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Orthogonality in the N-way Nested Classification

Ortogonalność w N-krotnej klasyfikacji hierarchicznej

Ортогональность в N-факторной иерархической классификации

Introduction. Let us consider an N-way hierarchical classification in which classification A_N is nested within classification A_{N-1}, classification A_{N-1} is nested within A_{N-2} and so forth until classification A_1. Let $\eta_{i_1 i_2 \ldots i_N}$ denote the "true" mean of the $(i_1, i_2, \ldots, i_N)$th cell, i.e. the mean value of the yield obtained where classification A_1 is at the i_1-th level, classification A_2 is the (i_1, i_2)th level, ..., and classification A_N is at the $(i_1, i_2, \ldots, i_N)$th level. The mean $\eta i_1 i_2 \ldots i_N$ is usually broken up into a general mean μ, an effect $a^1_{i_1}$ due to the i_1-th first stage class $A^1_{i_1}$, an effect $a^2_{i_1 i_2}$ due to the (i_1, i_2)th second stage class $A^2_{i_1 i_2}$, ... and an effect $a^N_{i_1 i_2 \ldots i_N}$ due to the $(i_1, i_2, \ldots, i_N)$th N-th stage class $A_{i_1 i_2 \ldots i_N}$, i.e.:

(1) $$\eta_{i_1 i_2 \ldots i_N} = \mu + a^1_{i_1} + a^2_{i_1 i_2} + \ldots + a^N_{i_1 i_2 \ldots i_N}$$

where $i_1 = 1, 2, \ldots a^1$; $i_p = 1, 2, \ldots, a^p_{i_1 i_2 \ldots i_{p-1}}$ $(p = 2, 3, \ldots, N)$.
The $a^p_{i_1 i_2 \ldots i_{p-1}}$ is the number of levels of the classification A_p within the $(i_1, i_2, \ldots, i_{p-1})$th class of the classification A_{p-1}.
If nothing more is stated about the decomposition, these components of the decomposition are not uniquely defined. It is for this reason to impose some constraints among these components. In order to seek for a set of reasonable and intuitively acceptable constraints, we introduce for every class $(i_1, i_2, \ldots, i_p)$ of classification $A_p (p = 1, 2, \ldots, N)$ a positive weight $w^p_{i_1 i_2 \ldots i_p}$. The purpose of introducing such weights is to develop a unified treatment of the identification problem in the decomposition (1) of the mean $\eta_{i_1 i_2 \ldots i_N}$. The constraints are then as follows:

(2) $$\sum_{i_p} w^p_{i_1 i_2 \ldots i_p} a^p_{i_1 i_2 \ldots i_p} = 0 \text{ for all } i_1, i_2, \ldots, i_{p-1} \quad (p = 1, 2, \ldots, N).$$

Without loss of generality we may assume that:

(3) $$\sum_{i_p} w^p_{i_1 i_2 \ldots i_p} = 1 \text{ for all } i_1, i_2, \ldots, i_{p-1};\ p = 1, 2, \ldots, N.$$

The restrictions (2) and decomposition (1) give the following definitions of the general mean μ and the effects $a^p_{i_1 i_2 \ldots i_p}$:

(4) $$\mu = \sum_{i_1} \sum_{i_2} \ldots \sum_{i_N} w^1_{i_1} w^2_{i_1 i_2} \ldots w^N_{i_1 i_2 \ldots i_N} \eta_{i_1 i_2 \ldots i_N}$$

$$a^p_{i_1 i_2 \ldots i_p} = \sum_{i_{p+1}} \sum_{i_{p+2}} \ldots \sum_{i_N} w^{p+1}_{i_1 i_2 \ldots i_{p+1}} w^{p+2}_{i_1 i_2 \ldots i_{p+2}} \ldots$$

$$\ldots w^N_{i_1 i_2 \ldots i_N} \eta_{i_1 i_2 \ldots i_N} - \sum_{i_p} \ldots \sum_{i_N} w^p_{i_1 i_2 \ldots i_p} \ldots w^N_{i_1 i_2 \ldots i_N} \eta_{i_1 i_2 \ldots i_N}$$

$$(p = 1, 2, \ldots, N-1)$$

$$a^N_{i_1 i_2 \ldots i_N} = \eta_{i_1 i_2 \ldots i_N} - \sum_{i_N} w^N_{i_1 i_2 \ldots i_N} \eta_{i_1 i_2 \ldots i_N}.$$

Let $y_{i_1 i_2 \ldots i_{N+1}}$ denote the i_{N+1}th observation in the $(i_1, i_2, \ldots, i_N)$th subclass. The mathematical model of the N-way nested classification may be expressed as:

(5) $$y_{i_1 i_2 \ldots i_{N+1}} = \theta_{i_1 i_2 \ldots i_{N+1}} + e_{i_1 i_2 \ldots i_{N+1}},$$

$$i_{N+1} = 1, 2, \ldots, n_{i_1 i_2 \ldots i_N},\ n_{i_1 i_2 \ldots i_N} > 0.$$

The random error connected with the observation $y_{i_1 i_2 \ldots i_{N+1}}$ is denoted as $e_{i_1 i_2 \ldots i_{N+1}}$. We assume that the random variables $e_{i_1 i_2 \ldots i_{N+1}}$ have normal independent distributions with zero means and the same variances σ^2_e.

In thus expressed model the true mean of the $(i_1, i_2, \ldots i_N)$th cell is equal to:

(6) $$\eta_{i_1 i_2 \ldots i_N} = \overline{\theta}_{i_1 i_2 \ldots i_N} = (n_{i_1 i_2 \ldots i_r})^{-1} \sum_{i_{N+1}} \theta_{i_1 i_2 \ldots i_{N+1}}$$

We now consider testing the following hypotheses $H_1, H_2, \ldots, H_{N+1}$ against $\mathscr{G}$ where:

$$\mathscr{G}:\ v_{i_1 i_2 \ldots i_{N+1}} = \theta_{i_1 i_2 \ldots i_{N+1}} - \overline{\theta}_{i_1 i_2 \ldots i_N} = 0$$

(7) $$H_t : v_{i_1 i_2 \ldots i_{N+1}} = 0,\ a^{N-t+1}_{i_1 i_2 \ldots i_{N-t+1}} = 0.\ (t = 1, 2, \ldots, N)$$

$$H_{N+1}: v_{i_1 i_2 \ldots i_{N+1}} = 0,\ \mu = 0.$$

For further considerations we find matrices $A, A_t (t = 1, 2, \ldots, N+1)$ which permit to introduce assumptions $\mathscr{G}$ and the hypotheses H_t $(t = 1, 2, \ldots, N+1)$ in the form:

$$\text{(8)} \qquad \begin{aligned} &\mathscr{G} : \theta \in \Omega \ \text{ where } \Omega = \{\theta : A\theta = 0\} \\ &H_t : \theta \in \omega_t \text{ where } \omega_t = \{\theta : A\theta = 0 \text{ and } A_t\theta = 0\}. \end{aligned}$$

The elements of the vector θ occurring in the formulas (8) are the values $\theta_{i_1 i_2 \ldots i_{N+1}}$.

The matrix A will have n rows and n columns where

$$n = \sum_{i_1} \sum_{i_2} \ldots \sum_{i_N} n_{i_1 i_2 \ldots i_N}.$$

The element in the $(i_1, i_2, \ldots, i_{N+1})$th row and in the $(j_1, j_2, \ldots, j_{N+1})$th column of the matrix A is equal to

$$\text{(9)} \qquad \delta_{i_1 j_1} \delta_{i_2 j_2} \ldots \delta_{i_{N+1} j_{N+1}} - \frac{1}{n_{j_1 j_2 \ldots j_N}} \delta_{i_1 j_1} \delta_{i_2 j_2} \ldots \delta_{i_N j_N}$$

where σ_{ij} is the Kronecker delta.

Similarly the matrix A_1 will have $a^N_{\cdot}$ rows and n columns where

$$a^N_{\cdot} = \sum_{i_1} \sum_{i_2} \ldots \sum_{i_N} a^N_{i_1 i_2 \ldots i_N}.$$

The element in the $(i_1, i_2, \ldots, i_N)$th row and $(j_1, j_2, \ldots j_{N+1})$th column of the matrix A_1 is equal to

$$\text{(10)} \qquad \frac{1}{n_{j_1 j_2 \ldots j_N}} (\delta_{i_1 j_1} \delta_{i_2 j_2} \ldots \delta_{i_N j_N} - w^N_{j_1 j_2 \ldots j_N} \delta_{i_1 j_1} \ldots \delta_{i_{N+1} j_{N+1}}).$$

The matrix $A_t (t = 2, 3, \ldots, N)$ will be $a^{N-t+1}_{\cdot}$ x n where

$$a^{N-t+1}_{\cdot} = \sum_{i_1} \sum_{i_2} \ldots \sum_{i_{N-t+1}} a^{N-t+1}_{i_1 i_2 \ldots i_{N-t+1}}$$

and the $(i_1, i_2, \ldots, i_{N-t+1})$th row will have the $(j_1, j_2, \ldots, j_{N+1})$th element of the form

$$\text{(11)} \qquad \frac{1}{n_{j_1 j_2 \ldots j_N}} (w^{N-t+2}_{j_1 j_2 \ldots j_{N-t+2}} \ldots w^N_{j_1 j_2 \ldots j_N} \delta_{i_1 j_1} \ldots \delta_{i_{N-t+1} j_{N-t+1}} - w^{N-t+1}_{j_1 j_2 \ldots j_{N-t+1}} \ldots$$
$$\ldots w^N_{j_1 j_2 \ldots j_N} \delta_{i_1 j_1} \ldots \delta_{i_{N-t} j_{N-t}}).$$

The element in the $(j_1, j_2, \ldots j_{N+1})$th column of the matrix A_{N+1}, which is 1 x n, will be equal to

$$\text{(12)} \qquad \frac{1}{n_{j_1 j_2 \ldots j_N}} w^1_{j_1} w^2_{j_1 j_2} \ldots w^N_{j_1 j_2 \ldots j_N}.$$

The identity of the expressions (7) and (8) can be easily proved by multiplying any row of the matrices A, $A_t (t = 1, 2, \dots N+1)$ and the vector θ. This gives us the definition of the $\nu_{i_1 i_2 \dots i_{N+1}}$ or the effect $a^{N-t+1}_{i_1 i_2 \dots i_{N-t+1}}$ respectively. For the above defined matrices A, A_t the following relations are satisfied:

Lemma 1. *The matrices A, A_t hold the conditions*

$$A_t A' = 0 \quad (t = 1, 2, \dots, N+1). \tag{13}$$

Proof: For the proof it is enough to show that the product of any row of the matrix A_t and of any row of the matrix A equals zero. This consist in multiplying each of the expressions (10), (11), (12) by (9) and summing on $j_1, j_2, \dots, j_{N+1}$.

Orthogonality. According to the definition of Darroch and Silvey [1], an experimental design (5) is orthogonal relative to a general linear model $\mathcal{G}$ and linear hypotheses $H_1, H_2, \dots, H_{N+1}$ [see (8)], if and only if, with this design, the subspaces $\Omega, \omega_1, \dots, \omega_{N+1}$ satisfy the conditions

$$\omega_t^{\perp} \cap \Omega \perp \omega_r^{\perp} \cap \Omega \text{ for all } t, r, \tag{14}$$

$t \neq r$, i.e. the orthogonal complements of ω_t, ω_r with respect to Ω, are mutually orthogonal. Seber [4] showed that the conditions (14) are equivalent to

$$A_t A_r' = 0, \text{ for all } t, r;\ t \neq r, \tag{15}$$

where the matrices A_t, $A_r (t, r = 1, 2, \dots, N+1)$ defined by the formulas (10) – (12) satisfy Lemma 1.

Using the conditions (15), we derive necessary and sufficient conditions for this system of hypotheses to be orthogonal.

Theorem. *N-way hierarchical classification, in which all $n_{i_1 i_2 \dots i_N} \neq 0$, is orthogonal relative to a general linear model $\mathcal{G}$ and the hypotheses $H_1, H_2, \dots, H_{N+1}$ if and only if*

$$w^p_{i_1 i_2 \dots i_p} = \frac{n^p_{i_1 i_2 \dots i_p}}{n^{p-1}_{i_1 i_2 \dots i_{p-1}}} \quad (p = 1, 2, \dots, N) \tag{16}$$

where $n^0 = n$, $n^N_{i_1 i_2 \dots i_N} = n_{i_1 i_2 \dots i_N}$,

$$n^p_{i_1 i_2 \dots i_N} = \sum_{i_{p+1}} \sum_{i_{p+2}} \dots \sum_{i_N} n_{i_1 i_2 \dots i_N}.$$

Proof: Let that design be orthogonal, i.e.

$$A_q A_r' = 0,\ q \neq r;\ q, r = 1, 2, \dots, N+1.$$

Then the product of the $(i_1, i_2, \dots, i_{N-p+1})$th row of the matrix $A_p (p = 1, 2, \dots, N)$ and the matrix A'_{N+1} gives the condition

$$w^{N-p+1}_{i_1 i_2 \dots i_{N-p+1}} S^{N-p+1}_{i_0 i_1 \dots i_{N-p+1}} = S^{N-p}_{i_0 i_1 \dots i_{N-p}} \quad (p = 1, 2, \dots, N) \tag{w}$$

where $S^{N}_{i_0 i_1 \ldots i_N} = n_{i_1 i_2 \ldots i_N}$,

$$S^{N-p}_{i_0 i_1 \ldots i_{N-p}} = \sum_{j_{N-p+1}} \ldots \sum_{j_N} (n_{i_1 i_2 \ldots i_{N-p} j_{N-p+1} j_{N-p+2} \ldots j_N}) - 1$$

$$\times (w^{N-p+1}_{i_1 i_2 \ldots i_{N-p} j_{N-p+1}} w^{N-p+2}_{i_1 i_2 \ldots i_{N-p} j_{N-p+1} j_{N-p+2}} \ldots w^{N}_{i_1 i_2 \ldots i_{N-p} j_{N-p+1} \ldots j_N})^2 .$$

It will be proved now that from the condition (w) the following condition can be derived

$$(w') \qquad S^{N-p}_{i_0 i_1 \ldots i_{N-p}} = \frac{1}{n^{N-p}_{i_1 i_2 \ldots i_{N-p}}} \quad (p = 1, 2, \ldots, N).$$

To complete the proof it will be proved that the condition (w') is satisfied for $p = 1$. The condition (w) for $p = 1$ is expressed as

$$\frac{w^{N}_{i_1 i_2 \ldots i_N}}{n_{i_1 i_2 \ldots i_N}} = S^{N-p}_{i_0 \ldots i_{N-p}} .$$

Multiplying this equation by $n_{i_1 i_2 \ldots i_N}$ and summing on i_N gives

$$S^{N-1}_{i_0 i_1 \ldots i_{N-1}} = \frac{1}{n^{N-1}_{i_1 i_2 \ldots i_{N-1}}} .$$

Similarly it can be proved that if the condition (w') is satisfied for a $p < N-1$, it is also satisfied for $p+1$.
For the conditions (w) and (w') we have

$$\frac{w^{N-p+1}}{n^{N-p+1}_{i_1 i_2 \ldots i_{N-p+1}}} = \frac{1}{n^{N-1}_{i_1 i_2 \ldots i_{N-p}}} \qquad (p = 1, 2, \ldots, N)$$

and hence the dependence (16) is directly derived. Let us now suppose that the conditions (14) hold. Then the elements of the matrix $A_t (t = 1, 2, \ldots, N+1)$ are as follows

$$A_1 \colon \delta_{i_1 j_1} \ldots \delta_{i_N j_N} (n_{j_1 j_2 \ldots j_N})^{-1} - \delta_{i_1 j_1} \ldots \delta_{i_{N-1} j_{N-1}} (n^{N-1}_{j_1 j_2 \ldots j_{N-1}})^{-1},$$

$$A_p \colon \delta_{i_1 j_1} \ldots \delta_{i_{N-p+1} j_{N-p+1}} (n^{N-p+1}_{j_1 j_2 \ldots j_{N-p+1}})^{-1} - \delta_{i_1 j_1} \delta_{i_2 j_2} \ldots \delta_{i_{N-p} j_{N-p}} (n^{N-p}_{j_1 j_2 \ldots j_{N-p}})^{-1} \quad (p = 1, 2, \ldots, N),$$

$A_{N+1} \colon 1/n$. It can be easily proved that for these matrices hold

$$A_p A'_q = 0 \;(p \neq q,\; p, q = 1, 2, \ldots, N+1).$$

We shall prove the condition for $p = 1$, $q = 2$.
The product of the $(i_1, i_2, \ldots, i_N)$th row of the matrix A_1 and the $(i'_1, i'_2, \ldots$

$\ldots, i'_{N-1}$)th row of the matrix A_2 is equal to

$$\sum_{j_1}\ldots\sum_{j_N}[\delta_{i_1j_1}\ldots\delta_{i_Nj_N}(n_{j_1\ldots j_N})^{-1}-\delta_{i_1j_1}\ldots$$
$$\ldots\delta_{i_{N-1}j_{N-1}}(n^{N-1}_{j_1j_2\ldots j_{N-1}})^{-1}][\delta_{i'_1j_1}\ldots\delta_{i'_{N-1}j_{N-1}}(n^{N-1}_{j_1\ldots j_{N-1}})^{-1}-\delta_{i'_1j_1}\ldots$$
$$\ldots\delta_{i'_{N-2}j_{N-1}}(n_{j_1\ldots j_{N-2}})^{-1}]\, n_{j_1j_2\ldots j_N}$$
$$=\sum_{j_1}\ldots\sum_{j_{N-1}}[\delta_{i_1j_1}\ldots\delta_{i_{N-1}j_{N-1}}-\delta_{i_1j_1}\ldots$$
$$\ldots\delta_{i_{N-1}j_{N-1}}(n^{N-1}_{j_1\ldots j_{N-1}})^{-1}n^{N-1}_{j_1\ldots j_{N-1}}][\delta_{i'_1j_1}\ldots\delta_{i'_{N-1}j_{N-1}}(n^{N-1}_{j_1\ldots j_{N-1}})^{-1}-\delta_{i'_1j_1}\ldots$$
$$\ldots\delta_{i'_{N-2}j_{N-2}}(n_{j_1\ldots j_{N-2}})^{-1}].$$

It is easy to see that it is equal to zero. In the same way we can prove the remaining conditions.

Thus we have shown that if all $n_{i_1i_2\ldots i_N} > 0$, the orthogonality of an N-way nested classification depends only on the choice of weights occurring in restictions. It is easy to see that the assumption $n_{i_1i_2\ldots i_N} > 0$ does not limit the generality of the theorem.

Analysis of variance. In further considerations our attention will be focused on an orthogonal case, i.e. on the case when all $n_{i_1i_2\ldots i_N} > 0$ and the weights satisfy the conditions (16). To find the sums of squares due to the hypotheses $H_1, H_2, \ldots, H_{N+1}$ the two following Lemmas are indispensable:

Lemma 2. *If the matrices A, $A_t (t = 1, 2, \ldots, N+1)$ satisfy the conditions* (8), (13) *and* (15), *the least squares estimate of the vector $A_t\theta (t = 1, 2, \ldots, N+1)$ is*

$$A_t\hat{\theta} = A_t y.$$

Proof. From the Gauss-Markov theorem (see theorem 3.51 [6]) we have $A_t\hat{\theta} = A_tPy$ but $P = I-\bar{A}'(\bar{A}\bar{A}')^{-1}\bar{A}$ where $\bar{A}$ is the matrix of lineary independent rows of A. Hence $A_t\hat{\theta} = A_t(I-\bar{A}'(\bar{A}\bar{A}')^{-1}\bar{A})y = A_ty$.

Lemma 3. *If the matrices A, $A_t(t = 1, 2, \ldots, N+1)$ satisfy the conditions* (8), (13) and (15), *the acceptance of any of the hypotheses $H_j : \theta \in \omega_j$ $(j = 1, 2, \ldots, N+1)$ does not cause any change of the least squares estimates of the vectors $A_t\theta (t \neq j, t = 1, 2, \ldots, N+1)$.*

Proof. Let the hypothesis H_j: $\theta \in \omega_j$ be true, where $\omega_j = N\begin{bmatrix} A \\ A_j \end{bmatrix}$, i.e. ω_j is the null space of $\begin{bmatrix} A \\ A_j \end{bmatrix}$. If we denote the projection operator to the ω_j as P_j, then I-P_j is the projection operator to the $\omega_j^{\perp} = \left\{N\begin{bmatrix} A \\ A_j \end{bmatrix}\right\}^{\perp} = R[A',$

$A_j']$ where $R[A]$ denotes the range space of A. On the other hand, it appears from the conditions $A_j'A_t' = 0$ $(t \neq j)$ and $AA_t' = 0$ $(t = 1, 2, \ldots, N+1)$, that $R[A_t']$ is orthogonal to $R[A', A_j']$. Hence we have that $(I-P_j)A_t' = 0$ or $A_t(I-P_j) = 0$. The least squares estimate of the vector $A_t\theta$(t $\neq j$) for $\theta \epsilon \omega_j$ is

$$A_tP_jy = A_ty - A_t(I-P_j)y = A_ty,$$

but A_ty is the least squares estimate of the vector A_t for $\theta \epsilon \Omega$.
It follows from lemma 2 and lemma 3 that the least squares estimates μ, $a^p_{i_1i_2\ldots i_p}$ in the orthogonal N-way nested classification can be derived immediately from the definitions (4), (6), (13) of the parameters, namely

$$\hat{\mu} = \overline{y}$$

$$\hat{a}^p_{i_1i_2\ldots i_p} = \overline{y}^p_{i_1i_2\ldots i_p} - \overline{y}^{p-1}_{i_1i_2\ldots i_{p-1}} \quad (p = 1, 2, \ldots, N)$$

where

$$y^0 = \overline{y} = \frac{1}{n}\sum_{i_1}\sum_{i_2}\ldots\sum_{i_{N+1}} y_{i_1i_2\ldots i_{N+1}},$$

$$\overline{y}^p_{i_1i_2\ldots i_p} = (n^p_{i_1i_2\ldots i_p})^{-1}\sum_{i_{p+1}}\sum_{i_{p+2}}\ldots\sum_{i_{N+1}} y_{i_1i_2\ldots i_{N+1}} \quad (p = 1, 2, \ldots, N).$$

The likelihood ratio criterion for testing $H_t(t = 1, 2, \ldots, N+1)$ is equivalent to

$$F_t = \frac{y'(P_\Omega - P_{\omega t})y}{\nu_t} : \frac{y'(I-P_\Omega)y}{\nu_e}$$

where $\nu_e = n$-dimension (Ω) and $\nu_t =$ dimension (Ω)-dimension $(\omega_t)\cdot F_t$ has a central F distribution under the hypothesis H_t and a non-central F distribution under the alternative with ν_t, ν_e degrees of freedom.
The sums of squares $SS_t = y'(P_\Omega - P_{\omega t})y$ and $SS_e = y'(I-P_\Omega)y$ can be found by means of Lemma 3, namely

$$SS_t = (\hat{\theta}_\Omega - \hat{\theta}_{\omega_t})'(\hat{\theta}_\Omega - \hat{\theta}_{\omega_t}) = \sum_{i_1}\sum_{i_2}\ldots\sum_{i_{N-t+1}} n^{N-t+1}_{i_1i_2\ldots i_{N-t+1}}(\hat{a}^{N-t+1}_{i_1i_2\ldots i_{N-t+1}})^2.$$

It is so because the $(i_1, i_2, \ldots, i_{N+1})$th element of the vector $\hat{\theta}_\Omega$ is equal to $\hat{\mu} + \sum_{p=1}^{N} \hat{a}^p_{i_1i_2\ldots i_p}$, whereas the $(i_1, i_2, \ldots, i_{N+1})$th element of the vector $\hat{\theta}_{\omega_t}$ is equal to $\hat{\mu} + \sum_{\substack{p=1 \\ p\neq N-t+1}}^{N} \hat{a}^p_{i_1i_2\ldots i_p}$.

For the same reason

$$SS_{N+1} = n\hat{\mu}^2$$

and

$$SS_e = \sum_{i_1} \sum_{i_2} \ldots \sum_{i_{N+1}} (y_{i_1 i_2 \ldots i_{N+1}} - \overline{y}^N_{i_1 i_2 \ldots i_N})^2.$$

From the above results we obtain the following table of analysis of variance.

Table 1. Null-hypotheses, degrees of freedom and sums of squares for orthogonal N-way nested classification.

Null-hypothesis	Degrees of freedom	Sum of squares
$H_t : a^t_{i_1 i_2 \ldots i_t} = 0$ for all $i_1 i_2, \ldots, i_t$ (t = 1, 2, ..., N)	$\nu_t = a^t_{\cdot} - a^{t-1}_{\cdot}$	$SS_t = \sum_{i_1} \ldots \sum_{i_t} n^t_{i_1 i_2 \ldots i_t} (\hat{a}^t_{i_1 i_2 \ldots i_t})^2$
$H_{N+1} : \mu = 0$	$\nu_{N+1} = 1$	$SS_{N+1} = n\hat{\mu}^2$
Error	$\nu_e = n - a^N_{\cdot}$	$SS_e = \sum_{i_1} \ldots \sum_{i_{N+1}} (y_{i_1 i_2 \ldots i_{N+1}} - \overline{y}^N_{i_1 i_2 \ldots i_N})^2$

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REFERENCES

[1] Darroch, J. N.; Silvey, S.D. *On Testing one or More Hypothesis.* Ann. Math. Statist. 34 (1963), 555-567.

[2] Gates, C. E. and Shiue, C. *The Analysis ot Variance of the S-stage Hierarchical Classification.* Biometrics 18 (1962), 529-536.

[3] Gaylor, D. W.; Hartwell, T. D. *Expected Mean Squares for Nested Classifications,* Biometrics, vol. 25 (1969), 427-430.

[4] Seber, S. A. F. *Orthogonality in Analysis of Variance.* Ann. Math. Statist. 35 (1964). 705-710.

[5] Seber, G. A. F. *Linear Hypotheses and Induced Tests.* Biometrika 51 (1964), 41-47.

[6] Seber, G. A. F. *The Linear Hypotheses.* London 1966.

STRESZCZENIE

W pracy otrzymano warunki konieczne i dostateczne ortogonalności N-krotnej klasyfikacji hierarchicznej zgodnie z definicją ortogonalności podaną w pracy Dorroch i Silvey [1]. Dla ortogonalnej N-krotnej klasyfikacji hierarchicznej podano estymatory parametrów oraz tabele analizy wariancji.

РЕЗЮМЕ

Получены необходимые и достаточные условия ортогональности N-факторной иерархической классификации в смысле определения ортогональности, приведенной в работе [1].

В случае ортогональной N-факторной иерархической классификации получены оценки параметров и критерии значимости для проверки гипотез об эффектах исследуемых факторов.