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Moment Inequalities for Order and Record Statistics Under Restrictions on their Distributions

Abstract. Several inequalities for moments of order and record statistics are given under moment and symmetry restrictions on their distribution.

1. Introduction. Let $X_{1,n} \leq \ldots \leq X_{n,n}$ be the order statistics from i.i.d. random variables $X_1, \ldots, X_n$ with df F. Let $r, s, k, n \in N$ be such that $k \leq n$ and $r \leq s$.

In section 2 we present inequalities for the α-th moment of the order statistics $X_{k,n}$ under the condition $EX_{r,s} = 0$.In particular we prove the inequality

$$EX_{2r+1,2r+1} \leq \frac{r-1}{r} E\left[X^2_{2r+1,2r+1}\right]^{1/2},$$

which improves the classical inequality for moments.

In Section 3 we give analogous inequalities for k-th record statistics.

The bounds for moments of order and record statistics given here are more precise than their counterparts obtained by Lin (1988), Kamps (1990) and Gajek and Gather (1991).

The discussion of attainability of the bounds yields in special case new characterizations of the inverse gamma and other distributions. Since the discrete distributions are admitted in discussion, the results of the paper are applicable to the moments from a number of samples as well. Throughout the paper we assume that at least one side of considered inequalities is finite.

2. Inequalities for order statistics. The following inequality is an improvement of the classical moment inequality. Since we have not found it in the literature, a short proof is enclosed.

Lemma 1. *For every function $g : \mathbb{R} \to \mathbb{R}$ such that $\int_0^1 g^2(t)\,dt < \infty$, it holds*

$$\int_0^1 g^2(t)\,dt - \left(\int_0^1 g(t)\,dt\right)^2 \geq \int_0^1 \left[\frac{g(t) - g(1-t)}{2}\right]^2 dt\,. \tag{1}$$

Remark 1. Since $\int_0^1 (g(t) - g(1-t))^2\,dt = 2\int_0^1 g^2(t)\,dt - 2\int_0^1 g(t)g(1-t)\,dt$,

(1) is equivalent to the following inequality

$$\left(\int_0^1 g(t)\,dt\right)^2 - \int_0^1 g(t)g(1-t)\,dt \le \int_0^1 g^2(t)\,dt - \left(\int_0^1 g(t)\,dt\right)^2 . \tag{2}$$

Proof. We shall prove (2). Applying the Cauchy-Schwartz inequality, we have

$$\int_0^1 \left[\frac{g(t)+g(1-t)}{2}\right]^2 dt \ge \left[\int_0^1 \frac{g(t)+g(1-t)}{2}\,dt\right]^2 = \left[\int_0^1 g(t)\,dt\right]^2 . \tag{3}$$

On the other hand we have

$$\int_0^1 \left[\frac{g(t)+g(1-t)}{2}\right]^2 dt = \frac{1}{2}\int_0^1 g^2(t)\,dt + \frac{1}{2}\int_0^1 g(t)g(1-t)\,dt . \tag{4}$$

From (3) and (4), it holds

$$\int_0^1 g^2(t)\,dt + \int_0^1 g(t)g(1-t)\,dt \ge 2\left[\int_0^1 g(t)\,dt\right]^2$$

which is equivalent to (2). ■

Theorem 1. *Let* $r,s,k,n \in \mathbb{N}$ *be such that* $r \le k,\ s-r \le n-k$ *and let* $\alpha \ge 1$. *Then it holds*

$$\begin{aligned}\frac{2(EX^{\alpha-1}_{k-r+1,n-s+1})^2}{(k-r+1)^2\binom{n-s+1}{k-r+1}^2} &\le \frac{EX^{2\alpha-2}_{2k-2r+1,2n-2s+1}}{(2k-2r+1)\binom{2n-2s+1}{2k-2r+1}} \\ &+ \frac{E[F^{-1}(U_{n-s+1,2n-2s+1})F^{-1}(1-U_{n-s+1,2n-2s+1})]^{\alpha-1}}{(n-s+1)\binom{2n-2s+1}{n-s+1}},\end{aligned} \tag{5}$$

where $U_{i,j}$ *denotes the i-th order statistics from the sample* $U_1,\dots,U_j$ *of independent uniformly distributed random variables.*

Proof. From David (1981), p.47, we have

$$EX^{\alpha}_{k,n} = k\binom{n}{k}\int_0^1 [F^{-1}(t)]^{\alpha} t^{k-1}(1-t)^{n-k}\,dt,$$

where $F^{-1}(t) = \inf\{x : F(x) \ge t\}$, $k,n \in \mathbb{N}$, $k \le n$. Let us write the inequality (5) in the following equivalent form

$$\begin{aligned}&\left[\int_0^1 [F^{-1}(t)]^{\alpha-1} t^{k-r}(1-t)^{n-s-k+r}\,dt\right]^2 - \int_0^1 [F^{-1}(t)F^{-1}(1-t)]^{\alpha-1} t^{n-s}(1-t)^{n-s}\,dt \\ &\le \int_0^1 [F^{-1}(t)]^{2\alpha-2} t^{2k-2r}(1-t)^{2n-2s-2k+2r}\,dt - \left[\int_0^1 [F^{-1}(t)]^{\alpha-1} t^{k-r}(1-t)^{n-s-k+r}\,dt\right]^2\end{aligned}$$

After denoting

$$g(t) = [F^{-1}(t)]^{\alpha-1} t^{k-r}(1-t)^{n-s-k+r},$$

we get the above inequality from (2) . ■

Theorem 2. *Let $r, s, k, n \in \mathbb{N}$ be such that $r \le k$, $n-k \le s-r$ and suppose $\alpha \ge 1$. If $EX_{r,s} = 0$, then*

$$\frac{EX^{\alpha}_{k,n}}{k\binom{n}{k}} \le \left\{ \frac{EX^2_{2r-1,2s-1}}{(2r-1)\binom{2s-1}{2r-1}} \right\}^{1/2} \left\{ \frac{EX^{2\alpha-2}_{2k-2r+1,2n-2s+1}}{(2k-2r+1)\binom{2n-2s+1}{2k-2r+1}} \right.$$

$$\left. - \frac{\left[EX^{\alpha-1}_{k-r+1,n-s+1}\right]^2}{(k-r+1)^2\binom{n-s+1}{k-r+1}^2} \right\}^{1/2} . \tag{6}$$

Proof. Since $EX_{r,s} = 0$, for every $\gamma \in \mathbb{R}$

$$EX^{\alpha}_{k,n} = EX^{\alpha}_{k,n} - \gamma EX_{r,s}$$

$$= \int_0^1 F^{-1}(t)t^{r-1}(1-t)^{s-r} \left\{ k\binom{n}{k} [F^{-1}(t)]^{\alpha-1} t^{k-r}(1-t)^{n-k-r+s} - \gamma r \binom{s}{r} \right\} dt.$$

Applying now the Cauchy-Schwartz inequality, we get

$$EX^{\alpha}_{k,n} \le \left\{ \frac{EX^2_{2r-1,2s-1}}{(2r-1)\binom{2s-1}{2r-1}} \right\}^{1/2} \left\{ \frac{k^2\binom{n}{k}^2 EX^{2\alpha-2}_{2k-2r+1,2n-2s+1}}{(2k-2r+1)\binom{2n-2s+1}{2k-2r+1}} \right.$$

$$\left. + \gamma^2 r^2 \binom{s}{r}^2 - 2\gamma \frac{k\binom{n}{k} r\binom{s}{r}}{(k-r+1)\binom{n-s+1}{k-r+1}} EX^{\alpha-1}_{k-r+1,n-s+1} \right\}^{1/2} . \tag{7}$$

Let us denote

$$\varphi(\gamma) = \frac{k^2\binom{n}{k}^2 EX^{2\alpha-2}_{2k-2r+1,2n-2s+1}}{(2k-2r+1)\binom{2n-2s+1}{2k-2r+1}} + \gamma^2 r^2 \binom{s}{r}^2$$

$$-2\gamma \frac{k\binom{n}{k} r\binom{s}{r}}{(k-r+1)\binom{n-s+1}{k-r+1}} EX^{\alpha-1}_{k-r+1,n-s+1}.$$

The function $\varphi(\gamma)$ attains its minimum for

$$\gamma_0 = \frac{k\binom{n}{k} EX^{\alpha-1}_{k-r+1,n-s+1}}{(k-r+1)\binom{n-s+1}{k-r+1} r\binom{s}{r}} \tag{8}$$

which is equal to

$$\varphi(\gamma_0) = \frac{k^2\binom{n}{k}^2 EX^{2\alpha-2}_{2k-2r+1,2n-2s+1}}{(2k-2r+1)\binom{2n-2s+1}{2k-2r+1}} - \frac{k^2\binom{n}{k}^2 \left[EX^{\alpha-1}_{k-r+1,n-s+1}\right]^2}{(k-r+1)^2\binom{n-s+1}{k-r+1}^2}. \tag{9}$$

From (7) and (9) the result follows. ■

The assumption $EX_{r,s} = 0$ can be replaced by $EX_{r,s} = c$. To this end one should shift the sample $X_1, \ldots, X_n$ by $-c$ and next apply Theorem 2. An analogous remark concerns other results.

Let us compare inequality (6) with the bound given in Theorem 1 of [4] in the case where $p_1 = p_2 = 2, \alpha_1 = 1, \alpha_2 = \alpha - 1, k_1 = 2r - 1, k_2 = 2k - 2r + 1, n_1 = 2s - 1$ and $n_2 = 2n - 2s + 1$. Then we get from Theorem 1 of [4] the following inequality

$$\frac{EX^{\alpha}_{k,n}}{k\binom{n}{k}} \le \left\{\frac{EX^2_{2r-1,2s-1}}{(2r-1)\binom{2s-1}{2r-1}}\right\}^{1/2} \left\{\frac{EX^{2\alpha-2}_{2k-2r+1,2n-2s+1}}{(2k-2r+1)\binom{2n-2s+1}{2k-2r+1}}\right\}^{1/2},$$

which shows that the bound given in (6) is more precise due to the assumption $EX_{r,s} = 0$.

Remark 2. The equality sign in (6) is attained iff for some $c \in \mathbb{R}$ the following condition holds

$$F^{-1}(t)t^{r-1}(1-t)^{s-r} = ck\binom{n}{k}[F^{-1}(t)]^{\alpha-1}t^{k-r}(1-t)^{n-s-k+r} - cr\binom{s}{r}\gamma_0\,, \tag{10}$$

where γ_0 is given by (8).

From the Cauchy-Schwartz inequality, we have

$$EX_{k,n} \le \left[EX^2_{k,n}\right]^{1/2} \tag{11}$$

for any distribution function F . If we restrict the class of underlying distributions to the nondegenerate ones for which $EX_{r,s} = 0$, then (11) can be improved.

Corollary 1. *Let $r > 1$ and $EX_{r,r} = 0$. Then*

$$EX_{2r-1,2r-1} \le \left[EX^2_{2r-1,2r-1}\right]^{1/2} \frac{r-1}{r}\,. \tag{12}$$

The equality sign in (12) is attained iff

$$F(x) = \begin{cases} [r(1 - x/c(2r-1))]^{-1/(r-1)} & , \textit{for } x \in (-\infty, c(2r-1)(r-1)/r) \\ 1 & , \textit{otherwise}\,, \end{cases}$$

where c is a positive constant.

Proof. Applying Theorem 2 for $\alpha = 1$, $s = r$, $n = 2r - 1$, $k = 2r - 1$ we get (12).Equation (10) in this case takes the following form

$$F^{-1}(t)t^{r-1} = c(2r-1)t^{r-1} - c(2r-1)/r.$$

Solving this with respect to t and denoting $F(x) = t$ we get

$$F(x) = \left[r\left(1 - x/c(2r-1)\right)\right]^{1/(1-r)}$$

for $x \in (-\infty, c(2r-1)(r-1)/r)$, where $c > 0$. ■

The next result gives an analogous bound for the first order statistics.

Corollary 2. *For every $s > 1$,*

$$EX_{1,2s-1} \le \left[EX^2_{1,2s-1}\right]^{1/2} \frac{s-1}{s} \tag{13}$$

provided $EX_{1,s} = 0$. The equality sign in (13) is attained iff

$$F(x) = \begin{cases} 1 - [s(1 - x/c(2s-1))]^{1/(1-s)} & , \text{ for } x \in (c(2s-1)(s-1)/s, \infty) \\ 1 & , \text{ otherwise }, \end{cases}$$

where c is a negative constant.

Proof. Put $\alpha = k = r = 1$ and $n = 2s-1$ in Theorem 2 and use (10). ■

Finally, we derive the result of Hartley and David (1954).

Corollary 3. *For any $n \in \mathbb{N}$*

$$EX_{n,n} \le \frac{n-1}{\sqrt{2n-1}}$$

provided $EX = 0$ and $EX^2 = 1$ with equality iff

$$F(x) = \left[\frac{x+c}{cn}\right]^{1/(n-1)} , x \in (-c, c(n-1))$$

where $c = \sqrt{2n-1}/(n-1)$.

Proof. Put $k = n, r = s = 1$ and $\alpha = 1$ in Theorem 2 and use (10). ■

Theorem 3. *Let $r, s, k, n \in \mathbb{N}$ be such that $r \le k$, $s - r \le n - k$ and suppose $\alpha \ge 1$. Assume that the following symmetry condition holds:*

$$F^{-1}(t)t^{r-1}(1-t)^{s-r} = -F^{-1}(1-t)(1-t)^{r-1}t^{s-r} . \tag{14}$$

Then

$$\begin{aligned} \frac{EX^\alpha_{k,n}}{k\binom{n}{k}} \le & \left\{\frac{EX^2_{2r-1,2s-1}}{(2r-1)\binom{2s-1}{2r-1}}\right\}^{1/2} \left\{\frac{1}{2}\frac{EX^{2\alpha-2}_{2k-2r+1,2n-2s+1}}{(2k-2r+1)\binom{2n-2s+1}{2k-2r+1}}\right. \\ & \left. - \frac{1}{2}\frac{E[F^{-1}(U_{n-s+1,2n-2s+1})F^{-1}(1-U_{n-s+1,2n-2s+1})]^{\alpha-1}}{(n-s+1)\binom{2n-2s+1}{n-s+1}}\right\}^{1/2} \end{aligned} \tag{15}$$

Proof. Since

$$EX^{\alpha}_{k,n} = \int_{\frac{1}{2}}^{1} F^{-1}(1-t)(1-t)^{r-1}t^{s-r}k\binom{n}{k}[F^{-1}(1-t)]^{\alpha-1}(1-t)^{k-r}t^{n-k-s+r}\,dt$$

$$+\int_{\frac{1}{2}}^{1} F^{-1}(t)t^{r-1}(1-t)^{s-r}k\binom{n}{k}[F^{-1}(t)]^{\alpha-1}t^{k-r}(1-t)^{n-k-s+r}\,dt\,,$$

using (14), we get

$$EX^{\alpha}_{k,n} = \int_{\frac{1}{2}}^{1} F^{-1}(t)t^{r-1}(1-t)^{s-r}k\binom{n}{k}\Big\{[F^{-1}(t)]^{\alpha-1}t^{k-r}(1-t)^{n-k-s+r}$$
$$-[F^{-1}(1-t)]^{\alpha-1}(1-t)^{k-r}t^{n-k-s+r}\Big\}\,dt.$$

Now applying (14) and the Cauchy-Schwartz inequality, we get

$$EX^{\alpha}_{k,n} \le k\binom{n}{k}\left\{\frac{1}{2}\int_0^1 [F^{-1}(t)]^2t^{2r-2}(1-t)^{2s-2r}\,dt\right\}^{1/2}$$
$$\times\left\{\int_0^1 [F^{-1}(t)]^{2\alpha-2}t^{2k-2r}(1-t)^{2n-2k-2s+2r}\,dt\right.$$
$$\left.-\int_0^1 [F^{-1}(1-t)]^{\alpha-1}[F^{-1}(t)]^{\alpha-1}t^{n-s}(1-t)^{n-s}\,dt\right\}^{1/2},$$

which is equivalent to (15). ■

It is easy to see that (14) implies $EX_{r,s}=0$ so one can exepct that (15) is more precise than (6).

Let us write (15) in the following form

$$\frac{EX^{\alpha}_{k,n}}{k\binom{n}{k}} \le \left\{\frac{EX^2_{2r-1,2s-1}}{(2r-1)\binom{2s-1}{2r-1}}\right\}^{1/2}$$

$$(15')\qquad \times\left\{\frac{EX^{2\alpha-2}_{2k-2r+1,2n-2s+1}}{(2k-2r+1)\binom{2n-2s+1}{2k-2r+1}} - \frac{1}{2}\left(\frac{EX^{2\alpha-2}_{2k-2r+1,2n-2s+1}}{(2k-2r+1)\binom{2n-2s+1}{2k-2r+1}}\right.\right.$$
$$\left.\left.+\frac{E[F^{-1}(U_{n-s+1,2n-2s+1})F^{-1}(1-U_{n-s+1,2n-2s+1})]^{\alpha-1}}{(n-s+1)\binom{2n-2s+1}{n-s+1}}\right)\right\}^{1/2},$$

and compare it to the bound given in Theorem 2 of this paper. The bound (15′) is more precise than (6) because the following inequality holds (see Theorem 1)

$$\frac{1}{2}\left(\frac{EX^{2\alpha-2}_{2k-2r+1,2n-2s+1}}{(2k-2r+1)\binom{2n-2s+1}{2k-2r+1}}\right.$$
$$\left.+\frac{E[F^{-1}(U_{n-s+1,2n-2s+1})F^{-1}(1-U_{n-s+1,2n-2s+1})]^{\alpha-1}}{(n-s+1)\binom{2n-2s+1}{n-s+1}}\right)$$
$$\ge\frac{(EX^{\alpha-1}_{k-r+1,n-s+1})^2}{(k-r+1)^2\binom{n-s+1}{k-r+1}^2}.$$

Remark 3. If $s = 2r - 1$, then (14) is satisfied when $F^{-1}(t) = -F^{-1}(1-t)$, i.e. the parent distribution is symmetric about 0. The bound which corresponds to this case one can easy get from (15).

3. Inequalities for record statistics. Let $X_1, X_2, \dots$ be i.i.d. random variables. The k-th record statistics $Y_n^{(k)}$ from the sequence $X_1, X_2, \dots$ are defined by

$$Y_n^{(k)} = X_{L_k(n), L_k(n)+k-1}\ , \quad n = 0, 1, 2, \dots\ , \quad k \geq 1\ ,$$

where $L_k(0) = 1$, $L_k(n+1) = \min\{j : X_{L_k(n), L_k(n)+k-1} < X_{j, j+k-1}\}$ for $n = 0, 1, 2, \dots$ (c.f.Dziubdziela and Kopociński (1976)).

Properties of the k-th record statistics were investigated by Resnick (1973), Dziubdziela and Kopociński (1976), Grudzień (1979), Grudzień and Szynal (1983), Gajek (1985) and others. Some characterizations of the geometric, exponential and other distribution can be found in Srivastava (1978, 1979), Nagaraja (1978), Grudzień and Szynal (1983) and Gajek and Gather (1991).

The following formula for the α-th moment of the k-th record statistic was proven by Grudzień and Szynal (1983)

$$E\left(Y_n^{(k)}\right)^\alpha = \frac{k^{n+1}}{n!} \int_0^1 [F^{-1}(t)]^\alpha [-\log(1-t)]^n (1-t)^{k-1}\, dt\ .$$

Theorem 4. *Let $r, s, k, n \in \mathbb{N}$ be such that $r \leq k$ and $s \leq n$. Suppose $EY_s^{(r)} = 0$ and $\alpha \geq 1$. Then*

$$\begin{aligned} \frac{E\left(Y_n^{(k)}\right)^\alpha}{g(k,n)} \leq & \left\{ \frac{E\left(Y_{2s}^{(2r-1)}\right)^2}{g(2r-1, 2s)} \right\}^{1/2} \\ & \times \left\{ \frac{E\left(Y_{2n-2s}^{(2k-2r+1)}\right)^{2\alpha-2}}{g(2k-2r+1, 2n-2s)} - \left[\frac{E\left(Y_{n-s}^{(k-r+1)}\right)^{\alpha-1}}{g(k-r+1, n-s)} \right]^2 \right\}^{1/2}. \end{aligned} \tag{16}$$

where $g(k,n) = k^{n+1}/n!$.

Proof. Since $EY_s^{(r)} = 0$, for every $\gamma \in \mathbb{R}$

$$\begin{aligned} E\left(Y_n^{(k)}\right)^\alpha &= E\left(Y_n^{(k)}\right)^\alpha - \gamma EY_s^{(r)} = \int_0^1 F^{-1}(t)[-\log(1-t)]^s (1-t)^{r-1} \\ & \times \left\{ g(k,n)[F^{-1}(t)]^{\alpha-1}[-\log(1-t)]^{n-s}(1-t)^{k-r} - \gamma g(r,s) \right\} dt\ . \end{aligned}$$

Applying the Cauchy–Schwartz inequality and minimizing with respect to γ gives the result. ■

Remark 4. In (16) the equality holds iff

$$F^{-1}(t)[-\log(1-t)]^{s}(1-t)^{r-1}$$
$$= c\left\{g(k,n)[F^{-1}(t)]^{\alpha-1}[-\log(1-t)]^{n-s}(1-t)^{k-r} - \gamma_0 g(r,s)\right\} ,$$

for some $c \in \mathbb{R}$ and all $t \in (0,1)$, where

$$\gamma_0 = \frac{g(k,n)E(Y_{n-s}^{(k-r+1)})^{\alpha-1}}{g(r,s)g(k-r+1,n-s)} .$$

One gets from Theorem 4 an improvement of the moment inequality for record values.

Corollary 4. *Assume* $EY_s^{(1)} = 0$. *Then*

$$EY_{2s}^{(1)} \le \left[E\left(Y_{2s}^{(1)}\right)^2\right]^{1/2}\left[1-\frac{(s!)^2}{(2s)!}\right]^{1/2} .$$

The equality holds iff F *is the following inverse gamma distribution*

$$F(x) = 1-\exp\left[-\left(\frac{cs!}{c-(2s)!x}\right)^{1/s}\right],\ x \in (-\infty, c/(2s)!) ,$$

for any $c > 0$.

Proof. Put $k = r = \alpha = 1$ and $n = 2s$ in Theorem 4. The characterization follows from Remark 2. ■

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