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Manifold with the 3-structure

Rozmaitość z 3-strukturą

Introduction. In the paper [1], the authors studied properties of the Riemannian manifold M^{4n} with given 3-structure $\{\underset{\alpha}{\tilde{F}}\}$ which satisfies certain conditions and the properties of the 3-structure $\{\underset{\alpha}{F}, \underset{\alpha}{\omega}, \underset{\alpha}{\eta}\}$ induced on hypersurfaces M^{4n-1} immersed in M^{4n}.

In the present paper we shall give the relations between the Nijenhuis tensors of the 3-structure $\{\underset{\alpha}{\tilde{F}}\}$ on M^{4n} and the formulas for a linear connection satisfying the conditions : $\overset{0}{\tilde{\nabla}}\underset{\alpha}{\tilde{F}} = 0$, $\alpha = 1,2,3$. Further we shall give the formulas for the induced structure on the hypersurfaces M^{4n-1} immersed in M^{4n}.

1. Linear connections on differentiable manifold with the 3-structure. Let M^{4n}, $\{\underset{\alpha}{\tilde{F}}\}$ denote $4n$-dimensional differentiable manifold and the 3-structure on M^{4n} respectively ([1]). The $\underset{\alpha}{\tilde{F}}$ $(\alpha = 1,2,3)$ are tensor fields of the type (1,1) which satisfy the following conditions

$$\underset{\alpha}{\tilde{F}} \circ \underset{\alpha}{\tilde{F}} = \underset{\alpha}{\tilde{F}}^2 = \underset{\alpha}{\varepsilon}\tilde{I} \qquad \underset{\alpha}{\varepsilon} = \pm 1 , \tag{1.1}$$

$$\underset{\alpha}{\tilde{F}} \circ \underset{\beta}{\tilde{F}} = \underset{\alpha\beta}{\varepsilon} \cdot \underset{\gamma}{\tilde{F}} \qquad \underset{\alpha\beta}{\varepsilon} = \pm 1 , \ \alpha \neq \beta \neq \gamma \neq \alpha , \tag{1.2}$$

where $\tilde{I}$ denotes the identity mapping on TM^{4n}. The coefficients $\underset{\alpha}{\varepsilon}, \underset{\alpha\beta}{\varepsilon}$ satisfy the following identities

$$\underset{\alpha}{\varepsilon} \cdot \underset{\beta\gamma}{\varepsilon} = \underset{\alpha\beta}{\varepsilon} \cdot \underset{\gamma}{\varepsilon} , \tag{1.3}$$

$$\underset{\alpha\beta}{\varepsilon} \cdot \underset{\alpha\gamma}{\varepsilon} = \underset{\beta\alpha}{\varepsilon} \cdot \underset{\gamma\alpha}{\varepsilon} = \underset{\alpha}{\varepsilon} , \tag{1.4}$$

for $\alpha \neq \beta \neq \gamma \neq \alpha$ ([1]).

The Nijenhuis tensor for $\tilde{F}_{\alpha}, \tilde{F}_{\beta}$ is defined as follow

$$
\text{(1.5)}\qquad
\begin{aligned}
\tilde{N}_{\tilde{F}_{\alpha}\tilde{F}_{\beta}}(\tilde{X},\tilde{Y}) = {} & [\tilde{F}_{\alpha}\tilde{X},\tilde{F}_{\beta}\tilde{Y}] + [\tilde{F}_{\beta}\tilde{X},\tilde{F}_{\alpha}\tilde{Y}] - \tilde{F}_{\alpha}[\tilde{F}_{\beta}\tilde{X},\tilde{Y}] - \tilde{F}_{\beta}[\tilde{F}_{\alpha}\tilde{X},\tilde{Y}] - \\
& - \tilde{F}_{\alpha}[\tilde{X},\tilde{F}_{\beta}\tilde{Y}] - \tilde{F}_{\beta}[\tilde{X},\tilde{F}_{\alpha}\tilde{Y}] + (\tilde{F}_{\alpha}\circ\tilde{F}_{\beta} + \tilde{F}_{\beta}\circ\tilde{F}_{\alpha})[\tilde{X},\tilde{Y}] ,
\end{aligned}
$$

for $\tilde{X},\tilde{Y} \in TM^{4n}$ ([2]).

Let $\tilde{\nabla}$ denote the covariant differentiation of the linear connection $\tilde{\Gamma}$ without torsion on M^{4n}. For arbitrary vector fields $\tilde{X},\tilde{Y} \in TM^{4n}$ we have

$$[\tilde{X},\tilde{Y}] = \tilde{\nabla}_{\tilde{X}}\tilde{Y} - \tilde{\nabla}_{\tilde{Y}}\tilde{X} .$$

Moreover, we will make use of the relations

$$(\tilde{\nabla}_{\tilde{X}}\tilde{F}_{\alpha})(\tilde{Y}) = \tilde{\nabla}_{\tilde{X}}(\tilde{F}_{\alpha}\tilde{Y}) - \tilde{F}_{\alpha}\tilde{\nabla}_{\tilde{X}}\tilde{Y} .$$

Let $\tilde{N}_{\alpha\beta}(\tilde{X},\tilde{Y}) = \tilde{N}_{\tilde{F}_{\alpha}\tilde{F}_{\beta}}(\tilde{X},\tilde{Y})$. The formula (1.5) of Nijenhuis tensor and the above relations imply

$$
\text{(1.6)}\qquad
\begin{aligned}
\tilde{N}_{\alpha\beta}(\tilde{X},\tilde{Y}) = {} & (\tilde{\nabla}_{\tilde{F}_{\alpha}\tilde{X}}\tilde{F}_{\beta})(\tilde{Y}) + (\tilde{\nabla}_{\tilde{F}_{\beta}\tilde{X}}\tilde{F}_{\alpha})(\tilde{Y}) - \tilde{F}_{\alpha}(\tilde{\nabla}_{\tilde{X}}\tilde{F}_{\beta})(\tilde{Y}) - \tilde{F}_{\beta}(\tilde{\nabla}_{\tilde{X}}\tilde{F}_{\alpha})(\tilde{Y}) - \\
& - (\tilde{\nabla}_{\tilde{F}_{\alpha}\tilde{Y}}\tilde{F}_{\beta})(\tilde{X}) - (\tilde{\nabla}_{\tilde{F}_{\beta}\tilde{Y}}\tilde{F}_{\alpha})(\tilde{X}) + \tilde{F}_{\alpha}(\tilde{\nabla}_{\tilde{Y}}\tilde{F}_{\beta})(\tilde{X}) + \tilde{F}_{\beta}(\tilde{\nabla}_{\tilde{Y}}\tilde{F}_{\alpha})(\tilde{X}) .
\end{aligned}
$$

We have $\tilde{N}_{\alpha\beta} = \tilde{N}_{\beta\alpha}$. Thus for the 3-structure $\{\tilde{F}_{\alpha}\}$ on M^{4n} there exist 6 different Nijenhuis tensors.

Making use of (1.1)–(1.4) we immediately obtain the following theorem :

- The Nijenhuis tensors of the 3-structure $\{\tilde{F}_{\alpha}\}$ on M^{4n} satisfy the following identities :

(1.7.1) $\tilde{N}_{\alpha\alpha}(\tilde{F}_{\alpha}\tilde{X},\tilde{F}_{\alpha}\tilde{Y}) = \varepsilon_{\alpha}\tilde{N}_{\alpha\alpha}(\tilde{X},\tilde{Y})$

(1.7.2) $\tilde{N}_{\alpha\alpha}(\tilde{F}_{\alpha}\tilde{X},\tilde{Y}) = \tilde{N}_{\alpha\alpha}(\tilde{X},\tilde{F}_{\alpha}\tilde{Y}) = -\tilde{F}_{\alpha}\tilde{N}_{\alpha\alpha}(\tilde{X},\tilde{Y})$

(1.7.3) $\tilde{N}_{\alpha\alpha}(\tilde{F}_{\gamma}\tilde{X},\tilde{F}_{\gamma}\tilde{Y}) = \tilde{N}_{\beta\beta}(\tilde{X},\tilde{Y}) + \varepsilon_{\alpha}\tilde{N}_{\gamma\gamma}(\tilde{X},\tilde{Y}) - 2\varepsilon_{\alpha\gamma}\tilde{F}_{\alpha}\tilde{N}_{\beta\gamma}(\tilde{X},\tilde{Y})$

(1.7.4) $\tilde{N}_{\alpha\alpha}(\tilde{F}_{\beta}\tilde{X},\tilde{F}_{\gamma}\tilde{Y}) = -\varepsilon_{\alpha\gamma}\tilde{F}_{\gamma}\tilde{N}_{\alpha\alpha}(\tilde{F}_{\gamma}\tilde{X},\tilde{F}_{\gamma}\tilde{Y})$

(1.7.5) $\tilde{N}_{\alpha\alpha}(\tilde{F}_{\gamma}\tilde{X},\tilde{Y}) + \tilde{N}_{\alpha\alpha}(\tilde{X},\tilde{F}_{\gamma}\tilde{Y}) = 2[\varepsilon_{\alpha\gamma}\tilde{N}_{\alpha\beta}(\tilde{X},\tilde{Y}) - \tilde{F}_{\alpha}\tilde{N}_{\alpha\gamma}(\tilde{X},\tilde{Y})]$

(1.7.6) $\tilde{N}_{\alpha\alpha}(\tilde{F}_{\gamma}\tilde{X},\tilde{F}_{\alpha}\tilde{Y}) + \tilde{N}_{\alpha\alpha}(\tilde{F}_{\alpha}\tilde{X},\tilde{F}_{\gamma}\tilde{Y}) = -\tilde{F}_{\alpha}[\tilde{N}_{\alpha\alpha}(\tilde{F}_{\gamma}\tilde{X},\tilde{Y}) + \tilde{N}_{\alpha\alpha}(\tilde{X},\tilde{F}_{\gamma}\tilde{Y})]$

(1.7.7) $\tilde{N}_{\alpha\beta}(\tilde{F}_{\alpha}\tilde{X},\tilde{F}_{\alpha}\tilde{Y}) = \frac{1}{2}(\varepsilon_{\alpha\beta} + \varepsilon_{\beta\alpha})\tilde{F}_{\gamma}\tilde{N}_{\alpha\alpha}(\tilde{X},\tilde{Y}) - \varepsilon_{\beta\alpha}\tilde{F}_{\alpha}\tilde{N}_{\alpha\gamma}(\tilde{X},\tilde{Y})$

(1.7.8) $\tilde{N}_{\alpha\beta}(\tilde{F}_{\gamma}\tilde{X},\tilde{F}_{\gamma}\tilde{Y}) =$

$$= \varepsilon_{\gamma}\tilde{N}_{\alpha\beta}(\tilde{X},\tilde{Y}) - \varepsilon_{\alpha\gamma}\tilde{F}_{\beta}\tilde{N}_{\beta\gamma}(\tilde{X},\tilde{Y}) - \varepsilon_{\beta\gamma}\tilde{F}_{\alpha}\tilde{N}_{\alpha\gamma}(\tilde{X},\tilde{Y}) + \frac{1}{2}(\varepsilon_{\alpha\beta} + \varepsilon_{\beta\alpha})\tilde{F}_{\gamma}\tilde{N}_{\gamma\gamma}(\tilde{X},\tilde{Y})$$

(1.7.9) $\underset{\alpha\beta}{\tilde N}(\underset{\alpha}{\tilde F}\tilde X, \underset{\beta}{\tilde F}\tilde Y) + \underset{\alpha\beta}{\tilde N}(\underset{\beta}{\tilde F}\tilde X, \underset{\alpha}{\tilde F}\tilde Y) =$

$$= (\underset{\alpha\beta}{\varepsilon} + \underset{\beta\alpha}{\varepsilon})\underset{\gamma}{\tilde F}\,\underset{\alpha\beta}{\tilde N}(\tilde X,\tilde Y) - \underset{\beta\alpha}{\varepsilon}\,\underset{\alpha}{\tilde F}\,\underset{\beta\gamma}{\tilde N}(\tilde X,\tilde Y) - \underset{\alpha\beta}{\varepsilon}\,\underset{\beta}{\tilde F}\,\underset{\alpha\gamma}{\tilde N}(\tilde X,\tilde Y) + \underset{\alpha\beta}{\varepsilon}\cdot\underset{\beta\alpha}{\varepsilon}\,\underset{\gamma\gamma}{\tilde N}(\tilde X,\tilde Y)$$

(1.7.10) $\underset{\alpha\beta}{\tilde N}(\underset{\alpha}{\tilde F}\tilde X, \underset{\gamma}{\tilde F}\tilde Y) + \underset{\alpha\beta}{\tilde N}(\underset{\gamma}{\tilde F}\tilde X, \underset{\alpha}{\tilde F}\tilde Y) =$

$$= (\underset{\alpha\beta}{\varepsilon} + \underset{\beta\alpha}{\varepsilon})\underset{\gamma}{\tilde F}\,\underset{\alpha\gamma}{\tilde N}(\tilde X,\tilde Y) - \underset{\beta\alpha}{\varepsilon}\,\underset{\alpha}{\tilde F}\,\underset{\gamma\gamma}{\tilde N}(\tilde X,\tilde Y) - \underset{\beta\gamma}{\varepsilon}\,\underset{\alpha}{\tilde F}\,\underset{\alpha\alpha}{\tilde N}(\tilde X,\tilde Y) - \underset{\alpha\gamma}{\varepsilon}\,\underset{\beta}{\tilde F}\,\underset{\alpha\beta}{\tilde N}(\tilde X,\tilde Y) +$$
$$+ \underset{\alpha\gamma}{\varepsilon}\cdot\underset{\beta\alpha}{\varepsilon}\,\underset{\beta\gamma}{\tilde N}(\tilde X,\tilde Y)$$

(1.7.11) $\underset{\alpha\beta}{\tilde N}(\tilde X, \underset{\alpha}{\tilde F}\tilde Y) + \underset{\alpha\beta}{\tilde N}(\underset{\alpha}{\tilde F}\tilde X, \tilde Y) = \underset{\beta\alpha}{\varepsilon}\,\underset{\alpha\gamma}{\tilde N}(\tilde X,\tilde Y) - \underset{\beta}{\tilde F}\,\underset{\alpha\alpha}{\tilde N}(\tilde X,\tilde Y) - \underset{\alpha}{\tilde F}\,\underset{\alpha\beta}{\tilde N}(\tilde X,\tilde Y)$

(1.7.12) $\underset{\alpha\beta}{\tilde N}(\tilde X, \underset{\gamma}{\tilde F}\tilde Y) + \underset{\alpha\beta}{\tilde N}(\underset{\gamma}{\tilde F}\tilde X, \tilde Y) =$

$$= \underset{\beta\gamma}{\varepsilon}\,\underset{\alpha\alpha}{\tilde N}(\tilde X,\tilde Y) + \underset{\alpha\gamma}{\varepsilon}\,\underset{\beta\beta}{\tilde N}(\tilde X,\tilde Y) - \underset{\alpha}{\tilde F}\,\underset{\beta\gamma}{\tilde N}(\tilde X,\tilde Y) - \underset{\beta}{\tilde F}\,\underset{\alpha\gamma}{\tilde N}(\tilde X,\tilde Y)$$

$\alpha \neq \beta \neq \gamma \neq \alpha$, $\tilde X, \tilde Y \in TM^{4n}$.

Theorem 1. *Let the 3-structure* $\{\underset{\alpha}{\tilde F}\}$ *on* M^{4n} *be given. There exists the linear connection* $\overset{0}{\Gamma}$ *such that*

(1.8) $$\overset{\tilde 0}{\nabla}\underset{1}{\tilde F} = \overset{\tilde 0}{\nabla}\underset{2}{\tilde F} = \overset{\tilde 0}{\nabla}\underset{3}{\tilde F} = 0\,.$$

Proof. Let $\overset{\tilde 1}{\Gamma}$ denote an arbitrary linear connection without torsion on M^{4n}. We define the connection $\overset{\tilde 2}{\Gamma}$ as follow

(1.9) $\overset{\tilde 2}{\Gamma}(\tilde X,\tilde Y) =$

$$= \overset{\tilde 1}{\Gamma}(\tilde X,\tilde Y) + \tfrac14 \underset{1}{\varepsilon}\underset{1}{\tilde F}[(\overset{\tilde 1}{\nabla}_{\tilde X}\underset{1}{\tilde F})(\tilde Y) + (\overset{\tilde 1}{\nabla}_{\tilde Y}\underset{1}{\tilde F})(\tilde X)] - \tfrac14\underset{1}{\varepsilon}[(\overset{\tilde 1}{\nabla}_{\tilde X}\underset{1}{\tilde F})(\underset{1}{\tilde F}\tilde Y) - (\overset{\tilde 1}{\nabla}_{\underset{1}{\tilde F}\tilde Y}\underset{1}{\tilde F})(\tilde X)]$$

for arbitrary $\tilde X, \tilde Y \in TM^{4n}$. Then we have

$$(\overset{\tilde 2}{\nabla}_{\tilde X}\underset{1}{\tilde F})(\tilde Y) = 0\,.$$

Namely

$(\overset{\tilde 2}{\nabla}_{\tilde X}\underset{1}{\tilde F})(\tilde Y) =$

$$=(\overset{\tilde 1}{\nabla}_{\tilde X}\underset{1}{\tilde F})(\tilde Y) - \underset{1}{\tilde F}\Big[\tfrac14\underset{1}{\varepsilon}\underset{1}{\tilde F}\Big((\overset{\tilde 1}{\nabla}_{\tilde X}\underset{1}{\tilde F})(\tilde Y) + (\overset{\tilde 1}{\nabla}_{\tilde Y}\underset{1}{\tilde F})(\tilde X)\Big) - \tfrac14\underset{1}{\varepsilon}\Big((\overset{\tilde 1}{\nabla}_{\tilde X}\underset{1}{\tilde F})(\underset{1}{\tilde F}\tilde Y) - (\overset{\tilde 1}{\nabla}_{\underset{1}{\tilde F}\tilde Y}\underset{1}{\tilde F})(\tilde X)\Big)\Big] +$$
$$+ \tfrac14\underset{1}{\varepsilon}\underset{1}{\tilde F}\Big((\overset{\tilde 1}{\nabla}_{\tilde X}\underset{1}{\tilde F})(\underset{1}{\tilde F}\tilde Y) + (\overset{\tilde 1}{\nabla}_{\underset{1}{\tilde F}\tilde Y}\underset{1}{\tilde F})(\tilde X)\Big) - \tfrac14\underset{1}{\varepsilon}\Big((\overset{\tilde 1}{\nabla}_{\tilde X}\underset{1}{\tilde F})(\underset{1}{\tilde F}^2\tilde Y) - (\overset{\tilde 1}{\nabla}_{\underset{1}{\tilde F}^2\tilde Y}\underset{1}{\tilde F})(\tilde X)\Big) =$$
$$= (\overset{\tilde 1}{\nabla}_{\tilde X}\underset{1}{\tilde F})(\tilde Y) - \tfrac14(\overset{\tilde 1}{\nabla}_{\tilde X}\underset{1}{\tilde F})(\tilde Y) - \tfrac14(\overset{\tilde 1}{\nabla}_{\tilde Y}\underset{1}{\tilde F})(\tilde X) + \tfrac12\underset{1}{\varepsilon}\underset{1}{\tilde F}(\overset{\tilde 1}{\nabla}_{\tilde X}\underset{1}{\tilde F})(\underset{1}{\tilde F}\tilde Y) - \tfrac14(\overset{\tilde 1}{\nabla}_{\tilde X}\underset{1}{\tilde F})(\tilde Y) +$$
$$+ \tfrac14(\overset{\tilde 1}{\nabla}_{\tilde Y}\underset{1}{\tilde F})(\tilde X) =$$

$$= (\overset{\tilde 1}{\nabla}_{\tilde X}\underset{1}{\tilde F})(\tilde Y) - \tfrac12(\overset{\tilde 1}{\nabla}_{\tilde X}\underset{1}{\tilde F})(\tilde Y) + \tfrac12\underset{1}{\varepsilon}\underset{1}{\tilde F}(\overset{\tilde 1}{\nabla}_{\tilde X}\underset{1}{\tilde F}^2\tilde Y - \underset{1}{\tilde F}\overset{\tilde 1}{\nabla}_{\tilde X}(\underset{1}{\tilde F}\tilde Y)) =$$

$$= (\overset{\tilde 1}{\nabla}_{\tilde X}\underset{1}{\tilde F})(\tilde Y) - \tfrac12(\overset{\tilde 1}{\nabla}_{\tilde X}\underset{1}{\tilde F})(\tilde Y) + \tfrac12\underset{1}{\tilde F}(\overset{\tilde 1}{\nabla}_{\tilde X}\tilde Y) - \tfrac12(\overset{\tilde 1}{\nabla}_{\tilde X}\underset{1}{\tilde F}\tilde Y) =$$

$$= (\overset{\tilde 1}{\nabla}_{\tilde X}\underset{1}{\tilde F})(\tilde Y) - \tfrac12(\overset{\tilde 1}{\nabla}_{\tilde X}\underset{1}{\tilde F})(\tilde Y) - \tfrac12(\overset{\tilde 1}{\nabla}_{\tilde X}\underset{1}{\tilde F})(\tilde Y) = 0\,.$$

Now we are able to define the connection $\overset{\tilde 0}{\Gamma}$:

$$\overset{\tilde 0}{\Gamma}(\tilde X,\tilde Y) = \overset{\tilde 2}{\Gamma}(\tilde X,\tilde Y) + \tfrac12\underset{2}{\varepsilon}\underset{2}{\tilde F}(\overset{\tilde 2}{\nabla}_{\tilde X}\underset{2}{\tilde F})(\tilde Y) +$$

$$+ \frac{\underset{2}{\varepsilon}}{2(3\underset{13}{\varepsilon} + \underset{31}{\varepsilon})}\Big[\underset{13}{\varepsilon}\underset{2}{\tilde F}(\overset{\tilde 2}{\nabla}_{\tilde Y}\underset{2}{\tilde F})(\tilde X) + \underset{13}{\varepsilon}(\overset{\tilde 2}{\nabla}_{\underset{2}{\tilde F}\tilde Y}\underset{2}{\tilde F})(\tilde X) + \underset{3}{\tilde F}(\overset{\tilde 2}{\nabla}_{\underset{1}{\tilde F}\tilde Y}\underset{2}{\tilde F})(\tilde X) + \underset{21}{\varepsilon}(\overset{\tilde 2}{\nabla}_{\underset{3}{\tilde F}\tilde Y}\underset{3}{\tilde F})(\tilde X)\Big].$$

for arbitrary $\tilde X, \tilde Y \in TM^{4n}$. Let us note that

$$(\overset{\tilde 0}{\nabla}_{\tilde X}\underset{1}{\tilde F})(\tilde Y) = (\overset{\tilde 0}{\nabla}_{\tilde X}\underset{2}{\tilde F})(\tilde Y) = 0\,.$$

Namely, because $\overset{\tilde 2}{\nabla}_{\underset{1}{\tilde F}} = 0$, then

$$(\overset{\tilde 0}{\nabla}_{\tilde X}\underset{1}{\tilde F})(\tilde Y) = (\overset{\tilde 2}{\nabla}_{\tilde X}\underset{1}{\tilde F})(\tilde Y) - \underset{1}{\tilde F}\Big\{\tfrac12\underset{2}{\varepsilon}\underset{2}{\tilde F}(\overset{\tilde 2}{\nabla}_{\tilde X}\underset{2}{\tilde F})(\tilde Y) + \frac{\underset{2}{\varepsilon}}{2(3\underset{13}{\varepsilon} + \underset{31}{\varepsilon})}\Big[\underset{13}{\varepsilon}\underset{2}{\tilde F}(\overset{\tilde 2}{\nabla}_{\tilde Y}\underset{2}{\tilde F})(\tilde X) +$$

$$+ \underset{13}{\varepsilon}(\overset{\tilde 2}{\nabla}_{\underset{2}{\tilde F}\tilde Y}\underset{2}{\tilde F})(\tilde X) + \underset{3}{\tilde F}(\overset{\tilde 2}{\nabla}_{\underset{1}{\tilde F}\tilde Y}\underset{2}{\tilde F})(\tilde X) + \underset{21}{\varepsilon}(\overset{\tilde 2}{\nabla}_{\underset{3}{\tilde F}\tilde Y}\underset{3}{\tilde F})(\tilde X)\Big]\Big\} + \tfrac12\underset{2}{\varepsilon}\underset{2}{\tilde F}(\overset{\tilde 2}{\nabla}_{\tilde X}\underset{2}{\tilde F})(\underset{1}{\tilde F}\tilde Y) +$$

$$+ \frac{\underset{2}{\varepsilon}}{2(3\underset{13}{\varepsilon} + \underset{31}{\varepsilon})}\Big[\underset{13}{\varepsilon}\underset{2}{\tilde F}(\overset{\tilde 2}{\nabla}_{\underset{1}{\tilde F}\tilde Y}\underset{2}{\tilde F})(\tilde X) + \underset{13}{\varepsilon}(\overset{\tilde 2}{\nabla}_{\underset{2}{\tilde F}\underset{1}{\tilde F}\tilde Y}\underset{2}{\tilde F})(\tilde X) + \underset{3}{\tilde F}(\overset{\tilde 2}{\nabla}_{\underset{1}{\tilde F}^2\tilde Y}\underset{2}{\tilde F})(\tilde X) + \underset{21}{\varepsilon}(\overset{\tilde 2}{\nabla}_{\underset{3}{\tilde F}\underset{1}{\tilde F}\tilde Y}\underset{3}{\tilde F})(\tilde X)\Big] =$$

$$= -\tfrac12\underset{32}{\varepsilon}\underset{3}{\tilde F}(\overset{\tilde 2}{\nabla}_{\tilde X}\underset{2}{\tilde F})(\tilde Y) + \tfrac12\underset{2}{\varepsilon}\underset{2}{\tilde F}\Big(\underset{21}{\varepsilon}\overset{\tilde 2}{\nabla}_{\tilde X}\underset{3}{\tilde F}\tilde Y - \underset{2}{\tilde F}(\overset{\tilde 2}{\nabla}_{\tilde X}\underset{1}{\tilde F}\tilde Y)\Big) -$$

$$- \frac{\underset{2}{\varepsilon}}{2(3\underset{13}{\varepsilon} + \underset{31}{\varepsilon})}\Big[\underset{1}{\varepsilon}\underset{3}{\tilde F}(\overset{\tilde 2}{\nabla}_{\tilde Y}\underset{2}{\tilde F})(\tilde X) + \underset{13}{\varepsilon}\underset{1}{\tilde F}(\overset{\tilde 2}{\nabla}_{\underset{2}{\tilde F}\tilde Y}\underset{2}{\tilde F})(\tilde X) + \underset{13}{\varepsilon}\underset{2}{\tilde F}(\overset{\tilde 2}{\nabla}_{\underset{1}{\tilde F}\tilde Y}\underset{2}{\tilde F})(\tilde X) + \underset{21}{\varepsilon}\underset{1}{\tilde F}(\overset{\tilde 2}{\nabla}_{\underset{3}{\tilde F}\tilde Y}\underset{3}{\tilde F})(\tilde X) -$$

$$- \underset{13}{\varepsilon}\underset{2}{\tilde F}(\overset{\tilde 2}{\nabla}_{\underset{1}{\tilde F}\tilde Y}\underset{2}{\tilde F})(\tilde X) - \underset{13}{\varepsilon}\underset{21}{\varepsilon}(\overset{\tilde 2}{\nabla}_{\underset{3}{\tilde F}\tilde Y}\underset{2}{\tilde F})(\tilde X) - \underset{1}{\varepsilon}\underset{3}{\tilde F}(\overset{\tilde 2}{\nabla}_{\tilde Y}\underset{2}{\tilde F})(\tilde X) - \underset{1}{\varepsilon}(\overset{\tilde 2}{\nabla}_{\underset{2}{\tilde F}\tilde Y}\underset{3}{\tilde F})(\tilde X)\Big] =$$

$$= -\tfrac12\underset{32}{\varepsilon}\underset{3}{\tilde F}(\overset{\tilde 2}{\nabla}_{\tilde X}\underset{2}{\tilde F})(\tilde Y) + \tfrac12\underset{23}{\varepsilon}\underset{2}{\tilde F}\Big[(\overset{\tilde 2}{\nabla}_{\tilde X}\underset{3}{\tilde F})(\tilde Y) + \underset{3}{\tilde F}\overset{\tilde 2}{\nabla}_{\tilde X}\tilde Y\Big] - \tfrac12\Big[(\overset{\tilde 2}{\nabla}_{\tilde X}\underset{1}{\tilde F})(\tilde Y) + \underset{1}{\tilde F}(\overset{\tilde 2}{\nabla}_{\tilde X}\tilde Y)\Big] -$$

$$- \frac{\underset{2}{\varepsilon}}{2(3\underset{13}{\varepsilon} + \underset{31}{\varepsilon})}\Big\{\underset{13}{\varepsilon}\underset{1}{\tilde F}\underset{13}{\varepsilon}\Big[(\overset{\tilde 2}{\nabla}_{\underset{2}{\tilde F}\tilde Y}\underset{1}{\tilde F})\underset{3}{\tilde F}\tilde X + \underset{1}{\tilde F}(\overset{\tilde 2}{\nabla}_{\underset{2}{\tilde F}\tilde Y}\underset{3}{\tilde F})(\tilde X)\Big] - \underset{1}{\varepsilon}(\overset{\tilde 2}{\nabla}_{\underset{2}{\tilde F}\tilde Y}\underset{3}{\tilde F})(\tilde X) -$$

$$- \underset{13}{\varepsilon}\,\underset{21}{\varepsilon}\,\underset{13}{\varepsilon}\Big[(\overset{\tilde 2}{\nabla}_{\underset{3}{\tilde F}\tilde Y}\underset{1}{\tilde F})(\underset{3}{\tilde F}\tilde X) + \underset{1}{\tilde F}(\overset{\tilde 2}{\nabla}_{\underset{3}{\tilde F}\tilde Y}\underset{3}{\tilde F})(\tilde X)\Big] + \underset{21}{\varepsilon}\underset{1}{\tilde F}(\overset{\tilde 2}{\nabla}_{\underset{3}{\tilde F}\tilde Y}\underset{3}{\tilde F})(\tilde X)\Big\} =$$

$$= -\tfrac12\underset{32}{\varepsilon}\underset{3}{\tilde F}(\overset{\tilde 2}{\nabla}_{\tilde X}\underset{2}{\tilde F})(\tilde Y) + \tfrac12\underset{23}{\varepsilon}\underset{2}{\tilde F}(\overset{\tilde 2}{\nabla}_{\tilde X}\underset{2}{\tilde F})(\tilde Y) =$$

$$= -\tfrac{1}{2}\underset{32}{\varepsilon}\underset{3}{\tilde F}(\overset{\tilde 2}{\nabla}_{\tilde X}\underset{2}{\tilde F})(\tilde Y) + \tfrac{1}{2}\underset{23}{\varepsilon}\underset{2}{\tilde F}\underset{12}{\varepsilon}\Big[(\overset{\tilde 2}{\nabla}_{\tilde X}\underset{1}{\tilde F})(\underset{2}{\tilde F}\tilde Y) + \underset{1}{\tilde F}(\overset{\tilde 2}{\nabla}_{\tilde X}\underset{2}{\tilde F})(\tilde Y)\Big] =$$

$$= -\tfrac{1}{2}\underset{32}{\varepsilon}\underset{3}{\tilde F}(\overset{\tilde 2}{\nabla}_{\tilde X}\underset{2}{\tilde F})(\tilde Y) + \tfrac{1}{2}\underset{23}{\varepsilon}\underset{12}{\varepsilon}\underset{21}{\varepsilon}\underset{3}{\tilde F}(\overset{\tilde 2}{\nabla}_{\tilde X}\underset{2}{\tilde F})(\tilde Y) = 0\,,$$

and

$$(\overset{\tilde 0}{\nabla}_{\tilde X}\underset{2}{\tilde F})(\tilde Y) = (\overset{\tilde 2}{\nabla}_{\tilde X}\underset{2}{\tilde F})(\tilde Y) - \underset{2}{\tilde F}\Big\{\tfrac{1}{2}\underset{2}{\varepsilon}\underset{2}{\tilde F}(\overset{\tilde 2}{\nabla}_{\tilde X}\underset{2}{\tilde F})(\tilde Y) + \frac{\underset{2}{\varepsilon}}{2(3\underset{13}{\varepsilon}+\underset{31}{\varepsilon})}\Big[\underset{13}{\varepsilon}\underset{2}{\tilde F}(\overset{\tilde 2}{\nabla}_{\tilde Y}\underset{2}{\tilde F})(\tilde X) +$$

$$+\underset{13}{\varepsilon}(\overset{\tilde 2}{\nabla}_{\underset{3}{\tilde F}\tilde Y}\underset{2}{\tilde F})(\tilde X) + \underset{3}{\tilde F}(\overset{\tilde 2}{\nabla}_{\underset{1}{\tilde F}\tilde Y}\underset{2}{\tilde F})(\tilde X) + \underset{21}{\varepsilon}(\overset{\tilde 2}{\nabla}_{\underset{2}{\tilde F}\tilde Y}\underset{3}{\tilde F})(\tilde X)\Big]\Big\} + \tfrac{1}{2}\underset{2}{\varepsilon}\underset{2}{\tilde F}(\overset{\tilde 2}{\nabla}_{\tilde X}\underset{2}{\tilde F})(\underset{2}{\tilde F}\tilde Y) +$$

$$+\frac{\underset{2}{\varepsilon}}{2(3\underset{13}{\varepsilon}+\underset{31}{\varepsilon})}\Big[\underset{13}{\varepsilon}\underset{2}{\tilde F}(\overset{\tilde 2}{\nabla}_{\underset{3}{\tilde F}\tilde Y}\underset{2}{\tilde F})(\tilde X) + \underset{13}{\varepsilon}(\overset{\tilde 2}{\nabla}_{\underset{3}{\tilde F}^2\tilde Y}\underset{2}{\tilde F})(\tilde X) + \underset{3}{\tilde F}(\overset{\tilde 2}{\nabla}_{\underset{1}{\tilde F}\underset{2}{\tilde F}\tilde Y}\underset{2}{\tilde F})(\tilde X) + \underset{21}{\varepsilon}(\overset{\tilde 2}{\nabla}_{\underset{2}{\tilde F}\underset{2}{\tilde F}\tilde Y}\underset{3}{\tilde F})(\tilde X)\Big] =$$

$$= (\overset{\tilde 2}{\nabla}_{\tilde X}\underset{2}{\tilde F})(\tilde Y) - \tfrac{1}{2}(\overset{\tilde 2}{\nabla}_{\tilde X}\underset{2}{\tilde F})(\tilde Y) + \tfrac{1}{2}\underset{2}{\varepsilon}\underset{2}{\tilde F}\Big[\underset{2}{\varepsilon}\overset{\tilde 2}{\nabla}_{\tilde X}\tilde Y - \underset{2}{\tilde F}(\overset{\tilde 2}{\nabla}_{\tilde X}\underset{2}{\tilde F}\tilde Y)\Big] - \frac{\underset{2}{\varepsilon}}{2(3\underset{13}{\varepsilon}+\underset{31}{\varepsilon})}\underset{13}{\varepsilon}\underset{2}{\varepsilon}(\overset{\tilde 2}{\nabla}_{\tilde Y}\underset{2}{\tilde F})(\tilde X) +$$

$$+\underset{13}{\varepsilon}\underset{2}{\tilde F}(\overset{\tilde 2}{\nabla}_{\underset{3}{\tilde F}\tilde Y}\underset{2}{\tilde F})(\tilde X) + \underset{23}{\varepsilon}\underset{1}{\tilde F}(\overset{\tilde 2}{\nabla}_{\underset{1}{\tilde F}\tilde Y}\underset{2}{\tilde F})(\tilde X) + \underset{21}{\varepsilon}\underset{2}{\tilde F}(\overset{\tilde 2}{\nabla}_{\underset{3}{\tilde F}\tilde Y}\underset{3}{\tilde F})(\tilde X) - \underset{13}{\varepsilon}\underset{2}{\tilde F}(\overset{\tilde 2}{\nabla}_{\underset{3}{\tilde F}\tilde Y}\underset{2}{\tilde F})(\tilde X) -$$

$$-\underset{13}{\varepsilon}\underset{2}{\varepsilon}(\overset{\tilde 2}{\nabla}_{\tilde Y}\underset{2}{\tilde F})(\tilde X) - \underset{12}{\varepsilon}\underset{3}{\tilde F}(\overset{\tilde 2}{\nabla}_{\underset{3}{\tilde F}\tilde Y}\underset{2}{\tilde F})(\tilde X) - \underset{21}{\varepsilon}\underset{32}{\varepsilon}(\overset{\tilde 2}{\nabla}_{\underset{1}{\tilde F}\tilde Y}\underset{3}{\tilde F})(\tilde X) =$$

$$= \tfrac{1}{2}(\overset{\tilde 2}{\nabla}_{\tilde X}\underset{2}{\tilde F})(\tilde Y) + \tfrac{1}{2}\underset{2}{\tilde F}\overset{\tilde 2}{\nabla}_{\tilde X}\tilde Y - \tfrac{1}{2}(\overset{\tilde 2}{\nabla}_{\tilde X}\underset{2}{\tilde F}\tilde Y) - \frac{\underset{2}{\varepsilon}}{2(3\underset{13}{\varepsilon}+\underset{31}{\varepsilon})}\Big\{\underset{23}{\varepsilon}\underset{1}{\tilde F}(\overset{\tilde 2}{\nabla}_{\underset{2}{\tilde F}\tilde Y}\underset{2}{\tilde F})(\tilde X) +$$

$$+\underset{21}{\varepsilon}\underset{2}{\tilde F}\underset{12}{\varepsilon}\Big[(\overset{\tilde 2}{\nabla}_{\underset{2}{\tilde F}\tilde Y}\underset{1}{\tilde F})(\underset{2}{\tilde F}\tilde X) + \underset{1}{\tilde F}(\overset{\tilde 2}{\nabla}_{\underset{2}{\tilde F}\tilde Y}\underset{2}{\tilde F})(\tilde X)\Big] - \underset{13}{\varepsilon}\underset{3}{\tilde F}(\overset{\tilde 2}{\nabla}_{\underset{3}{\tilde F}\tilde Y}\underset{2}{\tilde F})(\tilde X) -$$

$$-\underset{21}{\varepsilon}\underset{32}{\varepsilon}\underset{12}{\varepsilon}\Big[(\overset{\tilde 2}{\nabla}_{\underset{1}{\tilde F}\tilde Y}\underset{1}{\tilde F})(\underset{2}{\tilde F}\tilde X) + \underset{1}{\tilde F}(\overset{\tilde 2}{\nabla}_{\underset{1}{\tilde F}\tilde Y}\underset{2}{\tilde F})(\tilde X)\Big]\Big\} =$$

$$= -\frac{\underset{2}{\varepsilon}}{2(3\underset{13}{\varepsilon}+\underset{31}{\varepsilon})}\Big[\underset{23}{\varepsilon}\underset{1}{\tilde F}(\overset{\tilde 2}{\nabla}_{\underset{1}{\tilde F}\tilde Y}\underset{2}{\tilde F})(\tilde X) + \underset{12}{\varepsilon}\underset{3}{\tilde F}(\overset{\tilde 2}{\nabla}_{\underset{3}{\tilde F}\tilde Y}\underset{2}{\tilde F})(\tilde X) - \underset{12}{\varepsilon}\underset{3}{\tilde F}(\overset{\tilde 2}{\nabla}_{\underset{3}{\tilde F}\tilde Y}\underset{2}{\tilde F})(\tilde X) - \underset{23}{\varepsilon}\underset{1}{\tilde F}(\overset{\tilde 2}{\nabla}_{\underset{1}{\tilde F}\tilde Y}\underset{2}{\tilde F})(\tilde X)\Big] = 0$$

This condition and (1.2) imply

$$(\overset{\tilde 0}{\nabla}_{\tilde X}\underset{3}{\tilde F})(\tilde Y) = \underset{12}{\varepsilon}[(\overset{\tilde 0}{\nabla}_{\tilde X}\underset{1}{\tilde F})(\underset{2}{\tilde F}\tilde Y) + \underset{1}{\tilde F}(\overset{\tilde 0}{\nabla}_{\tilde X}\underset{2}{\tilde F})(\tilde Y)] = 0\,.$$

Remark. The torsion tensor for the connections $\overset{\tilde 2}{\Gamma}$ and $\overset{\tilde 0}{\Gamma}$ are given by the following formulas

$$\overset{\tilde 2}{\Gamma}(\tilde X,\tilde Y) - \overset{\tilde 2}{\Gamma}(\tilde Y,\tilde X) = -\frac{1}{8}\underset{1}{\varepsilon}\underset{11}{\overset{\tilde 1}{N}}(\tilde X,\tilde Y)\,,$$

and

$$\overset{0}{\bar{\Gamma}}(\bar{X},\bar{Y})-\overset{0}{\bar{\Gamma}}(\bar{Y},\bar{X})=-\frac{1}{8}\underset{1}{\varepsilon}\underset{11}{\bar{N}}(\bar{X},\bar{Y})-\frac{\underset{3}{\varepsilon}\underset{13}{\varepsilon}}{4(3\underset{13}{\varepsilon}+\underset{31}{\varepsilon})}\underset{22}{\bar{N}}(\bar{X},\bar{Y})-$$

$$-\frac{\underset{33}{\varepsilon}}{4(3\underset{13}{\varepsilon}+\underset{31}{\varepsilon})}\underset{33}{\bar{N}}(\bar{X},\bar{Y})-\frac{\underset{2}{\varepsilon}}{2(3\underset{13}{\varepsilon}+\underset{31}{\varepsilon})}\underset{3}{\bar{F}}\underset{12}{\bar{N}}(\bar{X},\bar{Y})\,.$$

Definition . The linear connection $\overset{0}{\bar{\Gamma}}$ will be called $\{\underset{\alpha}{\bar{F}}\}$–connection.

2. Connections induced on submanifolds of Riemannian manifold with a 3–structure. Let us assume that M^{4n} is the $4n$–dimensional Riemannian manifold with the metric $\bar{g}$ and the 3–structure $\{\underset{\alpha}{\bar{F}}\}$. Moreover, let M^{4n-1} be smooth, oriented hypersurface immersed in M^{4n}. By N we denote the local vector field such that $N \notin TM^{4n-1}$ and $\bar{g}(N,N)=1$. Then for each vector field $\bar{X} \in TM^{4n}$ we have the following decomposition

(2.1) $$\underset{\alpha}{\bar{F}}\bar{X}=\underset{\alpha}{F}\bar{X}+\underset{\alpha}{\varepsilon}\,\underset{\alpha}{\omega}(\bar{X})N\,, \qquad \alpha=1,2,3,$$

where $\underset{\alpha}{F}$ is a tensor field of the type (1.1) , $\underset{\alpha}{F}\bar{X}\in TM^{4n-1}$, $\underset{\alpha}{\omega}$ is a tensor field of the type (0,1) ([1]).

We introduce notations

(2.2) $$\begin{cases} \underset{\alpha}{\eta} & =\underset{\alpha}{F}N\in TM^{4n-1} \\ \underset{\alpha}{\lambda} & =\underset{\alpha}{\omega}(N)\,. \end{cases}$$

We have

(2.3) $$\underset{\alpha}{\bar{F}}N=\underset{\alpha}{\eta}+\underset{\alpha}{\varepsilon}\,\underset{\alpha}{\lambda}\,N\,.$$

With respect to (2.1) we get

(2.4) $$\underset{\alpha}{\bar{F}}X=\underset{\alpha}{F}X+\underset{\alpha}{\varepsilon}\,\underset{\alpha}{\omega}\,(X)\,N$$

for $X\in TM^{4n-1}$.

Thus we obtained the 3-structure $\{\underset{\alpha}{F},\underset{\alpha}{\omega},\underset{\alpha}{\eta}\}$ on M^{4n-1}. It satisfies the following conditions:

(2.5) $$\begin{cases} \underset{\alpha}{F}^2 & =\underset{\alpha}{\varepsilon}(I-\underset{\alpha}{\omega}\otimes\underset{\alpha}{\eta}) \\ \underset{\alpha}{\omega}\circ\underset{\alpha}{F} & =-\underset{\alpha}{\varepsilon}\,\underset{\alpha}{\lambda}\,\underset{\alpha}{\omega} \\ \underset{\alpha}{F}\underset{\alpha}{\eta} & =-\underset{\alpha}{\varepsilon}\,\underset{\alpha}{\lambda}\,\underset{\alpha}{\eta} \\ \underset{\alpha}{\omega}(\underset{\alpha}{\eta}) & =1-\underset{\alpha}{\varepsilon}(\underset{\alpha}{\lambda})^2 \end{cases}$$

$$(2.6)\qquad \begin{cases} \underset{\alpha}{F}\circ\underset{\beta}{F} &= \underset{\alpha\beta}{\varepsilon}\,\underset{\gamma}{F}-\underset{\beta}{\varepsilon}\,\underset{\beta}{\omega}\otimes\underset{\alpha}{\eta}\\ \underset{\alpha}{\omega}\circ\underset{\beta}{F} &= \underset{\beta\gamma}{\varepsilon}\,\underset{\gamma}{\omega}-\underset{\beta}{\varepsilon}\,\underset{\alpha}{\lambda}\,\underset{\beta}{\omega}\\ \underset{\alpha}{F}\,\underset{\beta}{\eta} &= \underset{\alpha\beta}{\varepsilon}\,\underset{\gamma}{\eta}-\underset{\beta}{\varepsilon}\,\underset{\beta}{\lambda}\,\underset{\alpha}{\eta}\\ \underset{\alpha}{\omega}(\underset{\beta}{\eta}) &= \underset{\beta\gamma}{\varepsilon}\,\underset{\gamma}{\lambda}-\underset{\beta}{\varepsilon}\,\underset{\alpha}{\lambda}\,\underset{\beta}{\lambda} \end{cases}$$

(theorem 2, [1]).

Let $\bar{\nabla}$ denote the covariant differentiation of the linear connection $\bar{\Gamma}$ without torsion on M^{4n}. For all vector fields $X,Y\in TM^{4n-1}$ we have the decompositions

$$(2.7)\qquad \begin{cases} \bar{\nabla}_X Y &= \nabla_X Y + h(X,Y)\,N\\ \bar{\nabla}_X N &= -KX + k(X)\,N\,. \end{cases}$$

where ∇ is the covariant differentiation on M^{4n-1} and K,h,k are tensor fields on M^{4n-1} of the type (1.1), (0,2), (0,1) respectively. Moreover we have $h(X,Y) = h(Y,X)$ for all $X,Y\in TM^{4n-1}$ ([3]). The connection Γ will be called the induced connection by $\bar{\Gamma}$.

The condition (2.4) implies

$$(\bar{\nabla}_X\underset{\alpha}{\bar{F}})(Y) = (\bar{\nabla}_X\underset{\alpha}{F})(Y)+\underset{\alpha}{\varepsilon}(\bar{\nabla}_X\underset{\alpha}{\omega})(Y)\,N+\underset{\alpha}{\varepsilon}\,\underset{\alpha}{\omega}(Y)\bar{\nabla}_X N$$

Hence we get

$$(2.8)\qquad \begin{cases} (\bar{\nabla}_X\underset{\alpha}{\bar{F}})(Y) &= (\nabla_X\underset{\alpha}{F})(Y)-\underset{\alpha}{\varepsilon}\,\underset{\alpha}{\omega}(Y)KX-h(X,Y)\underset{\alpha}{\eta}+\\ &\quad+\underset{\alpha}{\varepsilon}\big[(\nabla_X\underset{\alpha}{\omega})(Y)+\underset{\alpha}{\varepsilon}\,h(X,\underset{\alpha}{F}Y)+\underset{\alpha}{\omega}(Y)k(X)-\underset{\alpha}{\lambda}\,h(X,Y)\big]N\\ (\bar{\nabla}_X\underset{\alpha}{\bar{F}})(N) &= \nabla_X\underset{\alpha}{\eta}-\underset{\alpha}{\varepsilon}\,\underset{\alpha}{\lambda}\,KX-k(X)\underset{\alpha}{\eta}+(\underset{\alpha}{F}\circ K)(X)+\\ &\quad+\underset{\alpha}{\varepsilon}\big[\partial_X\underset{\alpha}{\lambda}+\underset{\alpha}{\varepsilon}\,h(X,\underset{\alpha}{\eta})+(\underset{\alpha}{\omega}\circ K)(X)\big]N \end{cases}$$

Thus we have the following theorem:

Theorem 2. *If $\bar{\nabla}$ is a covariant differentiation of the linear connection without torsion on Riemannian manifold M^{4n} and ∇ is a covariant differentiation of the induced connection on oriented hypersurface M^{4n-1} immersed in M^{4n}, then the covariant derivatives of the 3-structure $\{\underset{\alpha}{\bar{F}}\}$ on M^{4n} and the induced 3-structure $\{\underset{\alpha}{F},\underset{\alpha}{\omega},\underset{\alpha}{\eta}\}$ on M^{4n-1} satisfy the relations (2.8).*

3. Riemannian connections without torsion. Let $\overset{*}{\nabla}$ denote a covariant differentiation of the Riemannian connection without torsion on Riemannian manifold with the 3-structure $\{\underset{\alpha}{\bar{F}}\}$ and with the metric $\bar{g}$ such that

$$(3.1)\qquad \bar{g}(\underset{\alpha}{\bar{F}}\bar{X},\underset{\alpha}{\bar{F}}\bar{Y})=\bar{g}(\bar{X},\bar{Y})\,,\qquad \alpha=1,2,3$$

for all $\tilde{X}, \tilde{Y} \in TM^{4n}$. The existence of such metric was given in [1].

Differentiating the above condition we get:

$$\tilde{g}((\tilde{\nabla}_{\tilde{Z}} \underset{\alpha}{\tilde{F}})(\tilde{X}) \, , \, \underset{\alpha}{\tilde{F}}\tilde{Y}) + \tilde{g}(\underset{\alpha}{\tilde{F}}\tilde{X} \, , \, (\tilde{\nabla}_{\tilde{Z}} \underset{\alpha}{\tilde{F}})(\tilde{Y})) = 0 \tag{3.2}$$

for arbitrary $\tilde{Z} \in TM^{4n}$.

Let M^{4n-1} denote the oriented hypersurface immersed in M^{4n}. We will assume that N is a normal unit vector field to M^{4n-1}, i.e. $\tilde{g}(N,N) = 1$, $\tilde{g}(X,N) = 0$ for $X \in TM^{4n-1}$. Then we have $k(X) = 0$ and

$$\begin{cases} \tilde{\overset{\bullet}{\nabla}}_X Y &= \overset{\bullet}{\nabla}_X Y + \overset{\bullet}{h}(X,Y)N \\ \tilde{\overset{\bullet}{\nabla}}_X N &= -\overset{\bullet}{K}\, X \end{cases} \tag{3.3}$$

for $X, Y \in TM^{4n-1}$. Thus the formulas (2.8) have the form

$$\begin{cases} (\tilde{\overset{\bullet}{\nabla}}_X \underset{\alpha}{\tilde{F}})(Y) &= (\overset{\bullet}{\nabla}_X \underset{\alpha}{F})(Y) - \underset{\alpha}{\varepsilon}\, \underset{\alpha}{\omega}(Y) \overset{\bullet}{K}\, X - \overset{\bullet}{h}(X,Y)\underset{\alpha}{\eta} + \\ &\quad + \underset{\alpha}{\varepsilon}\big[(\overset{\bullet}{\nabla}_X \underset{\alpha}{\omega})(Y) + \underset{\alpha}{\varepsilon}\, \overset{\bullet}{h}(X, \underset{\alpha}{F}Y) - \underset{\alpha}{\lambda}\, \overset{\bullet}{h}(X,Y)\big] N \\ (\tilde{\overset{\bullet}{\nabla}}_X \underset{\alpha}{\tilde{F}})(N) &= \overset{\bullet}{\nabla}_X \underset{\alpha}{\eta} - \underset{\alpha}{\varepsilon}\, \underset{\alpha}{\lambda}\, \overset{\bullet}{K}\, X + (\underset{\alpha}{F} \circ \overset{\bullet}{K})(X) + \\ &\quad + \underset{\alpha}{\varepsilon}\big[\partial_X \underset{\alpha}{\lambda} + \underset{\alpha}{\varepsilon}\, \overset{\bullet}{h}(X, \underset{\alpha}{\eta}) + (\underset{\alpha}{\omega} \circ \overset{\bullet}{K})(X)\big] N \end{cases} \tag{3.4}$$

Substituting (3.4) to (3.2) and making use of (2.1) and the conditions $\tilde{g}(X,Y) = g(X,Y)$ for $X, Y \in TM^{4n-1}$ $\quad g(X, \underset{\alpha}{\eta}) = \underset{\alpha}{\omega}(X)$ ([1]) and (2.5) we get the following relation for the induced metric g on M^{4n-1}

$$\begin{aligned} & g((\overset{\bullet}{\nabla}_Z \underset{\alpha}{F})(X), \underset{\alpha}{F}Y) + g(\underset{\alpha}{F}X, (\overset{\bullet}{\nabla}_Z \underset{\alpha}{F})(Y)) + \\ & + \underset{\alpha}{\omega}(X)\big[(\overset{\bullet}{\nabla}_Z \underset{\alpha}{\omega})(Y) + \underset{\alpha}{\varepsilon}\, \overset{\bullet}{h}(Z, \underset{\alpha}{F}Y) + (\underset{\alpha}{\varepsilon} - 1)\underset{\alpha}{\lambda}\, \overset{\bullet}{h}(Z,Y) - \underset{\alpha}{\varepsilon}\, g(\overset{\bullet}{K}\, Z, \underset{\alpha}{F}Y)\big] + \\ & + \underset{\alpha}{\omega}(Y)\big[(\overset{\bullet}{\nabla}_Z \underset{\alpha}{\omega})(X) + \underset{\alpha}{\varepsilon}\, \overset{\bullet}{h}(\underset{\alpha}{F}X, Z) + (\underset{\alpha}{\varepsilon} - 1)\underset{\alpha}{\lambda}\, \overset{\bullet}{h}(X,Z) - \underset{\alpha}{\varepsilon}\, g(\underset{\alpha}{F}X, \overset{\bullet}{K}\, Z)\big] = 0\,. \end{aligned}$$

4. On some connections on a manifold connected with a 3-structure.

Let $\tilde{\overset{S}{\nabla}}$ denote the covariant differentiation of the linear connection on Riemannian manifold M^{4n}, which satisfies the condition

$$\tilde{\overset{S}{\nabla}}_{\tilde{X}} \underset{\alpha}{\tilde{F}} = a_{\alpha}^{\beta}(\tilde{X})\, \underset{\beta}{\tilde{F}}\,, \qquad \tilde{X} \in TM^{4n} \tag{4.1}$$

where a_α^β are local 1-forms. By $\overset{S}{\nabla}$ we denote the covariant differentiation on M^{4n-1} of the induced connection. We have

$$
(4.2)\quad \begin{cases}
(\overset{S}{\nabla}_X \underset{\alpha}{F})(Y) &= \underset{\alpha}{\varepsilon}\, \underset{\alpha}{\omega}(Y)\overset{S}{K}\,X + \overset{S}{h}(X,Y)\underset{\alpha}{\eta} + a_\alpha^\beta(X)\underset{\beta}{F}Y \\
(\overset{S}{\nabla}_X \underset{\alpha}{\omega})(Y) &= \underset{\alpha}{\lambda}\, \overset{S}{h}(X,Y) - \underset{\alpha}{\varepsilon}\, \overset{S}{h}(X,\underset{\alpha}{F}Y) - \underset{\alpha}{\omega}(Y)\overset{S}{k}(X) + \underset{\alpha}{\varepsilon}\cdot\underset{\beta}{\varepsilon}\cdot a_\alpha^\beta(X)\underset{\beta}{\omega}(Y) \\
\overset{S}{\nabla}_X \underset{\alpha}{\eta} &= \underset{\alpha}{\varepsilon}\, \underset{\alpha}{\lambda}\, \overset{S}{K}\,X + \overset{S}{k}(X)\underset{\alpha}{\eta} - (\underset{\alpha}{F}\circ \overset{S}{K})(X) + a_\alpha^\beta(X)\underset{\beta}{\eta} \\
\partial_X \underset{\alpha}{\lambda} &= -\underset{\alpha}{\varepsilon}\, \overset{S}{h}(X,\underset{\alpha}{\eta}) - (\underset{\alpha}{\omega}\circ \overset{S}{K})(X) + \underset{\alpha}{\varepsilon}\cdot\underset{\beta}{\varepsilon}\, a_\alpha^\beta(X)\underset{\beta}{\lambda}
\end{cases}
$$

$X,Y \in TM^{4n-1}$.

Hence we get the following theorem:

Theorem 3. *If $\overset{\tilde{S}}{\nabla}$ is a covariant differentiation of a linear connection on Riemannian manifold which satisfies the condition (4.1), then the induced connection on the hypersurface satisfies the relations (4.2).*

In the particular case we get

Corollary. *If $\overset{\tilde{0}}{\nabla}$ is a covariant differentiation of the $\{\underset{\alpha}{\tilde{F}}\}$-connection on Riemannian manifold $(\overset{\tilde{0}}{\nabla}\underset{\alpha}{\tilde{F}} = 0$, $\alpha = 1,2,3)$, then the induced connection on hypersurface satisfies the following formulas:*

$$
(4.3)\quad \begin{aligned}
(\overset{0}{\nabla}_X \underset{\alpha}{F})(Y) &= \underset{\alpha}{\varepsilon}\, \underset{\alpha}{\omega}(Y)\overset{0}{K}X + \overset{0}{h}(X,Y)\underset{\alpha}{\eta} \\
(\overset{0}{\nabla}_X \underset{\alpha}{\omega})(Y) &= \underset{\alpha}{\lambda}\, \overset{0}{h}(X,Y) - \underset{\alpha}{\varepsilon}\, \overset{0}{h}(X,\underset{\alpha}{F}Y) - \underset{\alpha}{\omega}(Y)\overset{0}{k}(X) \\
\overset{0}{\nabla}_X \underset{\alpha}{\eta} &= \underset{\alpha}{\varepsilon}\, \underset{\alpha}{\lambda}\, \overset{0}{K}X + \overset{0}{k}(X)\underset{\alpha}{\eta} - (\underset{\alpha}{F}\circ \overset{0}{K})(X) \\
\partial_X \underset{\alpha}{\lambda} &= \underset{\alpha}{\varepsilon}\, \overset{0}{h}(X,\underset{\alpha}{\eta}) - (\underset{\alpha}{\omega}\circ \overset{0}{K})(X)
\end{aligned}
$$

for $X,Y \in TM^{4n-1}$.

Theorem 4. *Let $\overset{\tilde{0}}{\nabla}$ denote the covariant differentiation of the $\{\underset{\alpha}{\tilde{F}}\}$-connection on Riemannian manifold with the metric $\tilde{g}$, which satisfies the condition (3.1). Then we have*

$$
(\overset{\tilde{0}}{\nabla}_{\tilde{Z}}\tilde{g})(\underset{\alpha}{\tilde{F}}\tilde{X},\underset{\alpha}{\tilde{F}}\tilde{Y}) = (\overset{\tilde{0}}{\nabla}_{\tilde{Z}}\tilde{g})(\tilde{X},\tilde{Y}),
$$

$$
(\overset{\tilde{0}}{\nabla}_{\tilde{Z}}\tilde{g})(\underset{\alpha}{\tilde{F}}\tilde{X},\underset{\beta}{\tilde{F}}\tilde{Y}) = \underset{\alpha\gamma}{\varepsilon}\cdot(\overset{\tilde{0}}{\nabla}_{\tilde{Z}}\tilde{g})(\tilde{X},\underset{\gamma}{\tilde{F}}\tilde{Y})
$$

for arbitrary vector fields $\tilde{X}, \tilde{Y}, \tilde{Z} \in TM^{4n}$.

The first identity follows from the assumption and the condition (3.1). Substituting $\underset{\gamma}{\tilde{F}}\tilde{Y}$ instead of $\tilde{Y}$ we get the second identity.

5. Riemannian manifold with Tachibana 3–structure. Let us consider the Riemannian manifold M^{4n} with the metric $\tilde{g}$ and with the 3–structure $\{\underset{\alpha}{\tilde{F}}\}$. We assume that condition (3.1) is satisfied:

$$\tilde{g}(\underset{\alpha}{\tilde{F}}\tilde{X}, \underset{\alpha}{\tilde{F}}\tilde{Y}) = \tilde{g}(\tilde{X}, \tilde{Y}) , \qquad \alpha = 1,2,3$$

for arbitrary $\tilde{X}, \tilde{Y} \in TM^{4n}$. By $\overset{\tilde{T}}{\nabla}$ we denote the covariant differentiation of Riemannian connection without torsion with respect to the metric $\tilde{g}$.

The manifold M^{4n} is said to be a manifold with Tachibana 3–structure, if

$$(5.1) \qquad (\overset{\tilde{T}}{\nabla}_{\tilde{X}} \underset{\alpha}{\tilde{F}})(\tilde{Y}) + (\overset{\tilde{T}}{\nabla}_{\tilde{Y}} \underset{\alpha}{\tilde{F}})(\tilde{X}) = 0 , \qquad \alpha = 1,2,3$$

for all vector fields $\tilde{X}, \tilde{Y} \in TM^{4n}$.

Let us consider a smooth oriented hypersurface M^{4n-1} immersed in M^{4n}. By N we denote the normal unit vector field on M^{4n-1} (with respect to $\tilde{g}$). Then we get the decomposition (2.1) :

$$\underset{\alpha}{\tilde{F}}\tilde{X} = \underset{\alpha}{F}\tilde{X} + \underset{\alpha}{\varepsilon}\,\underset{\alpha}{\omega}(\tilde{X})\, N , \qquad \alpha = 1,2,3 .$$

The condition (5.1) implies

$$(5.2) \qquad (\overset{\tilde{T}}{\nabla}_X \underset{\alpha}{\tilde{F}})(Y) + (\overset{\tilde{T}}{\nabla}_Y \underset{\alpha}{\tilde{F}})(X) = 0 \qquad \text{for } X, Y \in TM^{4n-1}$$

The above formula and the formulas (3.4) imply the following conditions for the induced connection on M^{4n-1} :

$$(5.3) \qquad \begin{aligned} (\overset{T}{\nabla}_X \underset{\alpha}{F})(Y) + (\overset{T}{\nabla}_Y \underset{\alpha}{F})(X) &= 2\, \overset{T}{h}(X,Y)\underset{\alpha}{\eta} + \underset{\alpha}{\varepsilon}\big[\underset{\alpha}{\omega}(Y)\overset{T}{K}X + \underset{\alpha}{\omega}(X)\overset{T}{K}Y\big] \\ (\overset{T}{\nabla}_X \underset{\alpha}{\omega})(Y) + (\overset{T}{\nabla}_Y \underset{\alpha}{\omega})(X) &= 2\, \overset{T}{h}(X,Y)\underset{\alpha}{\lambda} - \underset{\alpha}{\varepsilon}\big[\, \overset{T}{h}(X, \underset{\alpha}{F}Y) + \overset{T}{h}(Y, \underset{\alpha}{F}X)\big] \end{aligned}$$

Theorem 5. *The induced connection on hypersurface of the manifold with Tachibana 3–structure fulfills the relations* (5.3).

6. Riemannian manifold with almost Sasaki 3–structure. Let us consider the Riemannian manifold M^{4n} with the metric $\tilde{g}$ and the 3–structure $\{\underset{\alpha}{\tilde{F}}\}$, which satisfies the condition (3.1). By $\overset{\tilde{s}}{\nabla}$ we denote the covariant differentiation of the Riemannian connection without torsion wih respect to $\tilde{g}$.

We define on M^{4n} the tensor field $\underset{\alpha}{\bar{\Phi}}$ of the type (0,2) as follow

(6.1) $$\underset{\alpha}{\bar{\Phi}}(\bar{X},\bar{Y}) = \bar{g}(\underset{\alpha}{\bar{F}}\bar{X},\bar{Y}), \qquad \alpha = 1,2,3$$

for arbitrary vector fields $\bar{X},\bar{Y} \in TM^{4n}$.

The manifold M^{4n} is said to be a manifold with almost Sasaki 3-structure if

(6.2) $$\bar{\nabla}^{\circ}_{\bar{X}}(\underset{\alpha}{\bar{\Phi}}(\bar{Y},\bar{Z})) + \bar{\nabla}^{\circ}_{\bar{Y}}(\underset{\alpha}{\bar{\Phi}}(\bar{Z},\bar{X})) + \bar{\nabla}^{\circ}_{\bar{Z}}(\underset{\alpha}{\bar{\Phi}}(\bar{X},\bar{Y})) = 0$$

for $\bar{X},\bar{Y},\bar{Z} \in TM^{4n}$. The condition (6.2) is equivalent to

(6.2') $$\bar{\nabla}^{\circ}_{\bar{X}}(\bar{g}(\underset{\alpha}{\bar{F}}\bar{Y},\bar{Z})) + \bar{\nabla}^{\circ}_{\bar{Y}}(\bar{g}(\underset{\alpha}{\bar{F}}\bar{Z},\bar{X})) + \bar{\nabla}^{\circ}_{\bar{Z}}(\bar{g}(\underset{\alpha}{\bar{F}}\bar{X},\bar{Y})) = 0$$

Let M^{4n-1} denote a smooth oriented hypersurface immersed in M^{4n} with induced metric g , i.e.

$$g(X,Y) = \bar{g}(X,Y)$$

for $X,Y \in TM^{4n-1}$. By N we denote the normal unit vector field to M^{4n-1} (with respect to $\bar{g}$). Using the decompostion (2.1) we get

$$\bar{\nabla}^{\circ}_{X}(\bar{g}(\underset{\alpha}{F}Y,Z)) + \bar{\nabla}^{\circ}_{Y}(\bar{g}(\underset{\alpha}{F}Z,X)) + \bar{\nabla}^{\circ}_{Z}(\bar{g}(\underset{\alpha}{F}X,Y)) = 0$$

for all $X,Y,Z \in TM^{4n-1}$. Hence we obtain

$$\begin{aligned}&\bar{g}(\bar{\nabla}^{\circ}_{X}\underset{\alpha}{F}Y,Z) + \bar{g}(\underset{\alpha}{F}Y,\bar{\nabla}^{\circ}_{X}Z) + \bar{g}(\bar{\nabla}^{\circ}_{Y}\underset{\alpha}{F}Z,X)+\\ &+\bar{g}(\underset{\alpha}{F}Z,\bar{\nabla}^{\circ}_{Y}X) + \bar{g}(\bar{\nabla}^{\circ}_{Z}\underset{\alpha}{F}X,Y) + \bar{g}(\underset{\alpha}{F}X,\bar{\nabla}^{\circ}_{Z}Y) = 0\,.\end{aligned}$$

The induced metric g on M^{4n-1} and the induced connection ∇° satisfy the condition

$$\begin{aligned}&g(\nabla^{\circ}_{X}\underset{\alpha}{F}Y,Z) + g(\underset{\alpha}{F}Y,\nabla^{\circ}_{X}Z) + g(\nabla^{\circ}_{Y}\underset{\alpha}{F}Z,X)+\\ &+g(\underset{\alpha}{F}Z,\nabla^{\circ}_{Y}X) + g(\nabla^{\circ}_{Z}\underset{\alpha}{F}X,Y) + g(\underset{\alpha}{F}X,\nabla^{\circ}_{Z}Y) = 0\,.\end{aligned}$$

The above condition is equivalent to the condition

$$\nabla_{X}(g(\underset{\alpha}{F}Y,Z)) + \nabla_{Y}(g(\underset{\alpha}{F}Z,X)) + \nabla_{Z}(g(\underset{\alpha}{F}X,Y)) = 0$$

or

$$\nabla_{X}(\underset{\alpha}{\Phi}(Y,Z)) + \nabla_{Y}(\underset{\alpha}{\Phi}(Z,X)) + \nabla_{Z}(\underset{\alpha}{\Phi}(X,Y)) = 0\,,$$

where

(6.3) $$\underset{\alpha}{\Phi}(X,Y) = g(\underset{\alpha}{F}X,Y)\,, \qquad X,Y \in TM^{4n-1}\,.$$

Thus we have

Theorem 6. *The almost Sasaki 3–structure $\{\underset{\alpha}{F}\}$ on Riemannian manifold M^{4n} induces the almost Sasaki 3–structure $\{\underset{\alpha}{F}\}$ on an oriented hypersurface of M^{4n-1} with the induced metric and the induced connection.*

Let us consider the tensor field $\underset{\alpha}{\Phi}$ on hypersurface M^{4n-1} defined by (6.3). The formula (theorem 3, [1]) :

$$g(\underset{\alpha}{F}X, \underset{\alpha}{F}Y) = g(X,Y) - \underset{\alpha}{\omega}(X) \cdot \underset{\alpha}{\omega}(Y)$$

and the relation (2.5) imply

$$\begin{aligned} g(\underset{\alpha}{F}X,Y) &= g(\underset{\alpha}{F}^2X, \underset{\alpha}{F}Y) + (\underset{\alpha}{\omega} \circ \underset{\alpha}{F})(X)\underset{\alpha}{\omega}(Y) = \\ &= \underset{\alpha}{\varepsilon}\, g(X, \underset{\alpha}{F}Y) - \underset{\alpha}{\varepsilon}\, \underset{\alpha}{\omega}(X)(\underset{\alpha}{\omega} \circ \underset{\alpha}{F})(Y) + (\underset{\alpha}{\omega} \circ \underset{\alpha}{F})(X)\underset{\alpha}{\omega}(Y) = \\ &= \underset{\alpha}{\varepsilon}\, g(X, \underset{\alpha}{F}Y) + \underset{\alpha}{\lambda}\, \underset{\alpha}{\omega}(X)\underset{\alpha}{\omega}(Y) - \underset{\alpha}{\varepsilon}\, \underset{\alpha}{\lambda}\, \underset{\alpha}{\omega}(X)\underset{\alpha}{\omega}(Y)\,. \end{aligned}$$

Hence we get

$$\underset{\alpha}{\Phi}(X,Y) = \underset{\alpha}{\varepsilon}\, \underset{\alpha}{\Phi}(Y,X) + (1 - \underset{\alpha}{\varepsilon})\underset{\alpha}{\lambda}\, \underset{\alpha}{\omega}(X)\underset{\alpha}{\omega}(Y)\,.$$

If $\underset{\alpha}{\varepsilon} = 1$, then $\underset{\alpha}{\Phi}$ are symmetric

$$\underset{\alpha}{\Phi}(X,Y) = \underset{\alpha}{\Phi}(Y,X)\,.$$

If $\underset{\alpha}{\varepsilon} = -1$, then we have

$$\underset{\alpha}{\Phi}(X,Y) + \underset{\alpha}{\Phi}(Y,X) = 2\, \underset{\alpha}{\lambda}\, \underset{\alpha}{\omega}(X)\underset{\alpha}{\omega}(Y)$$

Calculating covariant derivatives from (6.1) with respect to $\{\underset{\alpha}{F}\}$–connection $\overset{\bar{0}}{\nabla}$ (p. 60) we get

$$\begin{aligned} &(\overset{\bar{0}}{\nabla}_{\bar{Z}} \underset{\alpha}{\bar{\Phi}})(\bar{X},\bar{Y}) + \underset{\alpha}{\bar{\Phi}}(\overset{\bar{0}}{\nabla}_{\bar{Z}}\bar{X},\bar{Y}) + \underset{\alpha}{\bar{\Phi}}(\bar{X},\overset{\bar{0}}{\nabla}_{\bar{Z}}\bar{Y}) = \\ &= (\overset{\bar{0}}{\nabla}_{\bar{Z}}\bar{g})(\underset{\alpha}{\bar{F}}\bar{X},\bar{Y}) + g(\underset{\alpha}{\bar{F}}\overset{\bar{0}}{\nabla}_{\bar{Z}}\bar{X},\bar{Y}) + \bar{g}(\underset{\alpha}{\bar{F}}\bar{X},\overset{\bar{0}}{\nabla}_{\bar{Z}}\bar{Y}) \end{aligned}$$

for arbitrary $\bar{X},\bar{Y},\bar{Z} \in TM^{4n}$.
The above relation and the formula (6.1) yield

$$\begin{aligned} (\overset{\bar{0}}{\nabla}_{\bar{Z}} \underset{\alpha}{\bar{\Phi}})(\bar{X},\bar{Y}) = &- \bar{g}(\underset{\alpha}{\bar{F}}(\overset{\bar{0}}{\nabla}_{\bar{Z}}\bar{X}),\bar{Y}) - \bar{g}(\underset{\alpha}{\bar{F}}\bar{X},\overset{\bar{0}}{\nabla}_{\bar{Z}}\bar{Y}) + (\overset{\bar{0}}{\nabla}_{\bar{Z}}\bar{g})(\underset{\alpha}{\bar{F}}\bar{X},\bar{Y}) + \\ &+ \bar{g}(\underset{\alpha}{\bar{F}}(\overset{\bar{0}}{\nabla}_{\bar{Z}}\bar{X}),\bar{Y}) - \bar{g}(\underset{\alpha}{\bar{F}}\bar{X},\overset{\bar{0}}{\nabla}_{\bar{Z}}\bar{Y}) \end{aligned}$$

and we have

$$(\overset{\bar{0}}{\nabla}_{\bar{Z}} \underset{\alpha}{\bar{\Phi}})(\bar{X},\bar{Y}) = (\overset{\bar{0}}{\nabla}_{\bar{Z}}\bar{g})(\underset{\alpha}{\bar{F}}\bar{X},\bar{Y})\,.$$

Thus we have proved the following theorem:

Theorem 7. *The tensor field* $\underset{\alpha}{\bar{\Phi}}$ *on Riemannian manifold* M^{4n} *with a metric* $\bar{g}$ *given by the formula*

$$\underset{\alpha}{\bar{\Phi}}(\bar{X},\bar{Y}) = \bar{g}(\underset{\alpha}{\bar{F}}\bar{X},\bar{Y})\,, \qquad \bar{X},\bar{Y} \in TM^{4n}$$

satisfies the relation

$$(\overset{\bar{0}}{\nabla}_{\bar{Z}}\underset{\alpha}{\bar{\Phi}})(\bar{X},\bar{Y}) = (\overset{\bar{0}}{\nabla}_{\bar{Z}}\bar{g})(\underset{\alpha}{\bar{F}}\bar{X},\bar{Y})\,,$$

where $\overset{\bar{0}}{\nabla}$ *is a covariant differentiation with respect to* $\{\underset{\alpha}{\bar{F}}\}$*-connection* (p.60).

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STRESZCZENIE

Niech M^{4n} będzie $4n$-wymiarową rozmaitością Riemanna z zadaną na niej 3-struktura $\{\underset{\alpha}{F}\}$, $\alpha=1,2,3$ spełniającą określone warunki. W pracy tej zostały podane wszystkie zależności między tensorami Nijenhuisa danej 3-struktury oraz wzór na koneksję liniową zerującą tensory $\underset{\alpha}{F}$, $\alpha=1,2,3$ (twierdzenie 1 o istnieniu takiej koneksji). Uzyskane wyniki dla 3-struktury $\{\underset{\alpha}{F}\}$ na M^{4n} zostały zastosowane dla koneksji indukowanej na hiperpowierzchni zanurzonej w M^{4n}, na której określona jest odpowiednia 3-struktura $\{\underset{\alpha}{F},\underset{\alpha}{\omega},\underset{\alpha}{\eta}\}$ generowana przez $\{\underset{\alpha}{\bar{F}}\}$. W dalszej części pracy otrzymane wcześniej zależności zostały przeniesione na pewne specjalne rozmaitości i zanurzone w nich hiperpowierzchnie.

SUMMARY

Let M^{4n} be a $4n$-dimensional Riemannian manifold with a given 3-structure $\{\underset{\alpha}{\bar{F}}\}$, $\alpha=1,2,3$ subject to some conditions. In this paper the relations between Nijenhuis tensor of the given 3-structure and a formula for the linear connection annihilating tensors $\underset{\alpha}{\bar{F}}$, $\alpha=1,2,3$ are given. The existence of such connection is established in Thm 1. The results obtained for the 3-structure $\{\underset{\alpha}{\bar{F}}\}$ on M^{4n} are applied to an induced connection on a hypersurface immersed in M^{4n} where a suitable 3-structure induced by $\{\underset{\alpha}{\bar{F}}\}$ can be defined. These results are applied to some special manifolds and immersed hypersurfaces.