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π-Geodesics on Hypersurfaces

π-geodezyjne na hiperpowierzchniach

π-геодезические на хиперповерхностях

In this paper we deal with the problem of the coincidence of π-geodesics with geodesics on a hypersurface $M^n \subset E^{n+1}$. In the first part of this paper we'll consider those π-geodesics which are determined by a tensor π of the type (0,2) that is associated in a natural way with hypersurfaces e.g. the third and fourth fundamental tensors of hypersurfaces. In the second part we define π-geodesics determined by a tensor field π of the type (1,1).

1. π-GEODESICS WITH π BEING OF THE TYPE (0,2).

First we recall some fundamental definitions and theorems.

DEFINITION 1 [8]. A vector field $\overline{w}$ on a manifold M with a given linear connection Γ is said to be π-geodesic if:

$$\nabla_{\bar{w}} \pi^{\bar{w}} = \lambda \pi^{\bar{w}} ; \qquad \pi^{\bar{w}} : \bar{v} \longmapsto \pi(\bar{v}, \bar{w})$$

where π is a symmetric, non-singular tensor field on M of the type (0,2) and λ is a real differentiable function on M. The integral curve of the π-geodesic vector field on M is called the π-geodesic line.

In a local map U, the equation of this line is:

$$(1) \qquad \frac{d^2u^i}{dt^2} + (\Gamma^i_{ks} + \tilde{\pi}^{ip} \nabla_k \pi_{ps}) \frac{du^k}{dt} \frac{du^s}{dt} = \lambda \frac{du^i}{dt}$$

THEOREM 1 [6], [7]. The necessary and sufficient condition for π-geodesics on a manifold M to coincide with geodesics of the connection Γ is:

$$(2) \qquad \nabla_k \pi_{ij} + \nabla_i \pi_{kj} = p_k \pi_{ij} + p_i \pi_{kj}$$

In particular, if π is symmetric, then (2) becomes:

$$(3) \qquad \nabla_k \pi_{ij} = p_k \pi_{ij}$$

where p_k is some covector field on M.

THEOREM 2 [3]. If, on a surface $M^2 \subset E^3$ with $K \neq 0$, symmetric tensor fields g and $\hat{g}$ are the solutions of the equation: $\nabla_k g_{ij} = 0$ and $\det(g_{ij}) \neq 0$, $\det(\hat{g}_{ij}) \neq 0$, then $\hat{g} = \alpha g$, $\alpha = \text{const.}$ (∇ - the Levi-Civita connection).

DEFINITION 2 [5]. Let V be an n-dimensional vector space and $R \in \mathrm{Hom}(V \wedge V, \mathrm{Hom}(V,V))$. The mapping R is said to be regular if and only if $R(X \wedge Y) \neq 0$ for each bivector $X \wedge Y \in V \wedge V - \{0\}$.

DEFINITION 3. A point $x \in M$ is said to be regular for the curvature tensor R of a differentiable manifold M with a linear connection Γ if

$$R_x \in \mathrm{Hom}(T_xM \wedge T_xM, \mathrm{Hom}(T_xM, T_xM))$$

is a regular mapping.

We have:

THEOREM 3 [5]. Suppose, that two Riemannian connections given on a connected differentiable manifold M of dimension $n \geq 3$ with metric tensors g and $\hat{g}$, have the same curvature tensors and the set of regular points of these tensors is dense in M. Then $\hat{g} = \lambda g$ where $\lambda = \mathrm{const}$.

REMARK 1. Observe, that in case of a surface $M^2 \subset E^3$, the dimension of $T_xM^2 \wedge T_xM^2$ is one and the condition of regularity of the curvature tensor R of the Levi-Civita connection of M^2 at any x is equivalent to the non-vanishing of the Gaussian curvature of M^2 at this point, namely: if g_{ij} and b_{ij} are components of the first and the second fundamental tensors of M^2 respectively, then:

$$R_{jklp} = R^i_{jkl} g_{ip} = b_{jp} b_{kl} - b_{kp} b_{jl}$$

or

$$R^i_{jkl} = (b_{jp} b_{kl} - b_{kp} b_{jl}) g^{pi}$$

and if $\bar{x}_1, \bar{x}_2$ are vectors of the natural basis of T_xM^2, then:

$$R(\bar{x}_1 \wedge \bar{x}_2) = (b_{1i} b_{2k} - b_{2i} b_{1k}) g^{ri}$$

Since $\det(g^{ri}) \neq 0$, then $R(\bar{x}_1 \wedge \bar{x}_2) = 0$ if and only if:

$b_{1i}b_{2k} - b_{2i}b_{1k} = 0$ for each $i,k = 1,2$, what is equivalent to $\det(b_{ij}) = 0$. And so $R(\bar{x}_1 \wedge \bar{x}_2) \neq 0$ is equivalent to $K \neq 0$.

When we combine theorems 2 and 3 we get:

THEOREM 4. If Γ is the Levi-Civita connection of a hypersurface $M^n \subset E^{n+1}$, $n \geqslant 2$ such, that the set of regular points of the curvature tensor R is dense in M^n and g, $\hat{g}$ are symmetric, non-singular tensors with $\nabla_i g_{jk} = \nabla_i \hat{g}_{jk} = 0$ then $\hat{g}_{ij} = \lambda g_{ij}$ with $\lambda = \text{const}$.

Now we can prove two more theorems:

THEOREM 5. Suppose, that on a Riemannian manifold M with the metric tensor g, there is given a non-singular, symmetric tensor field π of the type $(0,2)$ satisfying $\pi_{ij}g^{ij} \neq 0$ everywhere on M. Then, if π-geodesics coincide with geodesics on M, we have: $\nabla_k \pi_{ij} = p_k \pi_{ij}$ and $p_k = \partial_k \ln|\pi^g|$ where $\pi^g = \pi_{ij}g^{ij}$ and there exists a scalar function $\lambda \neq 0$ such, that $\nabla_k(\lambda \pi_{ij}) = 0$ and $\lambda = c(\pi^g)^{-1}$, $c = \text{const} \neq 0$.

P r o o f. Since π-geodesics are geodesics on M, thus:

$$\nabla_k \pi_{ij} = p_k \pi_{ij}$$

In virtue of the fact that $\nabla_k g^{js} = 0$ we have:

$$\nabla_k(\pi_{ij}g^{js}) = p_k \pi_{ij}g^{js}$$

Putting $s = i$ and summing with respect to i, we get:

$$\nabla_k \pi^g = p_k \pi^g$$

or

$$\partial_k \pi^g = p_k \pi^g$$

Hence

$$p_k = \partial_k \ln |\pi^g|$$

Now, let $\hat{\pi} = \lambda \pi$, $\lambda \neq 0$ and $\nabla \hat{\pi} = 0$, then:

$$\nabla_k \hat{\pi}_{ij} = \partial_k \lambda \pi_{ij} + \lambda \nabla_k \pi_{ij} = \partial_k \lambda \pi_{ij} + \lambda p_k \pi_{ij} =$$

$$= \partial_k \lambda \pi_{ij} + \lambda \partial_k \ln |\pi^g| \pi_{ij} = 0$$

Hence

$$\partial_k \lambda + \lambda \partial_k \ln |\pi^g| = 0$$

or

$$\partial_k \ln |\lambda \pi^g| = 0$$

or

$$\lambda = c(\pi^g)^{-1}$$ Q.E.D.

THEOREM 6. Suppose that two Riemannian connections Γ and $\hat{\Gamma}$ are given on a differentiable, connected manifold M with the metric tensors g and $\hat{g}$, respectively. Assume that the set of regular points of the curvature tensor R of the connection Γ is dense in M and $\nabla_i \hat{g}_{kj} = \nabla_k \hat{g}_{ij}$, where ∇ is a differentiation operator with respect to Γ. Then Γ and $\hat{\Gamma}$ have the same geodesics if and only if $\hat{g} = \lambda g$ with $\lambda = \text{const}$.

P r o o f. From the condition (11) [8] we know that Γ and $\hat{\Gamma}$ determine the same family of geodesics if and only if:

$$\Gamma^i_{jk} = \hat{\Gamma}^i_{jk} + p_j \delta^i_k + p_k \delta^i_j$$

Now we can find: $\nabla_j \hat{g}_{kr}$:

$$\nabla_j \hat{g}_{kr} = \hat{\nabla}_j \hat{g}_{kr} - (p_j \delta_k^i + p_k \delta_j^i)\hat{g}_{ir} - (p_j \delta_r^i + p_r \delta_j^i)\hat{g}_{ik} =$$

$$= -(2p_j \hat{g}_{kr} + p_k \hat{g}_{jr} + p_r \hat{g}_{jk})$$

Since $\nabla_j \hat{g}_{kr} = \nabla_k \hat{g}_{jr}$, so we have:

$$2p_j \hat{g}_{kr} + p_k \hat{g}_{jr} + p_r \hat{g}_{jk} = 2p_k \hat{g}_{jr} + p_j \hat{g}_{kr} + p_r \hat{g}_{jk}$$

or

$$p_j \hat{g}_{kr} = p_k \hat{g}_{jr}$$

or

$$p_j \delta_k^s = p_k \delta_j^s$$

Putting $s = k$ and summing over k, we get:

$$p_j = 0$$

Hence

$$\Gamma_{jk}^i = \hat{\Gamma}_{jk}^i$$

Now, from the Theorem 4 it follows that:

$$\hat{g}_{ij} = \lambda g_{ij}, \qquad \lambda = \text{const.} \qquad \text{Q.E.D.}$$

The following will be useful:

THEOREM 7 [6]. Suppose, that Γ is a symmetric, linear connection on M with the symmetric Ricci tensor $\mathcal{R}$. If π is a non-singular tensor field of the type (0,2) and $\nabla_k \pi_{ij} = p_k \pi_{ij}$ where p is a covector field on M, then p is the gradient field (i.e. $p_k = \partial_k f$, where f is a scalar function on M).

REMARK 2. If the assumptions of this theorem hold, then one can find a scalar function $\lambda \neq 0$ such that

$\nabla_k(\lambda \pi_{ij}) = 0$ (compare the proof of the theorem 5).

At last, we deal with π-geodesics in case π is a tensor field associated in a natural way with a hypersurface $M^n \subset E^{n+1}$ i.e. π is either the third, or the fourth fundamental tensor of a hypersurface.

In the paper [2] the following has been proved:

THEOREM 8 [2]. Let $M^2 \subset E^3$ be a surface and $K \neq 0$ being its Gaussian curvature. The family of b-geodesics on M^2 coincides with the family of geodesics of this surface if and only if $K = \text{const}$ and $H = \text{const}$, where b is the second fundamental tensor of M^2.

We can prove:

THEOREM 9. On a surface $M^2 \subset E^3$ with the gaussian curvature $K \neq 0$ h-geodesics, where $h_{ij} = \alpha b_{ij} + \beta g_{ij}$, $\alpha \neq 0$, $\det(h_{ij}) \neq 0$, coincide with geodesics of this surface if and only if M^2 is a sphere (locally).

P r o o f.

$\Rightarrow$ On account of the Theorem 7 and the Remark 2, we have:

$$\nabla_k(\lambda h_{ij}) = 0$$

From the Theorem 2, we get:

$$\lambda h_{ij} = \mu g_{ij}$$

or

$$b_{ij} = \gamma g_{ij}$$

We now show that γ is constant.

From the above equality we get:

$$\nabla_k b_{ij} = \partial_k \gamma \, g_{ij}$$

Having used the Codazzi equations for b_{ij}, we obtain:

$$0 = \nabla_k b_{ij} - \nabla_i b_{kj} = \partial_k \gamma g_{ij} - \partial_i \gamma g_{kj}$$

or

$$\partial_k \gamma \delta^s_i - \partial_i \gamma \delta^s_k = 0$$

Putting $s = i$, we get:

$$\partial_k \gamma (n - 1) = 0$$

and γ = const for $n > 1$.

$\Leftarrow$ It is obvious. Q.E.D.

DEFINITION 4 [4]. Let $M^n \subset E^{n+1}$ be an orientable hypersurface and N denote the normal vector field to M^n, then a tensor field c of the type $(0,2)$ on M^n defined in the following way:

$$c(X,Y) = (dN)(X) \cdot (dN)(Y)$$

is said to be the third fundamental tensor of M^n.

If g_{ij} and b_{ij} are the components of the first and the second fundamental tensors of M^n respectively, then:

$$c_{ij} = b_{is} g^{st} b_{tj} \tag{4}$$

are the components of the third fundamental tensor of M^n. In particular, for $n = 2$:

$$c_{ij} = 2Hb_{ij} - Kg_{ij} \tag{5}$$

where K and H are the Gaussian and the mean curvatures of M^2 respectively.

Now, if we take $\alpha = 2H$ and $\beta = -K$ in the Theorem 9, we'll get:

THEOREM 10. *On a surface* $M^2 \subset E^3$ *with* $K \neq 0$ *and* $H \neq 0$, *c-geodesics coincide with geodesics on* M^2 *if and only if* $H = \text{const}$ *and* $K = \text{const}$ *or* M^2 *is a sphere (locally).*

For $n > 2$ we have:

THEOREM 11. *Suppose that the set of regular points of the curvature tensor* R *of the Levi-Civita connection* Γ *on a hypersurface* $M^n \subset E^{n+1}$, $n \geqslant 3$ *is dense in* M^n *and* $\det(b_{ij}) \neq 0$ *and*

(6) $$b_{ij}b_{km}\tilde{g}^{jk}\tilde{g}^{mi} = \text{const} \neq 0$$

Then, c-geodesics coincide with geodesics on M^n *if and only if* M^n *is a sphere (locally).*

P r o o f. Suppose, that c-geodesics coincide with geodesics on M^n. Then, from the Theorem 1, we have:
$\nabla_k c_{ij} = p_k c_{ij}$.
By virtue of the condition (6) and the Theorem 5 there exists $\lambda = \mu (c^g)^{-1} = \text{const}$, $c^g = b_{is}\tilde{g}^{st}b_{tj}\tilde{g}^{ji}$, $\mu = \text{const}$ such, that:

(7) $$\nabla_k(\lambda c_{ij}) = 0$$

From the Theorem 4 it follows that:

(8) $$c_{ij} = \alpha g_{ij}, \qquad \alpha = \text{const}$$

After having multiplied (8) by $\tilde{g}^{jk}$ and having substituted c_{ij} from (4), we get:

(9) $$b_{ik}\tilde{g}^{kl}b_{lj}\tilde{g}^{jr} = \alpha \delta_i^r$$

Because of (9), the tensor:

$$h^i_j = b_{js}\tilde{g}^{si} \qquad (\tilde{h}^j_p = \tilde{b}^{jr}g_{rp}) \tag{10}$$

satisfies the condition (2) of the Theorem 2 [1], i.e.

$$\nabla_s(\tilde{h}^j_p h^i_j) = 0$$

so it says, that there exists the connection $\tilde{\Gamma}$ such, that:

$$\tilde{\nabla}_s h^i_j = 0 \tag{11}$$

where $\tilde{\Gamma}^i_{jk} = \Gamma^i_{jk} + \frac{1}{2}\tilde{h}^i_p \nabla_j h^p_k = \Gamma^i_{jk} + \frac{1}{2}\tilde{b}^{ir}g_{rp}\nabla_j(b_{ks}\tilde{g}^{sp}) =$

$$= \Gamma^i_{jk} + \frac{1}{2}\tilde{b}^{ir}g_{rp}\tilde{g}^{sp}\nabla_j b_{ks} = \Gamma^i_{jk} + \frac{1}{2}\tilde{b}^{ir}\nabla_j b_{kr}$$

We now compute $\tilde{\nabla}_s b_{jk}$:

$$\begin{aligned} \tilde{\nabla}_s b_{jk} &= \partial_s b_{jk} - \tilde{\Gamma}^p_{sj}b_{pk} - \tilde{\Gamma}^p_{sk}b_{jp} = \nabla_s b_{jk} - \\ &- \frac{1}{2}\tilde{b}^{pr}\nabla_s b_{jr}b_{pk} - \frac{1}{2}\tilde{b}^{pr}b_{jp}\nabla_s b_{kr} = \nabla_s b_{jk} - \\ &- \frac{1}{2}\nabla_s b_{jk} - \frac{1}{2}\nabla_s b_{kj} = 0 \end{aligned} \tag{12}$$

Using (11) and (12), we have:

$$0 = \tilde{\nabla}_s h^i_j = \tilde{\nabla}_s(b_{jk}\tilde{g}^{ki}) = \tilde{g}^{ki}\tilde{\nabla}_s b_{jk} + b_{jk}\tilde{\nabla}_s\tilde{g}^{ki} =$$

$$= b_{jk}\tilde{\nabla}_s\tilde{g}^{ki}$$

Hence $\tilde{\nabla}_s\tilde{g}^{ki} = 0$

or

$$\tilde{\nabla}_s g_{jk} = 0 \tag{13}$$

It is easy to observe, that $\tilde{\Gamma}$ is torsionless (we use the Codazzi equations for b_{ij}), so we can apply the Theorem 3 and as a result we have: $b_{ij} = \gamma g_{ij}$, $\gamma = \text{const.}$ Q.E.D.

DEFINITION 5 [4]. A tensor field ρ on a hypersurface $M^n \subset E^{n+1}$ with the components:

$$\rho_{ij} = \frac{1}{2}(\varepsilon_{ik}h^k_j + \varepsilon_{jk}h^k_i) \tag{14}$$

where $h^k_i = b_{ip}\tilde{g}^{pk}$ and $\varepsilon_{ij} = \begin{cases} \sqrt{g} & \text{if} \quad i<j \\ -\sqrt{g} & \text{if} \quad i>j, \\ 0 & \text{if} \quad i=j \end{cases}$ $g = \det(g_{ij})$

is said to be the fourth fundamental tensor of $M^n \subset E^{n+1}$.

Now, let $n = 2$, then:

$$\rho_{11} = \frac{1}{2}(\varepsilon_{1k}h^k_1 + \varepsilon_{1k}h^k_1) = \varepsilon_{12}h^2_1 = \sqrt{g}\, h^2_1$$

$$\rho_{12} = \frac{1}{2}(\varepsilon_{1k}h^k_2 + \varepsilon_{2k}h^k_1) = \frac{1}{2}(\varepsilon_{12}h^2_2 + \varepsilon_{21}h^1_1) =$$

$$= \frac{1}{2}\sqrt{g}\,(h^2_2 - h^1_1)$$

$$\rho_{22} = \frac{1}{2}(\varepsilon_{2k}h^k_2 + \varepsilon_{2k}h^k_2) = -\sqrt{g}\, h^1_2$$

One knows, that:

$$2H = \tilde{g}^{\alpha\beta}b_{\alpha\beta} = \tilde{g}^{\alpha 1}b_{\alpha 1} + \tilde{g}^{\alpha 2}b_{\alpha 2} = h^1_1 + h^2_2 \tag{15}$$

and

$$K = \frac{b}{g} = \frac{b_{11}b_{22} - b_{12}b_{12}}{g} = \frac{1}{g}\begin{vmatrix} h^\alpha_1 g_{\alpha 1} & h^\alpha_2 g_{\alpha 1} \\ h^\alpha_1 g_{\alpha 2} & h^\alpha_2 g_{\alpha 2} \end{vmatrix} =$$

$$= \frac{1}{g}\begin{vmatrix} h^1_1 & h^1_2 \\ h^2_1 & h^2_2 \end{vmatrix}\begin{vmatrix} g_{11} & g_{12} \\ g_{12} & g_{22} \end{vmatrix} = h^1_1h^2_2 - h^1_2h^2_1$$

The determinant ρ is:

$$\rho = \det(\rho_{ij}) = \rho_{11}\rho_{22} - (\rho_{12})^2 = -gh^2_1h^1_2 -$$

$$- \frac{1}{4}g(h^2_2 - h^1_1)^2 = -\frac{1}{4}g\left[(h^2_2 - h^1_1)^2 + 4h^2_1h^1_2\right] =$$

$$= -\frac{1}{4}g\left[(h^1_1 + h^2_2)^2 - 4h^1_1h^2_2 + 4h^1_2h^2_1\right] =$$

$$= - \frac{1}{4} g\left[(h_1^1 + h_2^2)^2 - 4(h_1^1h_2^2 - h_2^1h_1^2)\right]$$

Using (15) and (16) we get:

$$\rho = - \frac{1}{4} g(4H^2 - 4K) = - g(H^2 - K)$$

$H^2 - K$ is always positive, provided that on M^2 there are no umbilical points. Hence ρ is negative. This means, it makes sense to investigate ρ-geodesics on hypersurfaces.

THEOREM 12. Suppose, that on a hypersurface $M^n \subset E^{n+1}$ the set of regular points of the curvature tensor R of the Levi-Civita connection Γ is dense in M^n and $\det(\rho_{ij}) \neq 0$. Then ρ-geodesics never coincide with geodesics on M^n. (In other words, there doesn't exist a hypersurface on which ρ-geodesics coincide with geodesics).

P r o o f. On account of the Theorems 4 and 7 we know, that ρ-geodesics should have coincided with geodesics if and only if

(18) $$\rho_{ij} = \alpha g_{ij}$$

Multiplying both sides of (18) on $\tilde{g}^{jk}$ and putting $k = i$, we get:

(19) $$\rho_{ij} g^{ji} = n\alpha$$

The left hand side of (19) is

$$\rho_{ij}\tilde{g}^{ji} = \frac{1}{2}(\varepsilon_{ip} b_{js} \tilde{g}^{sp} \tilde{g}^{ji} + \varepsilon_{jp} b_{is} \tilde{g}^{sp} \tilde{g}^{ji}) =$$

$$= \varepsilon_{ip} b_{js} \tilde{g}^{sp} \tilde{g}^{ji} = 0$$

because $\varepsilon_{ip} = - \varepsilon_{pi}$ and $b_{js}\tilde{g}^{sp}\tilde{g}^{ji} = b_{js}\tilde{g}^{si}\tilde{g}^{jp}$.

Hence $\alpha = 0$, what leads to a contradiction.

2. π-GEODESICS WITH π BEING OF THE TYPE (1,1).

Analogously to the definition of the π-geodesic given by K. Radziszewski [8] we introduce the definition of the π-geodesic in case π is of the type (1,1).

DEFINITION 6. A vector field $\overline{w}$ on a manifold M with a given linear connection Γ is said to be π-geodesic if: $\nabla_{\overline{w}} \pi_{\overline{w}} = \lambda \pi_{\overline{w}}$; $\pi_{\overline{w}} : \underline{f} \longmapsto \pi(\underline{f}, \overline{w})$, f - any covector field on M, where π is non-singular tensor field on M of the type (1,1) and λ is a real differentiable function on M. The integral curve of the π-geodesic vector field on M is called π-geodesic line.

It is easy to show, that in a local map U, the equation of this line is of the form:

$$\frac{d^2u^i}{dt^2} + (\Gamma^i_{kj} + \tilde{\pi}^i_p \nabla_k \pi^p_j) \frac{du^k}{dt} \frac{du^j}{dt} = \lambda \frac{du^i}{dt} \tag{20}$$

Suppose, that M is Riemannian n-dimensional manifold with the metric tensor g.

The tensor field π may be viewed as a linear transformation: $TM \longrightarrow TM$. Let π satisfy the following condition:

$$\bigwedge_{X,Y \in TM} g(\pi X, Y) = g(X, \pi Y) \tag{21}$$

or in a local map U:

$$g_{ij} \pi^i_k = g_{ik} \pi^i_j \tag{22}$$

Provided that (21) holds, it suffices to observe that the problem of coincidence of π-geodesics with geodesics on M is the same as the problem of covering geodesics with π^g-geodesics in the sense of the definition 1 (π^g is a tensor with components $\pi^i_j g_{ik}$).

We can prove:

THEOREM 13. Suppose, that the set of regular points of the curvature tensor R of a Riemannian, connected manifold M is dense in M and there is given a non-singular tensor field π of the type (1,1) on M satisfying (21). Then π-geodesics coincide with geodesics on M if and only if: $\pi = \lambda \cdot I$.

P r o o f. Since, the condition of coincidence of π-geodesice with geodesics provided that (21) holds, is equivalent to the problem of coincidence of π^g-geodesics with geodesics, then from the Theorems 3 and 7 it follows that:

$$\pi^i_j g_{ik} = \lambda g_{jk}$$

Hence

$$\pi^i_j = \lambda \delta^i_j \qquad \text{Q.E.D.}$$

As a special case of this theorem, we have:

THEOREM 14. If $h^j_k = b_{kl}\tilde{g}^{lj}$ are the components of the non-singular tensor field on a hypersurface $M^n \subset E^{n+1}$ with the dense set of regular points of the curvature tensor R, then h-geodesics are the same as geodesics on M^n if and only if M^n is a sphere (locally).

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STRESZCZENIE

W tej pracy zajmujemy się problemem pokrywania się π-geodezyjnych z geodezyjnymi na hiperpowierzchni $M^n \subset E^{n+1}$. W pierwszej części rozpatrujemy π-geodezyjne, gdzie π jest w naturalny sposób związany z hiperpowierzchnią. W przypadku gdy π jest trzecim podstawowym tensorem hiperpowierzchni,

wtedy π-geodezyjne pokrywają się z geodezyjnymi wtedy i tylko wtedy gdy hiperpowierzchnia jest lokalnie sferą, natomiast gdy π jest czwartym podstawowym tensorem powierzchni wtedy nie istnieje hiperpowierzchnia, na której π-geodezyjne pokrywają się z geodezyjnymi.
W drugiej części zajmujemy się π-geodezyjnymi wyznaczonymi przez pole tensorowe typu (1,1).

Резюме

В этой работе занимаемся проблемой совпадания π-геодезических с геодезическими поверхностями $M^M \subset E^{n+1}$. В первой части рассматривзем π-геодезические определимые тензорами, которые натуральным образом связаны с хиперповерхностью. Итак, когда является третим фундаментальным тензором хиперповерхности, тогда для того чтобы π-геодезические совпадали с геодезическими, необходимо и длстаточно, чтобы хиперповерхность была локально сферой, зато когда π является четвертым фундаментальным тензором, тогда не существует хиперповерхность, на которой π-геодезические совпадают с геодезическими.

Во второй части занимаемся π-геодезическими определенными тензорами типа (1,1).