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### Analytic Functions in Topological Vector Spaces

Funkcje analityczne w przestrzeni topologicznej wektorowej

Аналитические функции в топологическом векторном пространстве

Let  $E$  and  $F$  be topological vector spaces (tvs) over  $K$  ( $K = C$  or  $K = R$ ). A mapping  $f: E \rightarrow F$  is called a homogeneous polynomial of degree  $n$  if there exists an  $n$ -linear mapping  $\bar{f}: E^n \rightarrow F$  such that  $\bar{f}(x) = f(x, \dots, x)$ ,  $x \in E$ .

A continuous function  $f: U \rightarrow F$ , defined in an open subset  $U$  of  $E$ , is called analytic if for every point  $a \in U$  there exists a neighbourhood  $V$  of  $O \in E$  such that  $a + V \subset U$  and

$$f(a+x) = \sum_{n=0}^{\infty} f_n(x), \quad x \in V,$$

where  $f_n: E \rightarrow F$  is a continuous homogeneous polynomial of degree  $n$ .

An exposition of the theory of analytic functions in Banach spaces may be found in [1], [2]. Recently a still growing number of papers (especially in France, coworkers of professor Lelong) is being devoted to developing a theory of analytic functions in the case when  $E$  is rather arbitrary tvs and  $F$  is separated locally convex and sequentially complete.

A uniform exposition of basic concepts and facts concerning analytic functions in such topological vector spaces over  $K$  has been recently presented in [2], [3], [4]. A generalization of the Polynomial Lemma of Leja [6] to the case of polynomials of a complex variable with values in a locally tvs has found an essential application in this exposition.

#### REFERENCES

- [1] Alexiewicz, A., and Orlicz, W., *Analytic operations in real Banach spaces*, *Studia Math.* 14 (1953), 57–78.

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- [6] Siciak, J., *A generalization of a polynomial lemma of Leja*, Ann. Pol. Math. 25 (1971), 149-156.

### STRESZCZENIE

Autor przedstawił podstawowe pojęcia dotyczące teorii funkcji analitycznych w przestrzeniach topologicznych liniowych, zwracając szczególną uwagę na rolę lematu wielomianowego Leja. Szczegółowe wyniki są zawarte w pracach [2]-[6].

### РЕЗЮМЕ

Автор представил основные понятия, касающиеся теории аналитических функций в топологических линейных пространствах, обращая особенное внимание на роль многочленной леммы Ф. Лея. Подробные результаты даны в работах [2]-[6].