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# On Derivation of Model Equations for Cylindrical Gunn's Effect

Rozwiązywanie modelowych równań opisujących cylindryczny efekt Gunna

Решение модельных уравнений, описывающих цилиндрический эффект Гунна

The formation of domain and the current microwave oscillations in $A^3B^5$ semiconductors was discovered by Gunn [1]. Since this time dynamical behaviour of TE (transferred electron) type semiconductors has been discussed by many authors using different phenomenological and semi-phenomenological methods [2–5]. One of these methods is based on the full spatial charge dynamics, temporary and bound states on the assumption of special boundary conditions [2]. This approach allows specific results to be determined, but partial differential equation solutions may be only obtained numerically. Employing the other method in our paper made it possible to derive model equations which describe domain propagation. Simplifications are the neglect of diffusion in continuity equation and small initial amplitude of dipole spatial charge. An almost ideal sample of the semiconductor with excessive electrons in which small heterogeneity only near a cathode takes place is also a particular simplification.

We consider now the radial motion of charge domains in the infinite space of semiconductor. Our starting point is the following set of fundamental equations written in cylindrical geometry

$$n_t + \frac{1}{r}(rnV)_r = 0\,, \tag{1}$$

$$V_t + VV_r = E\,, \tag{2}$$

$$\frac{1}{r}(rE)_r = n - n_0\,, \tag{3}$$

$$n_{0r} + n_0 E = 0 \tag{4}$$

where dimensionless variables are introduced by the following transformation

$$\begin{aligned} r' &\to \sqrt{\frac{kT}{m_e}}\,r\,, \quad n' \to \frac{\varepsilon_1 m_e}{e^2}n\,, \quad E' \to \frac{\sqrt{m_e kT}}{e}E\,, \\ V' &\to \sqrt{\frac{kT}{m_e}}\,V\,, \quad n_0' \to \frac{\varepsilon_1 m_e}{e^2}n_0\,, \quad t' \to t\,, \end{aligned} \tag{5}$$

where $n'$ and $n_0'$ denote the concentration of electrons at the disturbed and undisturbed uniform states, respectively, $V'$ – the electron velocity, $E'$ – the electric field, $e$ – the elementary charge, $m_e$ – the electron mass, $\varepsilon_1$ – the dielectric constant, $T$ – the temperature, $k$ – the Boltzmann's constant, $r'$ – the space coordinate and $t'$ – the time. The indices $r$ and $t$ imply partial differentiation.

We investigate ingoing solutions of equations (1)–(4) in the small amplitude and weak dispersion approximations using the reductive Taniuti–Wei's method [6]. We introduce the following coordinate stretching

$$\xi = \sqrt{\varepsilon}(r - t)\,, \qquad \tau = \sqrt{\varepsilon^3}t\,, \tag{6}$$

where the small parameter $\varepsilon$ measures the weakness of dispersion [7]. Expansion of $n$, $n_0$, $V$, $E$ into power series of the same parameter $\varepsilon$

$$\begin{aligned} n_0 &= 1 + \varepsilon n_0^{(1)} + \varepsilon^2 n_0^{(2)} + \ldots\,, \\ n &= 1 + \varepsilon n^{(1)} + \varepsilon^2 n^{(2)} + \ldots\,, \\ E &= \sqrt{\varepsilon}(\varepsilon E_1 + \varepsilon^2 E_2 + \ldots)\,, \\ V &= \varepsilon V_1 + \varepsilon^2 V_2 + \ldots\,, \end{aligned} \tag{7}$$

leads to the following decomposition of equations (1)–(4) establishing the relationship among the first order perturbed quantities from collecting terms by $\varepsilon$:

$$V_1 = n^{(1)} = n_0^{(1)} \equiv \phi\,, \qquad E_1 = -\phi_\xi\,. \tag{8}$$

From the second order equations $\varepsilon^2$ the compatibility condition gives rise to the cylindrical Korteweg–de Vries equation

$$\phi_\tau + \phi\phi_\xi + \frac{1}{2}\phi_{\xi\xi\xi} + \frac{\phi}{2\tau} = 0\,. \tag{9}$$

This equation and its solutions have been obtained and discussed for the first time by Maxon and Viecelli [8] in connection with radial plasma waves propagation. In a different context, see also in [9–11]. We can go one step further up to the third order of $\varepsilon$ [12, 13], obtaining

$$\tau n^{(2)}_{0\xi\xi\xi} + 2\tau(\phi n^{(2)}_0)_\xi + 2\tau n^{(2)}_{0\tau} + n^{(2)}_0 = S(\phi, \tau, \xi)\,, \tag{10}$$

$$\begin{aligned} S \equiv \; & \tau\phi\phi_{\xi\xi\xi} - \xi\phi_{\xi\xi\xi} + \tau(\phi\phi_\xi)_{\xi\xi} + \tau\phi_{\xi\xi\tau} + \tau\phi_{\xi\xi} + \\ & -\phi_{\xi\xi} - 2\xi\phi\phi_\xi - \tau\phi^2\phi_\xi - 2\tau(\overline{\psi} + C)\phi_\xi - \xi\phi_\tau + \\ & -2\tau\phi\phi_\tau - \tau\overline{\psi}_\tau - \overline{\psi} - \phi^2 - C\,, \end{aligned}$$

$$\overline{\psi} \equiv \int \phi_\tau d\xi\,. \tag{11}$$

$C$ is an integration constant. The source term $S$ describes interaction effects between the fundamental nonlinear domain wave and effects of the higher order dispersion.

Now, turning our interest to a case of strongly dispersive waves, we develop Taniuti–Wei's method [14] to obtain the cylindrical nonlinear Schrödinger equation. For this purpose we expand the dependent variables around the undisturbed uniform state into power series

$$U \equiv \begin{bmatrix} n \\ n_0 \\ V \\ E \end{bmatrix} = U^{(0)} + \sum_{j=1}^{\infty} \varepsilon^j \sum_{l=-\infty}^{\infty} U_l^{(j)}(\xi, \tau) e^{il(kr - wt)}\,, \tag{12}$$

where $U^{(0)}$ represents an undisturbed column vector,

$$U^{(0)} \equiv \begin{bmatrix} 1 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \; U_l^{*(j)} \equiv U_{-l}^{(j)}\,;\; U_l^{(1)} = 0\,,\; m \neq \pm 1\,. \tag{13}$$

The asterisk denotes a complex conjugate. We introduce the following coordinate stretching

$$\xi = \varepsilon(r - \lambda t)\,, \qquad \tau = \varepsilon^2 t\,. \tag{14}$$

Substituting (12)–(14) into the fundamental set of equations (1)–(4) and equating terms of the same order of $\varepsilon$, we obtain from the first order of $\varepsilon$

$$V_1^{(1)} = \frac{w}{k} n_1^{(1)} \equiv c\phi\,, \tag{15}$$

$$E_1^{(1)} = \frac{-iw^2}{k}\phi\,, \tag{16}$$

$$n_{01}^{(1)} = (1-w^2)\phi\,, \tag{17}$$

$$w^2 = \frac{k^2}{1+k^2}\,. \tag{18}$$

From the second order of $\varepsilon$ the following relations are obtained

$$n_{00}^{(2)} = n_0^{(2)}\,, \qquad E_0^{(2)} = 0\,, \tag{19}$$

$$\lambda = w_k\,, \tag{20}$$

$$E_1^{(2)} = (w^2-1)\phi_\xi - ikn_{01}^{(2)}\,, \tag{21}$$

$$V_1^{(2)} = \frac{k}{w}n_{01}^{(2)} + \frac{1}{iw}(1-w^2-\lambda c)\phi_\xi\,, \tag{22}$$

$$n_1^{(2)} = \frac{i}{k}[k^2(w^2-1)-w^2]\phi_\xi + (1+k^2)n_{01}^{(2)}\,, \tag{23}$$

$$V_2^{(2)} = \frac{w(6k^4+9k^2+2)}{6k^3(1+k^2)}\phi^2\,, \tag{24}$$

$$E_2^{(2)} = -i\frac{2(3k^4+3k^2+1)}{3k(1+k^2)^2}\phi^2\,, \tag{25}$$

$$n_{02}^{(2)} = \frac{4k^6+3k^4+9k^2+2}{6k^2(1+k^2)^2}\phi^2\,, \tag{26}$$

$$n_2^{(2)} = \frac{12k^4+15k^2+2}{6k^2(1+k^2)}\phi^2\,. \tag{27}$$

From $\varepsilon^3$ we obtain

$$n_0^{(2)} = -\frac{(2+k^2)(1+k^2)}{k^2(k^4+3k^2+3)}|\phi|^2 + C_1\,, \tag{28}$$

$$V_0^{(2)} = -\frac{2k^6+6k^4+7k^2+2}{wk(1+k^2)(k^4+3k^2+3)}|\phi|^2 + C_2\,. \tag{29}$$

$C_1$, $C_2$ are arbitrary constants which may be determined from the boundary condition and a compatibility condition for the components of $\varepsilon^3$ which is reduced to the cylindrical nonlinear Schrödinger equation for the first order quantities perturbation $\phi$ as follows

$$i\phi_\tau + \alpha\phi_{\xi\xi} + \beta|\phi|^2 + i\frac{\phi}{2\tau} = \gamma\phi\,, \tag{30}$$

where:

$$\alpha = -\frac{3k^2}{2w(1+k^2)^3}\,, \tag{31}$$

$$\beta \equiv -\frac{w(36k^{12} + 177k^{10} + 390k^{8} + 561k^{6} + 414k^{4} + 219k^{2} + 42)}{12k^{2}(1+k^{2})^{2}(k^{4}+3k^{2}+3)}, \tag{32}$$

$$\gamma \equiv -\frac{1}{2}(w^{3}C_{1} + 2kC_{2}). \tag{33}$$

Based on the rigorous reductive Taniuti–Wei's method we have derived the cylindrical Korteweg–de Vries (9) and the nonlinear cylindrical Schrödinger (30) equations depending on whether the system is weakly or strongly dispersive. Also, we have proceeded with our analysis to the next order of approximation for a case of weakly dispersive waves obtaining the model equation (10) which describes an interaction of the fundamental nonlinear domain wave with the higher order one.

The current microwave oscillations appear on condition that $n_0 d$ exceeds critical value (for GaAs $n_0 d > 2.7 \times 10^{15}\mathrm{m}^{-2}$) where $n_0$ is the electron concentration of equilibrium and $d$ is the diode length [5]. The experiment with cylindrical Gunn's effect may be considered in semiconductor of cylindrical shape with two ohmic contacts placing concentricaly inside and outside the semiconductor.

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## STRESZCZENIE

W artykule rozwiązano metodami analitycznymi modelowe równania opisujące cylindryczny efekt Gunna przy uwzględnieniu odpowiednich założeń upraszczających.

Otrzymano rozwiązania w postaci cylindrycznego równania Korteweg–de Vriesa i nieliniowego cylindrycznego równania Schrödingera w zależności od tego, czy system jest słabo lub silnie dyspersyjny.

Z przeprowadzonych rozważań wynikałaby możliwość opisywania powstających w efekcie Gunna fal ładunku w oparciu o nieliniowe efekty dyspersyjne niższych i wyższych rzędów.

## РЕЗЮМЕ

В статье решили аналитическими методами модельные уравнения, описываыущие цилиндрический эффект Гунна, при учёте соответствующих упрощающих предпосылок.

Получили решения в виде цилиндрического уравнения Кортевега-де Фриса и нелинейного уравнения Шрёдингера в зависимости от того, слабо ли или сильно дисперсионна система.

Из проведённых рассуждений вытекала бы возможность описывания возникающих в эффекте Гунна волн груза на основе нелинейных дисперсионных эффектов низших и высших порядков.


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