauge $\vec{A} = B\,(z\sin\theta - y\cos\theta, 0, 0)$. This gauge is more convenient than that used in [7], as we can work only in the $\vec{r}$ representation without need of mixed representation. It leads very easily to the explicit expressions for the wave function also in $\vec{r}$ representation.

The resulting Hamiltonian commutes with the momentum operator p_x. Thus $p_x = \hbar k_x$ is the constant of motion, where k_x is the wave vector of the plane wave $\exp(ik_x x)$. Taking all this into account and shifting the y coordinate to $y' = y - l^2 k_x/\cos\theta$, with $l^2 = \hbar/eB$ we get

$$H_{k_x} = \frac{1}{2m_x}(eBz\sin\theta + eBy'\cos\theta)^2 - \frac{\hbar^2}{2m_y}\frac{\partial^2}{\partial y'^2} - \frac{\hbar^2}{2m_z}\frac{\partial^2}{\partial z^2} + \frac{m_z\Omega^2 z^2}{2} \qquad (2)$$

This Hamiltonian represents two coupled harmonic oscillators.

The coupling term $\frac{(eB)^2}{4m_x} zy'\sin(2\theta)$ can easily be eliminated

by proper rotation of scaled coordinates $\sqrt{m_z}z$ and $\sqrt{m_y}y'$ by an angle δ, i.e.

$$\begin{cases} x'' = x \\ y'' = \sqrt{m_y}y'\cos\delta + \sqrt{m_z}z\sin\delta \\ z'' = -\sqrt{m_y}y'\sin\delta + \sqrt{m_z}z\cos\delta \end{cases} \tag{3}$$

We get

$$\mathrm{tg}2\delta = \frac{\bar{\omega}_c^2\gamma\sin 2\theta}{\Omega^2 - \bar{\omega}_c^2(\cos^2\theta - \gamma^2\sin^2\theta)} \,, \tag{4}$$

where $\gamma^2 = m_y/m_z$ and $\bar{\omega}_c = eB/\sqrt{m_x m_y}$ is the parameter which for isotropic system reduces to the cyclotron frequency $\omega_c = eB/m$.

The spectrum of the problem consists of two harmonic oscillator branches

$$E_{NM} = (N + 1/2)\hbar\omega_1 + (M + 1/2)\hbar\omega_2 \,, \tag{5}$$

with frequencies

$$\omega_1^2 = (\Omega^2 + \bar{\omega}_c^2\gamma^2\sin^2\theta)\cos^2\delta + \bar{\omega}_c^2\cos^2\theta\sin^2\delta + \tfrac{1}{2}\bar{\omega}_c^2\gamma\sin 2\theta\sin 2\delta \,, \tag{6a}$$

$$\omega_2^2 = (\Omega^2 + \bar{\omega}_c^2\gamma^2\sin^2\theta)\cos^2\delta + \bar{\omega}_c^2\cos^2\theta\sin^2\delta - \tfrac{1}{2}\bar{\omega}_c^2\gamma\sin 2\theta\sin 2\delta \,. \tag{6b}$$

The eigenfunction of (1) can be written as

$$\Psi_{NM,k_x} = |N,M,k_x\rangle = \frac{1}{\sqrt{L_x}} e^{ik_x x} \phi_{n_1}(y''/\Lambda_1)\, \phi_{n_2}(z''/\Lambda_2), \tag{7}$$

where $\Lambda_{1,2} = \hbar/\omega_{1,2}$, L_x is the linear dimension of the system in the x-direction and for periodic in x direction boundary conditions we have $k_x = \frac{2\Pi}{L_x} n$, $n = 0, \pm 1, \pm 2 \ldots$. Energies (5) are degenerated, they do not depend on k_x while the wave functions

do. To calculate the degeneracy let us note, that the position of the center (y_o) of both oscillators depends on the k_x and z. Explicitly

$$(y_o)_1 = \frac{l^2 k_x}{\cos\theta} - \frac{z\,\mathrm{tg}\delta}{\gamma}$$

$$(y_o)_2 = \frac{l^2 k_x}{\cos\theta} - \frac{z\,\mathrm{ctg}\delta}{\gamma} \quad . \tag{8}$$

Applying the proper boundary conditions (the center of oscillator should not be outside the sample) to (3) we get the degeneracy g of levels to be

$$g = \frac{L_x L_y}{h} eB\cos\theta = \frac{L_x L_y}{h} eB_z \quad , \tag{9}$$

independently of the factor γ. The perpendicular component of the magnetic field controls the degeneracy of levels.

Inspection of the results for isotropic medium $m_i = m$ in the formulae (4) and (6) shows that the effect of anisotropy can be accounted for by replacing $\omega_c = eB/m$ and θ in isotropic expressions through

$$\omega_c \longrightarrow \tilde{\omega}_c = \omega_c(\gamma^2\sin^2\theta + \cos^2\theta)^{1/2} \quad , \tag{10a}$$

$$\theta \longrightarrow \tilde{\theta} = \mathrm{arctg}(\gamma\,\mathrm{tg}\theta) \quad . \tag{10b}$$

The cyclotron frequency in (10a) has a simple interpretation. It is just the cyclotron frequency of the bulk material with cyclotron effective mass $m_c(\theta)$, given by

$$\frac{1}{m_c(\theta)} = \frac{1}{\sqrt{m_x m_y}} \left[\frac{m_y}{m_z}\sin^2\theta + \cos^2\theta\right]^{1/2} \quad . \tag{11}$$

The dependence of $\tilde{\theta}$ on θ for the anisotropy coefficients corresponding to PbSe, Si, PbTe and Ge shows Fig.1. In calculations we have assumed that principal axis of the valley is along the z direction.

Formula (10b) shows that anisotropy effectively changes the tilt angle from θ to larger values $\tilde{\theta}$ for $\gamma > 1$ and to smaller values for $\gamma < 1$.

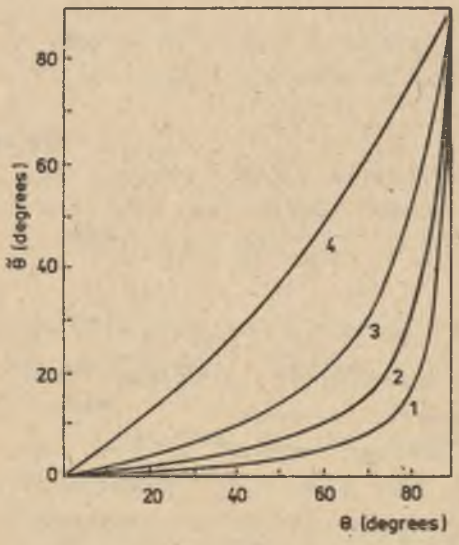

Figure 1.The effective tilt angle $\tilde{\theta}$ versus the actual θ for various ratio $K = m_z/m_y$ corresponding to known semiconductors. Curve 1 is for Ge (K=19.5); 2-PbTe (K=10.0); 3-Si (K=4.8); 4-PbSe (K=1.6).

The eigenenergies (5) for few first quantum numbers (N,M) are shown in figure 2 as a function of applied magnetic field (ω_c=eB/m) for tilt angle $\theta = 45^\circ$ (full lines) together with two dimensional calculations (dashed lines). As already mentioned, in a strictly two dimensional case the parallel field has no effect on the behaviour of the system and the lines cross. It has to be noted that the two-dimensional cyclotron frequency differs from the three dimensional ω_c used here. In two dimensions only the perpendicular component of the $\vec{B}$-field enters the expression for the cyclotron energy. In this figure, as well as in all the forthcoming results of numerical calculations we have set $m_x=m_y=m_z$.

Comparison of the above exact results with perturbative ones [8,10] can easily be done. It reveals that some of the effects obtained here, e.g. gap opening, could not be foreseen in perturbation theory. These will be discussed in Section 4.

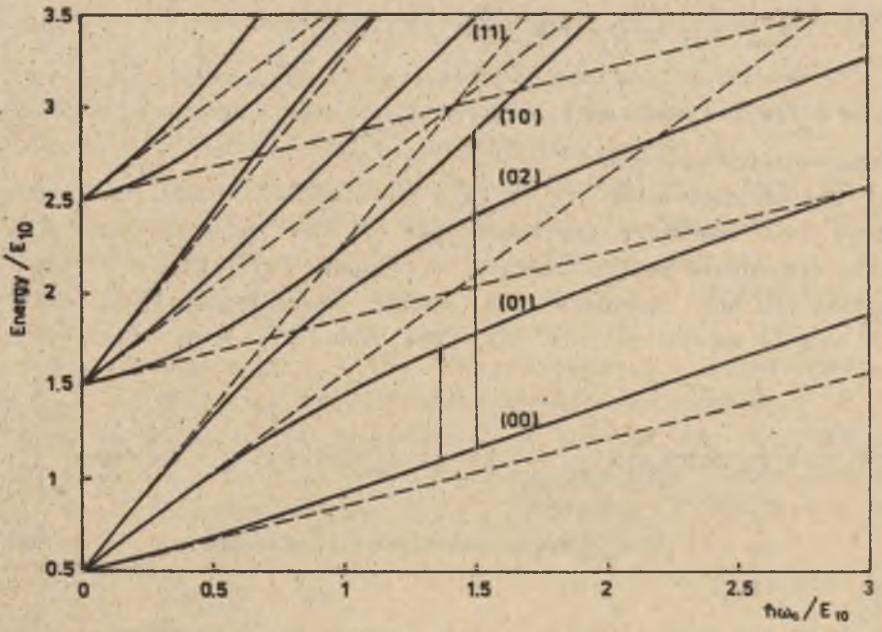

Figure 2. Energies (5) are plotted against cyclotron frequency (full curves). The numbers refer to (N,M).The dashed curves represent the spectrum of the two dimensional sample.

3. Cyclotron resonance

The knowledge of spectrum and wave functions enables us the calculation of optical absorption spectrum. For light polarized in x_i direction the strength of the transition is given by the real part of the dynamical conductivity $\sigma_{ii}(\omega)$. In linear response theory within dipole approximation it is given by [10]

$$\mathrm{Re}\sigma_{ii}(\omega) = \frac{e^2\pi}{2m_i}\sum_{\nu\mu}(N_\nu - N_\mu)\, f^{(i)}_{\mu\nu}\, \delta(E_\mu - E_\nu - \hbar\omega)\,, \qquad (12)$$

with

$$f_{\mu\nu}^{(i)} = \frac{2m_i}{(E_\mu - E_\nu)} |\langle \psi_\mu | v_i | \psi_\nu \rangle|^2 \quad . \tag{13}$$

where N_α is the electron concentration in α-th level and $\vec{v} = \frac{\partial H}{\partial \vec{p}}$ is the velocity operator.

The calculation of $\langle \psi_\mu | v_i | \psi_\nu \rangle$ is rather simple. To this end we shall transform the components of the velocity operator to the coordinate system denoted previously by (x",y",z") (see equation (3)) and calculate the matrix elements entering (13) with help of wave functions (7). The general result (for $\gamma \neq 0$) is

$$\langle k_x' N' M' | v_x | k_x, N, M \rangle = \delta_{k_x k_x'} \frac{\omega_c}{\sqrt{m_x}} \Big\{ \delta_{N'N} \langle y \rangle_{MM'} (\gamma \sin\delta \sin\theta - \cos\delta\cos\theta) + \delta_{M'M} \langle z \rangle_{N'N} (\gamma \sin\theta\cos\delta + \cos\theta\sin\delta) \Big\} , \tag{14a}$$

$$\langle k_x' N' M' | v_y | k_x, N, M \rangle = \frac{1}{\sqrt{m_y}} \Big\{ \delta_{N'N} \langle p_y \rangle_{MM'} \cos\delta - \delta_{M'M} \langle p_z \rangle_{NN'} \sin\delta \Big\}, \tag{14b}$$

$$\langle k_x' N' M' | v_z | k_x, N, M \rangle = \frac{1}{\sqrt{m_z}} \Big\{ \delta_{N'N} \langle p_y \rangle_{MM'} \sin\delta - \delta_{M'M} \langle p_z \rangle_{NN'} \cos\delta \Big\}, \tag{14c}$$

where for $r_i = y", z"$, $n = M, N$, $n' = M', N'$ we have

$$\langle r_i \rangle_{nn'} = \langle n' | r_i | n \rangle = \sqrt{\frac{n}{2}}\, \delta_{n',n-1} + \sqrt{\frac{n+1}{2}}\, \delta_{n',n+1} \quad , \tag{15a}$$

$$\langle p_i \rangle_{nn'} = \langle n' | p_i | n \rangle = -i\hbar \Big\{ \sqrt{\frac{n}{2}}\, \delta_{n',n-1} - \sqrt{\frac{n+1}{2}}\, \delta_{n',n+1} \Big\}. \tag{15b}$$

Note again that in order to get matrix elements for anisotropic system from that for isotropic one we have to multiply "isotropic" formulae for $\langle v_i \rangle$ by corresponding $\sqrt{m/m_i}$ and replace ω_c and θ by the values given on the right hand side of (10).

4. Results and discussion

Having determined the energy spectrum and matrix elements we are now in position to calculate the absorption spectrum described by the real part of the conductivity tensor (12).

The selection rules for the intra-band transitions may be obtained from (14). This equation shows that in the case of normal magnetic field ($\theta = 0$) the light polarized in the x-y plane induces only the cyclotron transitions $|N,M\rangle \to |N,M+1\rangle$ while the light polarized in the z direction couples to the intersubband transitions $|N,M\rangle \to |N+1,M\rangle$. Above simple picture breaks down when $\theta \neq 0$. Due to the subband-Landau level coupling induced by $\hat{p}_y$ the light polarized in the x-y plane (the z-direction) may also excite the intersubband (cyclotron) transitions. Consequently, the cyclotron (intersubband) resonance spectrum splits into two branches $\omega^{\pm} = (\mathcal{E}_{N+1,M}^{N,M+1} - \mathcal{E}_{00})/\hbar$. In the case of isotropic effective mass Eqs.5 and 6 give

$$(\omega^{\pm})^2 = \left\{\omega_c^2 + \Omega^2 \pm [(\omega_c^2 - \Omega^2)^2 + 4\omega_c^2\Omega^2\sin^2\theta]^{1/2}\right\}/2. \qquad (16)$$

The dispersion of these two branches calculated for $\theta = 30^\circ$, 45° and 60° shows Fig.3a. From this figure and Eq.16 we find that behaviour of ω^+ and ω^- is not consistent with perturbation theory for large angles θ in following points: i) as already noticed in [7] with increasing ω_c the lower branch (ω^-) tends to $\Omega\cos\theta$ (and not to Ω as would simple perturbation indicate) while the upper branch (ω^+) tends to ω_c (not to $\omega_c\cos\theta$), ii) the gap energy $\Delta E = \hbar(\omega^+ - \omega^-)$ takes on a minimum value equal $E_{10}\sin\theta$ when $\omega_c = \Omega\cos\theta$ (not when $\omega_c\cos\theta = \Omega$), iii) with increasing θ the gap center $\bar{E} = \hbar(\omega^+ + \omega^-)/2\,|_{\omega=\Omega\cos\theta}$ shifts from E_{10} to lower energy as $(1+3\cos^2\theta)^{1/2}/2$ and not like $\cos\theta$ given by perturbation theory.

Assume for simplicity that only ground level $|0,0\rangle$ is occupied by electrons. Then from (12-15) we find that in the case of the light polarized in the x_i direction the intensity

of the low (high) frequency resonance is proportional to the oscillator strength $f^{(i)}_{00,01}$ ($f^{(i)}_{00,01}$). Substituting (14) into (13) we obtain, for the isotropic case, the following result:

$$f^{(x)}_{00,01} = (\omega_c/\omega^+)^2 \cos^2(\delta - \theta) \quad , \tag{17a}$$

$$f^{(x)}_{00,10} = (\omega_c/\omega^+)^2 \sin^2(\delta - \theta) \quad , \tag{17b}$$

$$f^{(y)}_{00,01} = \cos^2\delta \quad , \tag{17c}$$

$$f^{(y)}_{00,10} = \sin^2\delta \quad , \tag{17d}$$

$$f^{(z)}_{00,NM} = f^{(y)}_{00,MN} \quad , \tag{17e}$$

where the angle δ is defined by (4).

We have calculated the oscillator strengths (for light polarized in the x and y directions) as a function of ω_c. Results are presented in Fig.3b,c. We see from this figure that similarly as in the perturbed theory the intensity of the branch ω^- (ω^+) decreases (increases) as ω_c increases. However, the dependence of the relative intensity of the low and high frequency transitions on ω_c for given θ reveals features that are not apparent in the results based on the perturbation theory. First of all we observe that even for normally incident light the shape of the absorption spectrum strongly depends on the light polarization. When the radiation is polarized in the x- direction the oscillator strengths of the two resonances are equal at $\omega_c = \Omega$. In the case of the y polarization the resonances have equal intensity at $\omega_c = \Omega/(\cos 2\theta)^{1/2}$ if $\theta < 45^\circ$. But when above condition is not fulfilled then $f^{(y)}_{00,01} > f^{(y)}_{00,10}$. Thus we see that the numerical value for the crossing point frequency (ω^*) determined by condition $f^{(i)}_{00,01} = f^{(i)}_{00,10}$ is independent on θ only in the case of the x polarization. It is interesting to note that the crossing point (ω^{**}) determined by the condition of minimum separation between ω^+ and ω^- is equal to $\Omega \cos\theta$. From above considerations we may conclude that in general $\omega^{**} < \Omega \leq \omega^*$ and the assumption commonly used in the interpretation of the experimental results, that

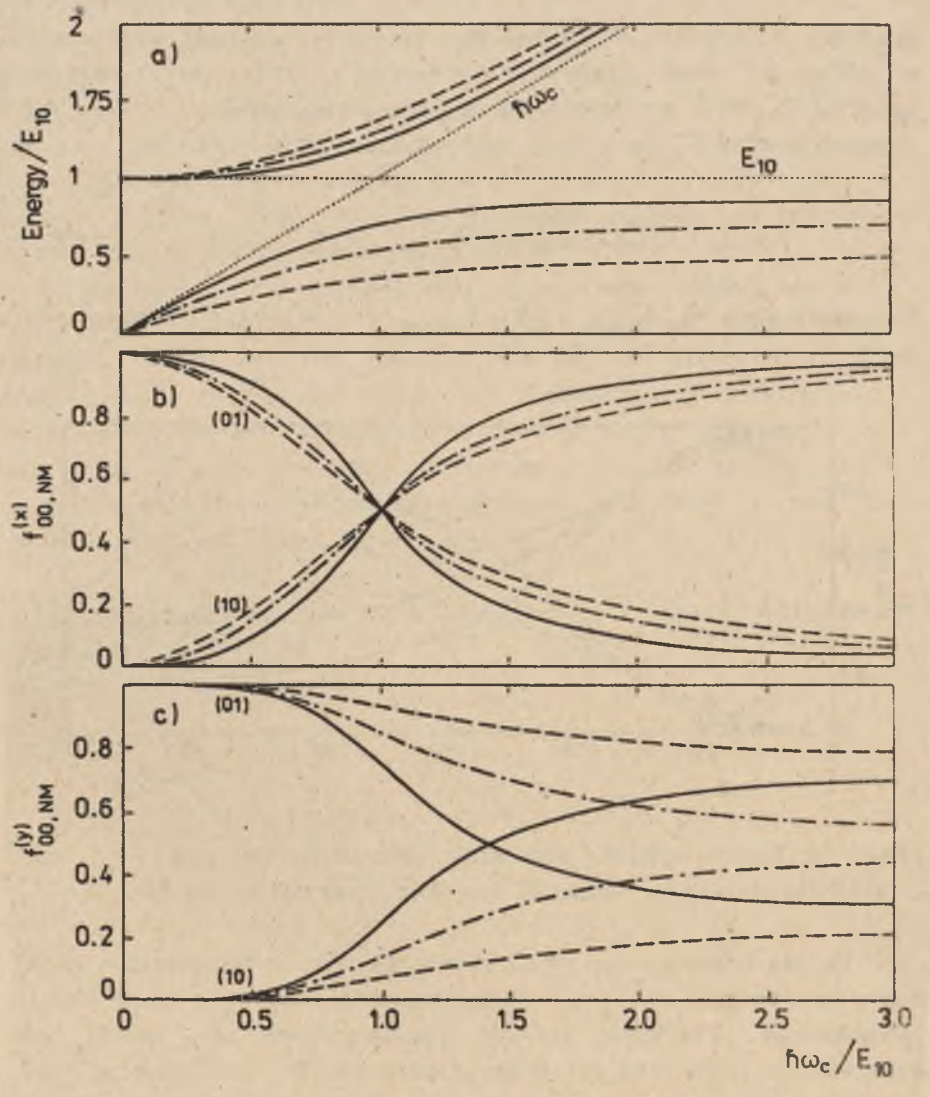

Figure 3. (a) Dispersion relation (16),(b) oscillator strength $f^{(x)}_{00,NM}$ (c) oscillator strength $f^{(y)}_{00,NM}$ calculated for the different tilt angles (------) $\theta = 30^{o}$, (- - - -) $\theta = 45^{o}$ and (————) $\theta = 60^{o}$.

$\bar{E}(\omega_c - \omega^*) = \bar{E}(\omega_c - \omega^{**}) = E_{10}$ is in principle correct only when θ is small. This conclusion is true also for nonpolarized light. In the case of the normal incidence the intensity of the $|0,0\rangle \rightarrow |N,M\rangle$ transition is then proportional to

$$\bar{f}_{00,NM} = \left[f^{(x)}_{00,NM} + f^{(y)}_{00,NM}\right]/2 \qquad (18)$$

The dependence of $\bar{f}_{00,01}$ and $\bar{f}_{00,10}$ on ω_c for different tilt angles is presented in the Fig.4.

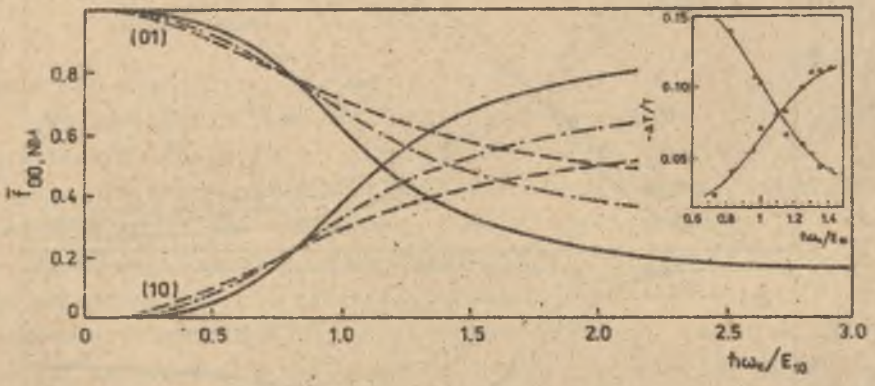

Figure 4. The averaged oscillator strength (18) *vs.* ω_c calculated for the same values of the tilt angle as in Fig.3.

We see that, as in the case of the y-polarization the difference between ω^* and Ω is positive and increases with increasing θ. The insert to this figure shows the height of experimental lines read off from Figure. 3 in [5] showing the far-infrared magneto transmission measurements on quasi-three dimensional modulation doped Al-Ga-As structure. The asymmetry in the heights of the lines with respect to the crossing point is in agreement with our results. The direct comparison of this experiment with our theory is not legitimate as we have omitted very important aspects of the phenomena - the depolarization and the excitonic effects which are known to strongly change

the frequencies of transitions. In perturbative regime it has been proved [10] that these effects merely lead to the renormalisation of frequencies without influence on the B-field dependence. Preliminary calculations of the depolarization effect in nonperturbative regime seem to indicate more complex influence of the many body interactions on the absorption spectrum. The problem is being studied now.

In summary, we have calculated the energy spectrum and velocity matrix elements for the parabolic quantum wells (with anisotropic effective mass tensor) placed in strong tilted magnetic field. We have found the pronounced polarization effects (see Fig.3) not predicted in perturbative approach. In view of this it would be very desirable to have experimental data performed with linearly polarized light (in the x and y directions) and at normal incidence.

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References

[1] Beinvogl W., Koch J. F., Phys. Rev. Lett. 40, 1736 (1978)
Schlesinger Z., Hwang J. C. M., Allen S. J., Phys. Rev. Lett. 50, 2098 (1983)

[2] Syphers D. A., Furneaux J. E., Solid State Comm. 65, 1513 (1988)

[3] Hang R. J., von Klitzing K., Nicolas R. J., Maan J. C., Weimann G., Surf. Sci. 196, 242 (1988)

[4] Eisenstein J. P., Stormer H. L., Pfeiffer L., West K. W., Phys. Rev. Lett. 62, 1540 (1989)

[5] Karrai K., Drew H. D., Lee M. W., Shayegan M. Phys. Rev. B 39, 1426 (1989)

[6] Maan J. C., in Two Dimensional Systems, Heterostructures, and Superlattices, edited by G. Bauer, F. Kuchar, and H. Heinrich (Springer-Verlag, Berlin, 1984), p. 183

[7] Merlin R. Solid State Commun. 64, 99 (1987)

[8] Huant S., Grynberg M., Martinez G., Etienne B., Solid State Comm. 65, 457 (1988)

[9] Huant S., Martinez G., Etienne B., Europhys. Lett. 9, 397 (1989)

[10] Załużny M., Solid State Commun 55, 747 (1985); 56, 235 (1985)