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Recurrence Relations for the Moments of the so-called Inflated Distributions

Wzory rekurencyjne na momenty tak zwanych rozkładów "nadętych"

Рекуррентные формулы для моментов так называемых „раздутых" распределений

1. Introduction and notations. A. Noack [3] and, next, G. P. Patil [5] have shown that a large class of random variables with the discrete probability distributions can be derived from certain power series

$$f(\theta) = \sum_x a(x)\,\theta^x$$

where the summation extends over a subset T of the set I of non-negative integers, $a(x) \geqslant 0$ and $\theta \in \Omega = \{\theta : 0 < \theta < R\}$, the parameter space, and R is the radius of the convergence of the power series of $f(\theta)$. A random variable X with the probability function

$$P[X = x] = p(x;\theta) = \frac{a(x)\,\theta^x}{f(\theta)},\ x \in T$$

is said to have the generalized power series distribution (GPSD).

Now we are going to introduce an inflated generalized power series distribution (IGPSD).

A random variable X is said to have the inflated (at the point $x = s$) generalized power series distribution, if

$$(1)\qquad P[X = x] = p(x;\theta, a) = \begin{cases} 1 - a + a\dfrac{a(x)\,\theta^x}{f(\theta)} & \text{for } x = s, \\[2ex] a\dfrac{a(x)\,\theta^x}{f(\theta)} & \text{for } x \neq s,\ x \in T \end{cases}$$

where $0 < a \leqslant 1$, $a(x) \geqslant 0$, $f(\theta) = \sum\limits_x a(x)\,\theta^x$ for $\theta \in \Omega = \{\theta : 0 < \theta < R\}$, the parameter space, and R is the radius of convergence of the power series of $f(\theta)$, T is a subset of the set non-negative integers, and $s \in T$.

A random variable X is said to have the inflated generalized power series distribution, truncated at point $x = x_0$, if

$$(2) \quad P[X = x] = p(x; \theta, a) = \begin{cases} \left[1 - a + a\dfrac{a(x)\theta^x}{f(\theta)}\right] \Big/ \left[1 - a\dfrac{a(x_0)\theta^{x_0}}{f(\theta)}\right] & \text{for } x = s \neq x_0, \\ a\dfrac{a(x)\theta^x}{f(\theta)} \Big/ \left[1 - a\dfrac{a(x_0)\theta^{x_0}}{f(\theta)}\right] & \text{for } x \in T,\ x \neq s,\ x \neq x_0 \end{cases}$$

$a, \theta, a(x)$ are defined as in above and $x_0 = \min_{x \in T} x$.

This note gives the recurrence relations for the moments of the random variable having IGPSD and the recurrence relations between the moments of GPSD and of ones of IGPSD. Moreover, we establish the recurrence relations for the moments of a truncated IGPSD. From some formulas given in this note one can obtain as particular cases, the formulas for the recurrence relations for the moments of the simple binomial, negative binomial and Poisson distributions. For instance, we get the formulas given in [1], [3], [6] and [10].

Through this note the following notations will be used:

m_r' – rth moment of GPSD,
m_r – rth moment of IGPSD,
μ_r' – rth central moment of GPSD,
μ_r – rth central moment of IGPSD.

2. The recurrence relations for the central moments of IGPSD and the recurrence relations between the moments of GPSD and the ones of IGPSD. We are going to prove

Theorem 1. *The* $(r+1)$ *-th central moment of a random variable* X *having the distribution* (1) *is expressed by*

$$(3) \quad \mu_{r+1} = \theta\left[\frac{d\mu_r}{d\theta} + r\frac{dm_1}{d\theta}\mu_{r-1}\right] - \frac{\beta}{a}(s - m_1)\mu_r + \frac{\beta}{a}(s - m_1)^{r+1},$$

where $\beta = 1 - a$.

Proof. The mathematical expectation and the rth moment of the distribution (1) are given by

$$(4) \quad m_1 = \beta s + a\theta\frac{f'(\theta)}{f(\theta)},$$

and

$$\mu_r = \beta(s-m_1)^r + a\sum_x (x-m_1)^r \frac{a(x)\theta^x}{f(\theta)},$$

respectively.

Differentiating the last formula with respect to θ, we get

$$\frac{d\mu_r}{d\theta} = -\beta r(s-m_1)^{r-1}\frac{dm_1}{d\theta} + \sum_x r(x-m_1)^{r-1}\frac{a(x)\theta^x}{f(\theta)}\frac{dm_1}{d\theta}$$

$$+a\sum_x x(x-m_1)^r \frac{a(x)\theta^{x-1}}{f(\theta)} - a\frac{f'(\theta)}{f(\theta)}\sum_x (x-m_1)^r\frac{a(x)\theta^x}{f(\theta)}.$$

Hence, we have

$$\theta\frac{d\mu_r}{d\theta} = -r\theta\frac{dm_1}{d\theta}\left[\beta(s-m_1)^{r-1} + a\sum_x (x-m_1)^{r-1}\frac{a(x)\theta^x}{f(\theta)}\right]$$

$$+a\sum_x (x-m_1)^{r+1}\frac{a(x)\theta^x}{f(\theta)} + a\sum_x (x-m_1)^r\frac{a(x)\theta^x}{f(\theta)}\left(m_1-\theta\frac{f'(\theta)}{f(\theta)}\right).$$

In view of (4), we get

$$m_1 - \theta\frac{f'(\theta)}{f(\theta)} = \frac{\beta}{a}(s-m_1).$$

Therefore

$$\theta\frac{d\mu_r}{d\theta} = -r\theta\frac{dm_1}{d\theta}\mu_{r-1} + \mu_{r+1} - \beta(s-m_1)^{r+1}$$

$$+\frac{\beta}{a}(s-m_1)[\mu_r - \beta(s-m_1)^r].$$

Hence, the formula (3) follows.

In the special case when T is the set of all non-negative integers and $s=0$, the formula (3) becomes

(3′) $$\mu_{r+1} = \theta\left[\frac{d\mu_r}{d\theta} + r\frac{dm_1}{d\theta}\mu_{r-1}\right] + \frac{\beta}{a}m_1\mu_r + \frac{\beta}{a}(-m_1)^{r+1}.$$

Moreover, if $a=1$ then the formula (3) describes the recurrence relations for the central moments of the power series distribution established by Noack [3].

The following theorem gives the recurrence relations between IGPSD and GPSD

Theorem 2. *If X is a random variable having the distribution* (1), *then*

$$\mu_r = A + a\theta \sum_{j=2}^{r} \binom{r}{j} \beta^{r-j} (m_1' - s)^{r-j} \left[\frac{d\mu_{j-1}'}{d\theta} + (j-1) \frac{dm_1'}{d\theta} \mu_{j-2}' \right], \tag{5}$$

where

$$A = a\beta(s - m_1')^r (a^{r-1} - (-\beta)^{r-1}).$$

Proof. It is obvious that

$$m_1 = \beta s + a m_1',$$

and

$$\mu_r = \beta(s - m_1)^r + a \sum_x (x - m_1)^r \frac{a(x)\theta^x}{f(\theta)}.$$

Using the fact that

$$\sum_x (x - m_1)^r \frac{a(x)\theta^x}{f(\theta)} = \sum_x \sum_{j=0}^{r} \binom{r}{j} (x - m_1')^j (m_1' - m_1)^{r-j} \frac{a(x)\theta^x}{f(\theta)}$$

$$= \sum_{j=0}^{r} \binom{r}{j} (\beta m_1' - \beta s)^{r-j} \mu_j',$$

we have

$$\mu_r = a^r \beta (s - m_1')^r + a \sum_{j=0}^{r} \binom{r}{j} \beta^{r-j} (m_1' - s)^{r-j} \mu_j'.$$

On the basis of the equalities $\mu_0' = 1$ and $\mu_1' = 0$, we have

$$\mu_r = a\beta(s - m_1')^r [a^{r-1} - (-\beta)^{r-1}]$$

$$+ \sum_{j=2}^{r} \binom{r}{j} \beta^{r-j} (m_1' - s)^{r-j} \mu_j'.$$

Using the formula for the rth central moment of simple distribution

$$\mu_r' = \theta \left[\frac{d\mu_{r-1}'}{d\theta} + (r-1) \frac{dm_1'}{d\theta} \mu_{r-2}' \right], \quad r = 2, 3, \ldots$$

and putting $A = a\beta(s - m_1')^r [a^{r-1} - (-\beta)^{r-1}]$, we get (5).

One can observe, that some formulas given in [9] are the particular cases of the formulas given in the Theorem 1.

Now we are going to consider some special cases of the distribution (1).

(a) Let $f(\theta) = (1+\theta)^n$, $\theta = \frac{p}{q}$. Then $a(x) = \binom{n}{x}$, $T = \{0, 1, \ldots, n\}$ and the distribution (1) has in this case the form of the so-called inflated binomial distribution

(6) $$P[X = x] = p(x; n, p, a)$$
$$= \begin{cases} 1-a+ap^x q^{n-x}\binom{n}{x} & \text{for } x = s, \\ a\binom{n}{x} p^x q^{n-x} & \text{for } x = 0, 1, \ldots, s-1, s+1, \ldots, n. \end{cases}$$

After simple calculations one can obtain the following relations:

(7) $$\mu_{r+1} = pq\left(\frac{d\mu_r}{dp} + anr\mu_{r-1}\right) - \beta(s-np)\mu_r + \beta a^r(s-np)^{r+1},$$

(8) $$\mu_r = A + apq\sum_{j=2}^{r}\binom{r}{j}\beta^{r-j}(np-s)^{r-j}\left[\frac{d\mu'_{j-1}}{dp} + n(j-1)\mu'_{j-2}\right]$$

where
$$A = a\beta(s-np)^r[a^{r-1} - (-\beta)^{r-1}].$$

(b) Setting $f(\theta) = e^\theta$, $\theta = \lambda$, $a(x) = \frac{1}{x!}$ and $T = \{0, 1, 2, \ldots\}$, we obtain the so-called inflated Poisson distribution

(9) $$P[X = x] = p(x; \lambda, a) = \begin{cases} 1-a+ae^{-\lambda}\frac{\lambda^x}{x!} & \text{for } x = s, \\ ae^{-\lambda}\frac{\lambda^x}{x!} & \text{for } x = 0, 1, \ldots, s-1, \\ s+1, \ldots, & \end{cases}$$

For the distribution (9), we have

(10) $$\mu_{r+1} = \lambda\left[\frac{d\mu_r}{d\lambda} + ar\mu_{r-1}\right] - \beta(s-\lambda)\mu_r + \beta a^r(s-\lambda)^{r+1},$$

(11) $$\mu_r = A + a\lambda\sum_{j=2}^{r}\binom{r}{j}\beta^{r-j}(\lambda-s)^{r-j}\left[\frac{d\mu'_{j-1}}{d\lambda} + (j-1)\mu'_{j-2}\right],$$

where
$$A = a\beta(s-\lambda)^r[a^{r-1} - (-\beta)^{r-1}].$$

(c) If $f(\theta) = (1-\theta)^{-n}$, $\theta = p$, $a(x) = (-1)^x\binom{-n}{x}$ and $T = \{0, 1, 2, \ldots\}$, then the distribution (1) is the so-called inflated negative binomial distribution

$$(12)\qquad P[X=x]=p(x;n,p,a)=\begin{cases}1-a+a(-1)^x\binom{-n}{x}p^xq^n & \text{for } x=s,\\ a(-1)^x\binom{-n}{x}p^xq^n & \text{for } x=0,1,\\ 2,\ldots,s-1,\ s+1,\ldots.\end{cases}$$

In the case of the distribution (12), we get

$$(13)\qquad \begin{aligned}\mu_{r+1}&=\frac{p}{q^2}\left(q^2\frac{d\mu_r}{dp}+arn\mu_{r-1}\right)-\beta\left(s-n\frac{p}{q}\right)\mu_r\\ &\quad+\beta a^r\left(s-n\frac{p}{q}\right)^{r+1},\end{aligned}$$

$$(14)\qquad \mu_r=A+ap\sum_{j=2}^{r}\binom{r}{j}\beta^{r-j}\left(n\frac{p}{q}-s\right)^{r-j}\left[\frac{d\mu'_{j-1}}{dp}+n(j-1)\frac{\mu'_{j-2}}{q^2}\right],$$

where

$$A=a\beta\left(s-n\frac{p}{q}\right)^r\left[a^{r-1}-(-\beta)^{r-1}\right].$$

(d) If $f(\theta)=(1+\theta)^n-1$, $\theta=\frac{p}{q}$, $a(x)=\binom{n}{x}$ and $T=\{1,2,\ldots,n\}$, then we obtain the so-called inflated truncated binomial distribution

$$(15)\qquad P[X=x]=p(x;n,p,a)$$

$$=\begin{cases}1-a+a\binom{n}{x}p^xq^{n-x}/(1-q^n) & \text{for } x=s,\\ a\binom{n}{x}p^xq^{n-x}/(1-q^n) & \text{for } x=1,2,\ldots,s-1,s+1,\\ s+2,\ldots,n.\end{cases}$$

For the distribution (15) we have

$$(16)\qquad \begin{aligned}\mu_{r+1}&=pq\left[\frac{d\mu_r}{dp}+arn\frac{1-q^n-npq^{n-1}}{(1-q^n)^2}\mu_{r-1}\right]\\ &\quad-\beta\left(s-\frac{np}{1-q^n}\right)\mu_r+\beta a^r\left(s-\frac{np}{1-q^n}\right)^{r+1},\end{aligned}$$

$$(17)\qquad \begin{aligned}\mu_r&=A+apq\sum_{j=2}^{r}\binom{r}{j}\beta^{r-j}\left(\frac{np}{1-q^n}-s\right)^{r-j}\\ &\quad\cdot\left[\frac{d\mu'_{j-1}}{dp}+n(j-1)\frac{1-q^n-npq^{n-1}}{(1-q^n)^2}\mu'_{j-2}\right],\end{aligned}$$

where

$$A = a\beta\left(s-\frac{np}{1-q^n}\right)^r [a^{r-1}-(-\beta)^{r-1}].$$

(e) If $f(\theta) = e^\theta - 1$, $\theta = \lambda$, $a(x) = \frac{1}{x!}$ and $T = \{1, 2, \ldots, \}$, then we have the so-called inflated truncated Poisson distribution

$$P[X = x] = p(x; \lambda, a) \tag{18}$$

$$= \begin{cases} 1-a+ae^{-\lambda}\frac{\lambda^x}{x!}/(1-e^{-\lambda}) \text{ for } x = s, \\ ae^{-\lambda}\frac{\lambda^x}{x!}/(1-e^{-\lambda}) \text{ for } x = 1, 2, \ldots, s-1, s+1, \ldots. \end{cases}$$

In the case of the distribution (18), we get

$$\mu_{r+1} = \lambda\left[\frac{d\mu_r}{d\lambda}+ar\frac{1-e^{-\lambda}-\lambda e^{-\lambda}}{(1-e^{-\lambda})^2}\mu_{r-1}\right] - \beta\left(s-\frac{\lambda}{1-e^{-\lambda}}\right)\mu_r+\beta a^r\left(s-\frac{\lambda}{1-e^{-\lambda}}\right)^{r+1}, \tag{19}$$

$$\mu_r = A+a\lambda\sum_{j=2}^{r}\binom{r}{j}\beta^{r-j}\left(\frac{\lambda}{1-e^{-\lambda}}-s\right)^{r-j} \cdot \left[\frac{d\mu'_{j-1}}{d\lambda}-(j-1)\frac{1-e^{-\lambda}-\lambda e^{-\lambda}}{(1-e^{-\lambda})^2}\mu'_{j-2}\right] \tag{20}$$

where

$$A = a\beta\left(1-\frac{\lambda}{1-e^{-\lambda}}\right)^r [a^{r-1}-(-\beta)^{r-1}].$$

(f) *If* $f(\theta) = (1-\theta)^{-n}-1$, $\theta = p$, $a(x) = (-1)^x\binom{-n}{x}$ and $T = \{1, 2, 3, \ldots\}$, then we have the so-called inflated truncated binomial distribution

$$P[X = x] = p(x; n, p, a) \tag{21}$$

$$= \begin{cases} 1-a+a(-1)^x\binom{-n}{x}p^x q^n/(1-q^n) \text{ for } x = s, \\ a(-1)^x\binom{-n}{x}p^x q^n/(1-q^n) \text{ for } x = 1, 2, \ldots, s-1, s+1, \ldots. \end{cases}$$

In this case we have

$$(22)\qquad \mu_{r+1} = \frac{p}{q^2}\left(q^2 \frac{d\mu_r}{dp} + arn\, \frac{1-q^n-npq^n}{(1-q^n)^2}\,\mu_{r-1}\right) - \beta\left[s - \frac{np}{q(1-q^n)}\right]\mu_r + \beta a^r\left[s - \frac{np}{q(1-q^n)}\right]^{r+1},$$

$$(23)\qquad \mu_r = A + ap\sum_{j=2}^{r}\binom{r}{j}\beta^{r-j}\left[\frac{np}{q(1-q^n)} - s\right]^{r-j} \cdot\left[\frac{d\mu'_{j-1}}{dp} + n(j-1)\,\frac{1-q^n-npq^n}{q^2(1-q^n)^2}\,\mu'_{j-2}\right],$$

where

$$A = a\beta\left(s - \frac{np}{q(1-q^n)}\right)^r [a^{r-1} - (-\beta)^{r-1}].$$

If in (7), (10), (13), (16), (19) and (22) we put $a = 1$, then we obtain the well known recurrence relations for the central moments of binomial, Poisson, negative binomial, truncated Poisson and truncated negative binomial distribution respectively.

3. The recurrence relations for the moments of negative order of the inflated generalized power series distributions. Now we can prove the following:

Theorem 3. *If X is a random variable having the distribution* (1) *for which $O \notin T$, then we have*

$$(24)\qquad \theta\frac{dm_{-r}}{d\theta} = m_{-r+1} - \theta\frac{f'(\theta)}{f(\theta)}\,m_{-r} - \beta s^{-r}\left[s - \theta\frac{f'(\theta)}{f(\theta)}\right].$$

Proof. It is obvious that the moment of negative order of X is given by

$$m_{-r} = \beta s^{-r} + a\sum_x x^{-r}\frac{a(x)\,\theta^x}{f(\theta)}.$$

Hence, we have

$$\frac{dm_{-r}}{d\theta} = \frac{1}{\theta}\left[\beta s^{-r+1} + a\sum_x x^{-r+1}\frac{a(x)\,\theta^x}{f(\theta)}\right] - \frac{f'(\theta)}{f(\theta)}\left[\beta s^{-r} + a\sum_x x^{-r}\frac{a(x)\,\theta^x}{f(\theta)}\right] + \frac{f'(\theta)}{f(\theta)}\beta s^{-r} - \frac{\beta s^{-r+1}}{\theta},$$

which proves (24).

It is easy to verify that:

(*i*) For the distribution (15), we have

$$pq\frac{dm_{-r}}{dp} = m_{-r+1} - \frac{np}{1-q^n}m_{-r} - \beta s^{-r}\left(s - \frac{np}{1-q^n}\right).$$

(*ii*) For the distribution (18), we have

$$\lambda\frac{dm_{-r}}{d\lambda} = m_{-r+1} - \frac{\lambda}{1-e^{-\lambda}}m_{-r} - \beta s^{-r}\left(s - \frac{\lambda}{1-e^{-\lambda}}\right).$$

(*iii*) For the distribution (21), we have

$$p\frac{dm_{-r}}{dp} = m_{-r+1} - \frac{np}{q(1-q^n)}m_{-r} - \beta s^{-r}\left(s - \frac{np}{q(1-q^n)}\right).$$

4. The recurrence relations for the positive and negative moments of the truncated inflated generalized power series distribution. Let X be a random variable having the distribution (2). By the similar considerations as in the proof of Theorem 3, we get

Theorem 4. *If X is a random variable having the distribution* (2), *then*

$$m_{r+1} = \theta\frac{dm_r}{d\theta} - \frac{ax_0\dfrac{a(x_0)\theta^{x_0}}{f(\theta)} - \theta\dfrac{f'(\theta)}{f(\theta)}}{1 - a\dfrac{a(x_0)\theta^{x_0}}{f(\theta)}}m_r + \frac{\beta s^r}{1 - a\dfrac{a(x_0)\theta^{x_0}}{f(\theta)}}\left[s - \theta\frac{f'(\theta)}{f(\theta)}\right], \tag{25}$$

and

$$\theta\frac{dm_{-r}}{d\theta} = m_{-r+1} + \frac{ax_0\dfrac{a(x_0)\theta^{x_0}}{f(\theta)} - \theta\dfrac{f'(\theta)}{f(\theta)}}{1 - a\dfrac{a(x_0)\theta^{x_0}}{f(\theta)}}m_{-r} - \frac{\beta s^{-r}}{1 - a\dfrac{a(x_0)\theta^{x_0}}{f(\theta)}}\left[s - \theta\frac{f'(\theta)}{f(\theta)}\right]. \tag{26}$$

In particular, one can obtain

(**A**) If $f(\theta) = (1+\theta)^n$ and $\theta = \dfrac{p}{q}$, then the random variable X has the truncated inflated binomial distribution

$$P[X = x] = p(x; n, p, a)$$

$$= \begin{cases} \left[1-a+a\binom{n}{x}p^x q^{n-x}\right] \Big/ (1-aq^n) & \text{for } x = s \neq 0, \\ a\binom{n}{x}p^x q^{n-x}/(1-aq^n) & \text{for } x = 1, 2, \ldots, s-1, s+1, \ldots, n, \end{cases}$$

and

$$m_{r+1} = pq\frac{dm_r}{dp} + \frac{np}{1-aq^n}m_r + \frac{\beta s^r}{1-aq^n}(s-np),$$

$$pq\frac{dm_{-r}}{dp} = m_{-r+1} - \frac{np}{1-aq^n}m_{-r} - \frac{\beta s^{-r}}{1-aq^n}(s-np).$$

(**B**) If $f(\theta) = e^\theta$ and $\theta = \lambda$, then the random variable X have the truncated inflated Poisson distribution

$$P[X = x] = p(x; \lambda, a)$$

$$= \begin{cases} \left[1-a+ae^{-\lambda}\frac{\lambda^x}{x!}\right] \Big/ (1-ae^{-\lambda}) & \text{for } x = s \neq 0, \\ ae^{-\lambda}\frac{\lambda^x}{x!} \Big/ (1-ae^{-\lambda}) & \text{for } x = 1, 2, \ldots, s-1, s+1, \ldots, \end{cases}$$

and

$$m_{r+1} = \lambda\frac{dm_r}{d\lambda} + \frac{1}{1-ae^{-\lambda}}m_r + \frac{\beta s^r}{1-ae^{-\lambda}}(s-\lambda),$$

$$\lambda\frac{dm_{-r}}{d\lambda} = m_{-r+1} - \frac{\lambda}{1-ae^{-\lambda}}m_{-r} - \frac{\beta s^{-r}}{1-ae^{-\lambda}}(s-\lambda).$$

(**C**) If $f(\theta) = (1-\theta)^{-n}$ nad $\theta = p$, then the random variable X has the truncated inflated negative binomial distribution

$$P[X = x] = p(x; n, p, a)$$

$$= \begin{cases} \left[1-a+a(-1)^x\binom{-n}{x}p^x q^n\right] \Big/ (1-aq^n) & \text{for } x = s \neq 0, \\ a(-1)^x\binom{-n}{x}p^x q^n/(1-aq^n) & \text{for } x = 1, 2, \ldots, s-1, \end{cases}$$

$s+1,\ s+2, \ldots,$

and

$$m_{r+1} = p\frac{dm_r}{dp} + \frac{np}{q(1-aq^n)}m_r + \frac{\beta s^r}{1-aq^n}\left(s-n\frac{p}{q}\right),$$

$$p\frac{dm_{-r}}{dp} = m_{-r+1} - \frac{np}{q(1-aq^n)}m_{-r} - \frac{\beta s^{-r}}{1-aq^n}\left(s-n\frac{p}{q}\right).$$

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STRESZCZENIE

W pracy podano wzory rekurencyjne na momenty „nadętych" rozkładów typu uogólnionych szeregów potęgowych.

РЕЗЮМЕ

В работе приводятся рекуррентные формулы для моментов „раздутых" распределений типа обобщенных степенных рядов.