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Semiclassics with Good Angular Momentum

Teorie semiklasyczne z dobrym momentem pędu

Полуклассика с хорошим моментом количества движения

1. Introduction

Thomas-Fermi theory or semiclassics is a convenient method to formulate average ground state properties of nuclei, like matter and level densities, ground state energy or kinetic energy density; for an overview, see Ring and Schuck[1]. The same methods were also applied to properties of excited nuclei, like multiphonon level densities[2,3], response functions[4], or the optical potential and quantities derived therefrom[5]. Only a few applications, however, treat good angular momentum like Jensen and Luttinger[6] who calculate angular momentum distributions in the Thomas-Fermi model. Other quantities like multiphonon excitation level densities and response functions are also experimentally accessible for given ℓ and correlated occupation numbers and the effective mass of ℓ-states could be compared with those of a Brueckner-Hartree-Fock calculation.

In this paper, we therefore formulate semiclassics for given angular momentum and given z-component or summed over the z-components. In section 2, we derive the propagator for given ℓ, m and other quantities. In section 3 the transformation to Wigner space is discussed. Section 4 contains applications to one-particle and one-particle one-hole level densities and to the imaginary part of the optical potential and the last section summarizes the paper. More details of the theory can be found in ref.[7].

2. Semiclassics in Coordinate Space

In semiclassical approximation the eigenfunctions of the Hamiltonian locally are plane waves

$$< \mathbf{r}|\mathbf{k} >= e^{i\mathbf{k}\cdot\mathbf{r}} \tag{1}$$

and

$$< \mathbf{r}|H|\mathbf{k} >= \frac{\hbar^2 k^2}{2m} + V(r) \tag{2}$$

are the eigenvalues of the Hamiltonian. For spherically symmetric potentials which commute with $\hat{\ell}^2$ and $\hat{\ell}_z$ the projection onto good angular momentum ℓ and its z-component m is achieved by employing the eigenstates of $\hat{\ell}^2$ and $\hat{\ell}_z$,

$$< \mathbf{r}|\mathbf{k}\ell m >= 4\pi i^\ell \, j_\ell(kr) \, Y^*_{\ell m}(\hat{k}) Y_{\ell m}(\hat{r}), \tag{3}$$

or, summed over the magnetic quantum numbers,

$$< \mathbf{r}|\mathbf{k}\ell >= (2\ell+1) i^\ell \, j_\ell(kr) \, P_\ell(\cos\theta), \tag{4}$$

where $\cos\theta = \mathbf{k}\cdot\mathbf{r}/kr$. Here and in the following indices ℓ, m or ℓ denote fixed modulus and z-component of the angular momentum or summed over the z-component, respectively. In semiclassics, the single particle propagator in coordinate representation

$$C^\beta(\mathbf{r},\mathbf{r}') =< \mathbf{r}|e^{-\beta H}|\mathbf{r}' > \tag{5}$$

is the central quantity. Here, β is a Lagrange parameter to constrain the energy, which mostly serves as Laplace-transformed energy, $\mathcal{L}^{-1}_{\beta\to\epsilon}$. For $\beta = 0$ one obtains the unit operator

$$C^{\beta=0}(\mathbf{r},\mathbf{r}') = \delta(\mathbf{r}-\mathbf{r}'). \tag{6}$$

For fixed ℓ, m or ℓ this becomes

$$C^{\beta=0}_{\ell m}(\mathbf{r},\mathbf{r}') = \mathcal{P}_{\ell m}(\mathbf{r},\mathbf{r}')\delta(r-r'),$$
$$C^{\beta=0}_{\ell}(\mathbf{r},\mathbf{r}') = \mathcal{P}_{\ell}(\mathbf{r},\mathbf{r}')\delta(r-r'). \tag{7}$$

Here we have defined the projectors

$$\mathcal{P}_{\ell m}(\mathbf{r},\mathbf{r}') = \frac{Y^*_{\ell m}(\hat{r}) Y_{\ell m}(\hat{r}')}{rr'},$$
$$\mathcal{P}_{\ell}(\mathbf{r},\mathbf{r}') = \frac{2\ell+1}{4\pi}\frac{P_\ell(\cos\chi)}{rr'}, \tag{8}$$

where $\cos\chi = \mathbf{r}\cdot\mathbf{r}'/rr'$. The same result is obtained from the constraint

$$< \mathbf{r}|(2\ell+1)\delta(\hat{\ell}^2 - \ell(\ell+1))\,\delta(\hat{\ell}_z - m)|\mathbf{r}' >= \mathcal{P}_{\ell m}(\mathbf{r},\mathbf{r}')\,\delta(r-r'). \tag{9}$$

As a result, the semiclassical traces become

$$\mathrm{Tr} = 4\int d^3r \int d^3r' \, \delta(\mathbf{r}-\mathbf{r}') \,,$$
$$\mathrm{Tr}_{\ell m} = 4 \int d^3r \int d^3r' \, \mathcal{P}_{\ell m}(\mathbf{r},\mathbf{r}') \, \delta(r-r') \,. \tag{10}$$

The factor of four comes from summation over spin and isospin whereas in the local quantities (not under traces) this is not taken into account. In treating the potential as locally constant the propagators can be evaluated by neglecting $\hbar$-corrections, i.e. using the asymptotic form of the Bessel function, see appendix, with the argument $mr^2/\beta\hbar^2$,

$$C^\beta(\mathbf{r},\mathbf{r}') = \left(\frac{m}{2\pi\hbar^2\beta}\right)^{3/2} \exp\left(-\frac{m}{2\beta\hbar^2}(\mathbf{r}-\mathbf{r}')^2 - \beta V(R)\right),$$
$$C^\beta_{\ell m}(\mathbf{r},\mathbf{r}') = \left(\frac{m}{2\pi\hbar^2\beta}\right)^{1/2} \mathcal{P}_{\ell m}(\mathbf{r},\mathbf{r}') \exp\left(-\frac{m}{\beta\hbar^2}(r-r')^2 - \beta\left(V(R) + \frac{\ell^2\hbar^2}{2mrr'}\right)\right). \tag{11}$$

Here and in the following $\ell(\ell+1)$ is replaced by ℓ^2 in the spirit of lowest order semiclassics but $2\ell+1$ is left because it facilitates summation over ℓ. From the propagator one derives the nonlocal spectral density

$$g^\epsilon(\mathbf{r},\mathbf{r}') = <\mathbf{r}|\delta(\epsilon - H)|\mathbf{r}'> = \mathcal{L}^{-1}_{\beta\to\epsilon}\, C^\beta(\mathbf{r},\mathbf{r}'), \tag{12}$$

the partition function

$$Z^\beta = \mathrm{Tr}\, C^\beta(\mathbf{r},\mathbf{r}') \,, \tag{13}$$

the density of states

$$g(\epsilon) = \mathrm{Tr}\, \delta(\epsilon - H) = \mathcal{L}^{-1}_{\beta\to\epsilon}\, Z^\beta, \tag{14}$$

and the nonlocal density

$$\rho(\mathbf{r},\mathbf{r}') = <\mathbf{r}|\Theta(\lambda - H)|\mathbf{r}'> = \mathcal{L}^{-1}_{\beta\to\lambda}\, C^\beta(\mathbf{r},\mathbf{r}')/\beta, \tag{15}$$

where λ is the Fermi energy measured from the bottom of the potential. Here, relative and center-of mass coordinates are defined as

$$\mathbf{R} = \frac{\mathbf{r}+\mathbf{r}'}{2}; \qquad \mathbf{s} = \mathbf{r}-\mathbf{r}', \tag{16}$$

and the potential $V(R)$ is normalized to $V(R=0) = 0$. With the propagators (11) the nonlocal densities becomes

$$\rho(\mathbf{r},\mathbf{r}') = \frac{p_F^2(R)}{2\pi^2\hbar^2}\, \frac{j_1(p_F(R)s/\hbar)}{s} \,,$$
$$\rho_{\ell m}(\mathbf{r},\mathbf{r}') = \frac{1}{\pi}\mathcal{P}_{\ell m}(\mathbf{r},\mathbf{r}')\, \frac{\sin\left(|r-r'|p_F^\ell(R)/\hbar\right)}{|r-r'|} \,. \tag{17}$$

where the local Fermi momenta are defined as

$$p_F(R) = \sqrt{2m(\lambda - V(R))}\; \Theta(\lambda - V(R)),$$
$$p_F^\ell(R) = \sqrt{2m\left(\lambda - V(R) - \frac{\ell^2\hbar^2}{2mR^2}\right)}\; \Theta\left(\lambda - V(R) - \frac{\ell^2\hbar^2}{2mR^2}\right), \tag{18}$$

and the same local assumption has been made as in the potential, i.e. rr' has been replaced by R^2. From this one derives the local densities

$$\rho(R) = \frac{p_F^3(R)}{6\pi^2\hbar^3}\,,$$
$$\rho_{\ell m}(R) = \frac{1}{4\pi^2\hbar}\,\frac{p_F^\ell(R)}{R^2}\,, \tag{19}$$

which vanish beyond the classical turning point R_λ where $p_F(R_\lambda) = 0$ and the kinetic energy densities

$$\tau(R) = \frac{\partial^2\rho(\mathbf{r},\mathbf{r}')}{\partial\mathbf{r}\partial\mathbf{r}'}\Big|_{\mathbf{r}=\mathbf{r}'=R} = \frac{p_F^5(R)}{10\pi^2\hbar^5}\,,$$
$$\tau_{\ell m}(R) = \frac{1}{2\pi^2\hbar^3}\,\frac{p_F^\ell(R)^3}{R^2}\,. \tag{20}$$

Since the quantities given above do not depend on the magnetic quantum number m, the corresponding m-summed quantities simply obtain by multiplying by a factor of $(2\ell+1)$. The single-particle level densities reads

$$g(\epsilon) = \frac{4}{\pi}\left(\frac{2m}{\hbar^2}\right)^{3/2}\int dr\, r^2\sqrt{\epsilon - V(r)}\;\Theta(\epsilon - V(r)),$$
$$g^\ell(\epsilon) = 2\frac{2\ell+1}{\pi\hbar}\sqrt{2m}\int dr\left(\epsilon - V(r) - \frac{\ell^2\hbar^2}{2mr^2}\right)^{-1/2}\Theta\left(\epsilon - V(r) - \frac{\ell^2\hbar^2}{2mr^2}\right). \tag{21}$$

The limits of integration are determined by the classically allowed region defined by the step function. The level density for given angular momentum can also be found in ref.[6] in integrated form.

3. Semiclassics in Wigner space

Semiclassics is also conveniently formulated in Wigner space $(\mathbf{R},\mathbf{P})$, where the relative coordinate $\mathbf{s}$ is Fourier transformed into $\mathbf{P}$,

$$f(\mathbf{R},\mathbf{P}) = \int d^3s\; e^{-i\mathbf{s}\cdot\mathbf{P}/\hbar} f(\mathbf{r},\mathbf{r}'). \tag{22}$$

The trace becomes

$$\mathrm{Tr} = 4\int d^3R\,\frac{d^3P}{(2\pi\hbar)^3}\,, \tag{23}$$

and the local limit is reached by integrating over all momenta. Since the Wigner transform of the Hamiltonian is equal to the classical Hamiltonian, the relevant quantities obtain simple forms. The propagator, for instance becomes

$$C^\beta(R,P) = e^{-\beta H(P,R)}\,, \tag{24}$$

where $H(P,R) = P^2/2m + V(R)$ is the classical Hamiltonian, and the nonlocal density reads

$$\rho(R,P) = \Theta(\lambda - H(P,R)). \tag{25}$$

Quantum mechanically and also semiclassically only ℓ and m are good quantum numbers whereas classically all three components of the angular momentum are fixed. This means that the quantum average over the x,y-components of the angular momentum is represented by the azimuthal average

$$\bar{\delta}(\mathbf{l}-\mathbf{l}') = \ell \int d(\ell_\phi - \ell'_\phi)\, \delta(\mathbf{l}-\mathbf{l}') = \delta(\ell-\ell')\delta(m-m'). \tag{26}$$

The classical limit then obtains from[8] $\ell_x, \ell_y \ll \ell_z \approx \ell$. The Wigner transform of eq.(9), hence is given by the classical counterpart

$$\bar{\delta}(\mathbf{R}\times\mathbf{P}/\hbar - \mathbf{l}) = \delta(RP\sin\theta/\hbar - \ell)\, \delta((\mathbf{R}\times\mathbf{P})_z/\hbar - m), \tag{27}$$

where $\cos\theta = \mathbf{R}\cdot\mathbf{P}/RP$. Then the trace becomes

$$\mathrm{Tr}_{\ell m} = 4\int d^3R \frac{d^3P}{(2\pi\hbar)^3}\, \bar{\delta}(\mathbf{R}\times\mathbf{P}/\hbar - \mathbf{l}). \tag{28}$$

Since in Wigner space and neglecting $\hbar$-corrections the expectation value of the product of two operators is equal to the product of the expectation values, all single particle quantities become proportional to $\bar{\delta}(\mathbf{R}\times\mathbf{P}/\hbar - \mathbf{l})$. The propagator for instance becomes

$$C^\beta_{\ell m}(\mathbf{R},\mathbf{P}) = \bar{\delta}(\mathbf{R}\times\mathbf{P}/\hbar - \mathbf{l})\, e^{-\beta H(P,R)}, \tag{29}$$

which, summed over m, can be rewritten as

$$C^\beta_\ell(\mathbf{R},\mathbf{P}) = \delta(PR\sin\theta/\hbar - \ell)\, \exp -\beta\left(\frac{P_R^2}{2m} + \frac{\ell^2\hbar^2}{2mR^2} + V(R)\right). \tag{30}$$

This now contains the familiar radial kinetic energy by virtue of $P_R = P\cos\theta$ and the centrifugal potential. The nonlocal spectral density

$$g^\epsilon_{\ell m}(\mathbf{R},\mathbf{P}) = \bar{\delta}(\mathbf{R}\times\mathbf{P}/\hbar - \mathbf{l})\, \delta(\epsilon - H(P,R)) \tag{31}$$

simply reflects energy and momentum conservation and the nonlocal density reads

$$\rho_{\ell m}(\mathbf{R},\mathbf{P}) = \bar{\delta}(\mathbf{R}\times\mathbf{P}/\hbar - \mathbf{l})\, \Theta(\lambda - H(P,R)). \tag{32}$$

Experimentally, however, only angle averaged quantities are accessible. Hence, averaging over the polar angles θ,ϕ the angular momentum constraint becomes

$$\delta^\ell(R,P) \equiv \int_{-1}^{+1} \frac{d\cos\theta}{2}\delta(RP\sin\theta/\hbar - \ell) = \frac{\hbar\ell\,\Theta(RP/\hbar - \ell)}{2RP\sqrt{R^2P^2/\hbar^2 - \ell^2}}. \tag{33}$$

The nonlocal density, for instance, averaged over all angles, becomes

$$< \rho_\ell(R,P) >= \delta^\ell(R,P)\ \Theta(\lambda - P^2/2m - V(R)), \tag{34}$$

from which one derives eq.(19) by momentum integration.

The quantities derived above by Wigner transforming eq.(9) can also be obtained by direct Wigner transform of the semiclassically asymptotic quantities (see appendix).

4. Densities and level densities

In this section, we calculate densities and level densities and the imaginary part of the optical potential for given angular momentum by employing an harmonic oscillator potential. The oscillator constant is denoted by $\hbar\omega = 41\text{MeV}/N^{1/3}$, and $R_\circ = 1.2\text{fm}N^{1/3}$ is used for the radius. The Fermi energy follows from number conservation,

$$\lambda = (3N/2)^{1/3}\hbar\omega. \tag{35}$$

The partition function can be evaluated to give

$$Z_\ell^\beta = 2\frac{2\ell+1}{\beta\hbar\omega}\,e^{-\beta\hbar\omega} \tag{36}$$

and the level density becomes

$$g^\ell(\epsilon) = 2\frac{2\ell+1}{\hbar\omega}\Theta\left(\frac{\epsilon}{\hbar\omega} - \ell\right). \tag{37}$$

The level density of the harmonic oscillator simply consists of step functions for given energy. The local densities of an harmonic oscillator for fixed angular momentum is shown in Fig. 1.

One notices that only the $\ell = 0$ states contribute at the center. The maximum angular momentum for given energy follow from the step function in eq.(18),

$$L(\epsilon) = \epsilon/\hbar\omega. \tag{38}$$

Next, we calculate the one-particle one-hole level density. Integrated over all angular momenta it is defined as

$$g_{1p1h}(\epsilon) = \mathrm{Tr}_1\mathrm{Tr}_2\ \Theta(\lambda - H_2)\ \Theta(H_1 - \lambda)\ \delta(\epsilon - H_1 + H_2). \tag{39}$$

For fixed ℓ, m this goes over into

$$\begin{aligned} g_{1p1h}^{\ell m}(\epsilon) &= \mathrm{Tr}_1\mathrm{Tr}_2 \int d\ell_1 \int d\ell_2 \int dm_1 \int dm_2 \Theta(\lambda - H_2)\ \Theta(H_1 - \lambda)\ \delta(\epsilon - H_1 + H_2) \\ &\times \bar\delta(\mathbf{R}_1 \times \mathbf{P}_1/\hbar - \mathbf{l}_1)\bar\delta(\mathbf{R}_2 \times \mathbf{P}_2/\hbar - \mathbf{l}_2)\bar\delta(\mathbf{l} - (\mathbf{l}_1 + \mathbf{l}_2)). \end{aligned} \tag{40}$$

By symmetry, the result does not depend on m and the dependence of the azimuthal angles of the traces is solely contained in the last delta function. Eq.(40) is therefore rewritten as

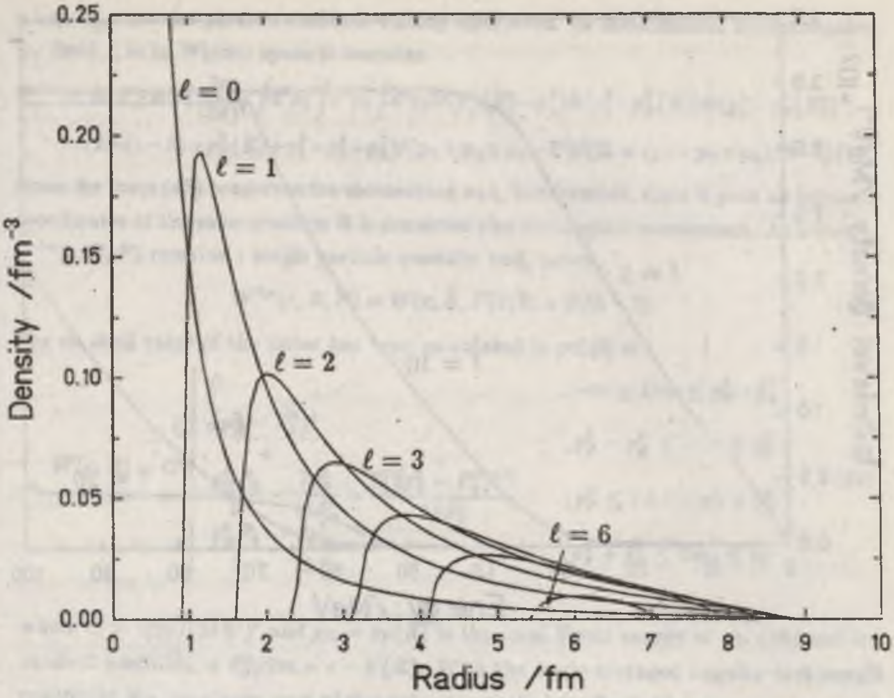

Figure 1:
Spin-isospin summed density of 200 particles for given angular momentum ($\ell = 0...6$).

$$
\begin{aligned}
g^{\ell m}_{1p1h}(\epsilon) &= \mathrm{Tr}_1\mathrm{Tr}_2 \int d\ell_1 \int d\ell_2 \int dm_1 \int dm_2 \Theta(\lambda - H_2)\ \Theta(H_1 - \lambda)\ \delta(\epsilon - H_1 + H_2) \\
&\quad \times \delta(R_1 P_1 \sin\theta_1/\hbar - \ell_1)\delta(R_2 P_2 \sin\theta_2/\hbar - \ell_2)\Delta(\ell_1\ell_2, \ell),
\end{aligned} \tag{41}
$$

where

$$
\begin{aligned}
\Delta(\ell_1\ell_2, \ell) &= \int \frac{d(\phi_2 - \phi_1)}{2\pi}\delta((\mathbf{l}_1 + \mathbf{l}_2)^2 - \mathbf{l}^2) \\
&= \frac{1}{\pi}\frac{\Theta(2\ell_1\ell_2 - |\ell^2 - \ell_1^2 - \ell_2^2|)}{\sqrt{4\ell_1^2\ell_2^2 - (\ell^2 - \ell_1^2 - \ell_2^2)^2}}
\end{aligned} \tag{42}
$$

is the angle averaged triangular relation.

By using the methods of refs.[2,3] it is transformed into a folding expression for fixed angular momentum

$$
g^{\ell}_{1p1h}(\epsilon) = \int d\epsilon_1 \int d\epsilon_2 \int d\ell_1 \int d\ell_2\ g^{\ell_1}_{1p}(\epsilon_1) g^{\ell_2}_{1h}(\epsilon_2)\delta(\epsilon - \epsilon_1 + \epsilon_2)\Delta(\ell_1\ell_2, \ell). \tag{43}
$$

Here g^{ℓ}_{1p} and g^{ℓ}_{1h} are the one-particle and one-hole level densities, i.e. the single particle level densities restricted to energies above and below the Fermi energy, respectively,

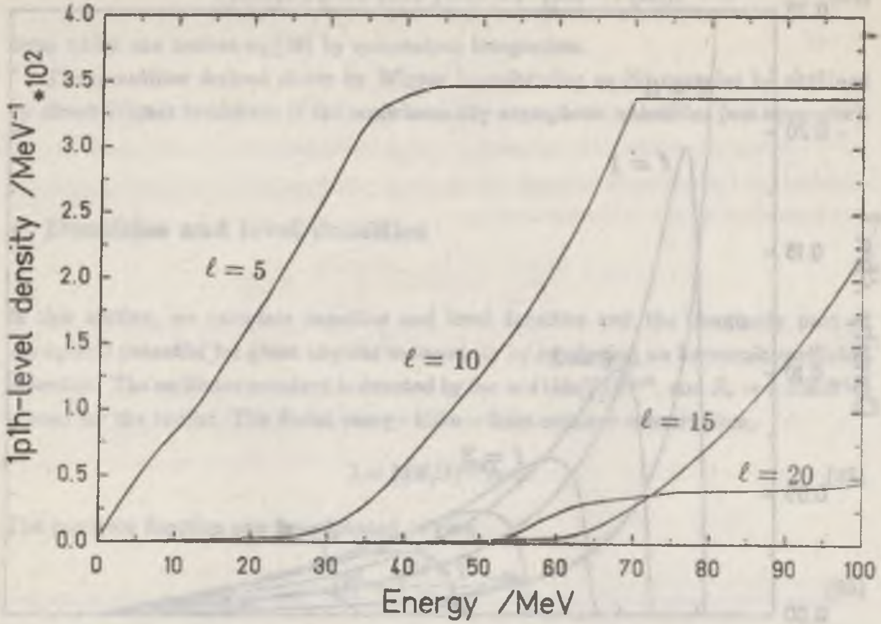

Figure 2:
One-particle one-hole excitation level density of 200 particles for given angular momentum ($\ell = 5...20$).

$$g^\ell_{1p}(\epsilon) = g^\ell(\epsilon)\,\Theta(\epsilon - \lambda),$$
$$g^\ell_{1h}(\epsilon) = g^\ell(\epsilon)\,\Theta(\lambda - \epsilon). \tag{44}$$

Note that our result eq.(43) differs from the expression recently proposed by Krappe[9] who did not account for the angle averaging in the angular momentum conserving delta function but it is conform with the semiclassical limit of the response function of Delafiore[10]. The result is shown in Fig. 2. The levelling off which starts at the Fermi energy and the later onset of higher angular momenta is due to the maximal angular momenta at the Fermi energy.

Finally, the on-shell value of the imaginary part of the optical potential including polarization and correlation graphs is calculated with a zero range two-body interaction

$$v(\mathbf{r} - \mathbf{r}') = -v_t\,\delta(\mathbf{r} - \mathbf{r}'). \tag{45}$$

Without angular momentum constraints in coordinate space it is obtained from the imaginary part of the mass operator[5],

$$W(\epsilon, \mathbf{r}, \mathbf{r}') = v_t^2 \frac{\pi}{2} \mathcal{L}^{-1}_{\beta\to\epsilon}\big(< \mathbf{r}|\rho e^{\beta H}|\mathbf{r}' >< \mathbf{r}|\bar{\rho} e^{-\beta H}|\mathbf{r}' >^2 + < \mathbf{r}|\bar{\rho} e^{-\beta H}|\mathbf{r}' >< \mathbf{r}|\rho e^{\beta H}|\mathbf{r}' >^2\big)\,, \tag{46}$$

where $\bar{\rho}, \rho$ are the particle and hole density operators. In semiclassical approximation for fixed ℓ, m in Wigner space it becomes

$$W^{\ell m}(\epsilon, R, P) = v_\delta^2 \frac{2m}{(2\pi)^6} \int d^3p_1 \int d^3p_2 \int d^3p_3 \Theta(p_F^2(R) - p_1^2)\Theta(p_2^2 - p_F^2(R))\Theta(p_3^2 - p_F^2(R))$$
$$\times \delta(2m(\epsilon - \lambda) + p_F^2(R) + p_1^2 - p_2^2 - p_3^2)\delta(\mathbf{p}_1 + \mathbf{p}_2 + \mathbf{p}_3 - \mathbf{P})\bar{\delta}(\mathbf{R} \times (\mathbf{p}_1 + \mathbf{p}_2 + \mathbf{p}_3)/\hbar - \mathbf{l}). \quad (47)$$

Since the force (45) conserves the momentum and, furthermore, since it puts all intrinsic coordinates at the same position **R** it conserves also the angular momentum. As a result, $W^{\ell m}(\epsilon, R, P)$ remains a single particle quantity and, hence,

$$W^{\ell m}(\epsilon, R, P) = W(\epsilon, R, P)\bar{\delta}(\mathbf{R} \times \mathbf{P}/\hbar - \mathbf{l}). \quad (48)$$

The on shell value of the latter has been calculated in ref.[5] as

$$W(\epsilon, R) = C \begin{cases} 0 & , -\infty \le 2m\epsilon \le p_F^2 - p_R^2 \\ \frac{(p_R^2 - P_0^2)^2}{4} & , p_F^2 - p_R^2 \le 2m\epsilon \le p_F^2 \\ \frac{p_R^3 P_0}{3} - \frac{7p_R^5}{15P_0} + \frac{2(2p_R^2 - P_0^2)^{5/2}}{15P_0} & , p_F^2 \le 2m\epsilon \le p_F^2 + p_R^2 \\ \frac{p_R^3 P_0}{3} - \frac{7p_R^5}{15P_0} & , p_F^2 + p_R^2 \le 2m\epsilon \le \infty \end{cases} \quad (49)$$

where $C = v_\delta^2 m/(2\pi\hbar^2)^3$ and $p_R = p_F(R)$ is the local Fermi energy of eq. (18) and the on-shell condition is $P_0^2/2m = \epsilon - V(R)$. With the angle averaged angular momentum constraint the imaginary part of the optical potential for fixed ℓ becomes

$$W^\ell(\epsilon, R) = \delta^\ell(P_0, R)\, W(\epsilon, R). \quad (50)$$

As strength for the two-body interaction, $v_\delta = 745$ MeV fm^3 is used which reproduces the experimentally known quadratic behaviour around the Fermi energy and a harmonic oscillator potential is employed. The result is shown in Fig.3.

One observes that due to the classical condition

$$\epsilon > V(R) + \frac{\hbar^2 \ell^2}{2mR^2} \quad (51)$$

for energies below the Fermi energy only a few angular momenta contribute. The higher partial bound states have a larger W^ℓ and since the single-particle spreading width is proportional to the imaginary part of the optical potential also a larger spreading width. This seems to be in agreement with recent observations of Langevin-Joliot et. al.[11] At high energy the imaginary part of the optical potential behaves like the centrifugal potential.

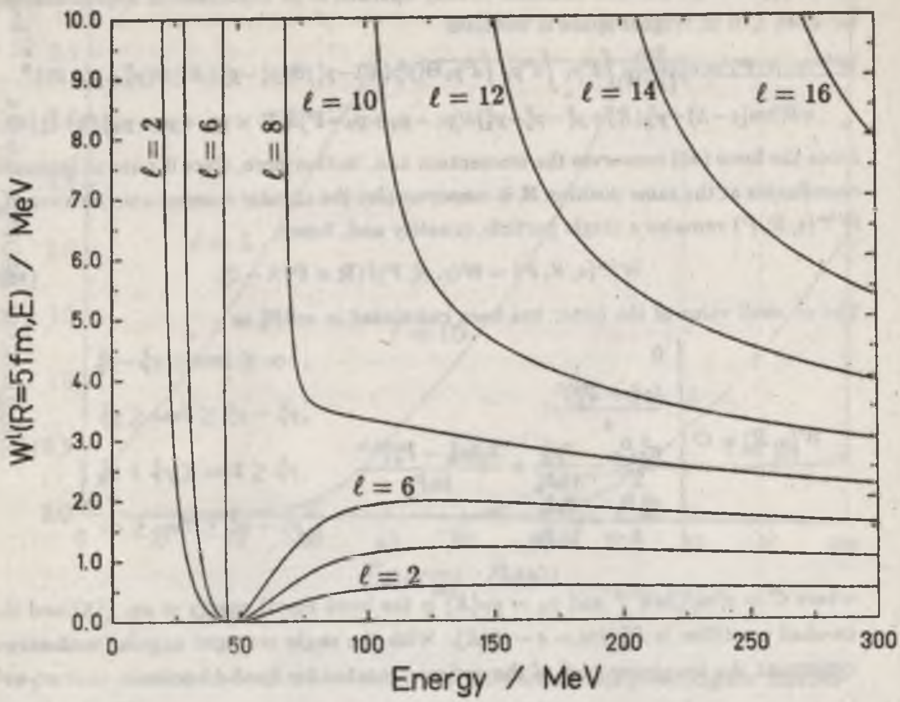

Figure 3:
Imaginary part of the optical potential of 200 particles at R=5 fm in a harmonic oscillator for given angular momentum ($\ell = 2...16$). The Fermi energy is at 47 MeV.

5. Summary and outlook

Semiclassics has been generalized to cases where the the modulus and the z-component of the angular momentum is fixed. We derived the propagator and other quantities in $(\mathbf{r}, \mathbf{r}')$ - space and in Wigner space $(\mathbf{R}, \mathbf{P})$. In coordinate space quantities under traces are easily evaluated because angular momentum projection can be done solely in the traces. In Wigner space all single-particle quantities are proportional to an angle averaged angular momentum conserving delta function. As applications, densities and one-particle one-hole excitation level densities have been calculated for an harmonic oscillator potential and the imaginary part of the optical potential has been obtained for a zero range two-body interaction and with an harmonic oscillator potential. The schemes outlined are very flexible and can be applied to arbitrary potentials and work is under way to employ realistic Woods-Saxon potentials and also deformed potentials which do not commute with $\hat{\ell}^2, \hat{\ell}_z$.

Furthermore, higher multiphonon excitation level densities will be calculated by using the analytic methods of refs.[2,3] as well as response functions. Also the real part of the optical potential for fixed ℓ and the therefrom derived correlated occupation numbers and the effective mass are of great interest in order to calculate realistic nuclear charge densities[12].

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The author likes to thank Dr. P. Schuck for the suggestion of this subject.

Appendix: Mathematical details

In the appendix we give some relevant formulas for the derivation of the asymptotic propagators.

Eq. (11) is derived by using the asymptotic expansion of the modified Bessel function of large argument

$$I_{\ell+1/2}(z) \to (2\pi z)^{-1/2}\, e^{z-\ell(\ell+1)/2z}, \tag{52}$$

which semiclassically is the first term of an $\hbar$- expansion in the large quantity $1/\hbar^2$.

Direct transforms from coordinate to Wigner space of the ℓ- projected quantities proceed as follows. First, the asymptotic expansion of the Legendre polynomials for large ℓ in the projector of eq.(8)

$$P_\ell^m(\cos\alpha) \to (-)^m \frac{(\ell+m)!}{\ell!} J_m((\ell+1/2)\chi) \tag{53}$$

is used to transform the projector from $(\mathbf{r}, \mathbf{r}')$ space to $(\mathbf{R}, \mathbf{s})$ space,

$$\mathcal{P}_\ell(\mathbf{R}, \mathbf{s}) = \frac{\ell}{2\pi R^2} J_0(\frac{\ell}{R} s \sin\sigma), \tag{54}$$

where $\cos\sigma = \mathbf{R}\cdot\mathbf{s}/Rs$. In taking the R-axis as integration axis it is noted that in cylindrical coordinates $\mathbf{s} = (s_\rho = s\sin\sigma, s_z = s\cos\sigma, \phi)$, the projector depends only on s_ρ, whereas the remaining integrands depend only on s_z, for instance

$$\delta(r - r') = \delta(s_z). \tag{55}$$

Then the Fourier transform $(\mathbf{s} \to \mathbf{P})$ separates and the integral representation of the Bessel function

$$J_m(u) = \int_0^{2\pi} \frac{d\phi}{2\pi} e^{iu\cos\phi + im(\phi - \pi/2)} \tag{56}$$

and the orthogonality relation of the Bessel function,

$$\int_0^{\infty} ds\; s\, J_\nu(us) J_\nu(vs) = \frac{\delta(u-v)}{\sqrt{uv}}, \tag{57}$$

are used to reduce the Wigner transform to a one-dimensional Fourier transform,

$$\int d^3s\; e^{i\mathbf{s}\cdot\mathbf{P}/\hbar} \frac{\ell}{2\pi R^2}\, J_0\left(\frac{\ell}{R} s_\rho\right) f(s_z) = \delta(PR\sin\theta/\hbar - \ell) \int ds_z\; e^{iPs_z\cos\theta/\hbar} f(s_z). \tag{58}$$

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STRESZCZENIE

Sformułowano wyrażenia semiklasyczne dla ustalonego modułu i z-owej składowej momentu pędu. Jako funkcji falowych użyto, wyrzutowanych na określony moment pędu, fal płaskich, uzyskując semiklasyczne wyrażenia dla jednocząstkowego propagatora i wynikających z niego wielkości, jak: partycja funkcji, nielokalna macierz gęstości, gęstość poziomów jednocząstkowych i gęstość poziomów cząstka-dziura. W przestrzeni Wignera są one proporcjonalne do kombinacji momentu pędu: $\delta(|\vec{R} \times \vec{P}|/\hbar - l)$ i $\delta((\vec{R} \times \vec{P})_z/h - m)$. Formalizm ten zastosowano do wyliczenia cząstka-dziura oraz urojonej części potencjału optycznego na bazie oscylatora harmonicznego z 2-ciałowym oddziaływaniem o zasięgu zerowym.

РЕЗЮМЕ

Сформулированы полуклассические выражения для определенного значения и z-составляющей момента количества движения. В качестве волновых функций применены плоские волны проектированные на заданный момент количества движения. Получены полуклассические выражения для одночастичного пропагатора и вытекающих из него величин: распределение функции, нелокальная матрица плотности, плотность одночастичных и частично-дырочных уровней. В пространстве Вигнера они пропорциональны комбинации момента количества движения: $\delta(|\vec{R} \times \vec{P}|/\hbar - l)$ и $\delta((\vec{R} \times \vec{P})_z(\hbar - m))$. Этот формализм применялся для расчета плотности одночастичных уровней и состояний частица-дырка, а также мнимой части оптического потенциала на основе гармонического осциллятора с двухтельным взаимодействием с нулевым радиусом действия.