ANNALES
UNIVERSITATIS MARIAE CURIE - SKŁODOWSKA
LUBLIN - POLONIA

VOL. LI.2, 9 SECTIO A 1997

JAROSŁAW GÓRNICKI

On Some Generalization of Lipschitzian Mappings in a Hilbert Space

ABSTRACT. In this paper, we present some generalization of Lipschitzian mappings in a Hilbert space. The existence of fixed points for this type of mappings is shown.

1. Introduction. The fixed point theory of nonlinear mappings is an important branch of nonlinear functional analysis which depends heavily upon the geometrical properties of the underlying space E.

Let E be a Banach space and C a weakly compact convex subset of E. A mapping $T : C \to C$ is called *nonexpansive* if

$$\|Tx - Ty\| \leq \|x - y\|$$

for every x, y in C. When E has some "nice" geometric properties, for instance E is uniformly convex, then mapping T has a fixed point [7], [1].

A more general concept is the notion of uniformly Lipschitz mapping introduced by Goebel and Kirk [3]. We recall, a mapping $T : C \to C$ is called *uniformly k-lipschitzian* on C if

$$\|T^n x - T^n y\| \leq k\|x - y\|$$

1991 *Mathematics Subject Classification.* 47H09, 47H10.
Key words and phrases. Lipschitzian mapping, fixed point, Hilbert space.

for every x, y in C and $n = 1, 2, \ldots$, or in the other words, if the *Lipschitz constant* of T^n,

$$\sup\left\{\frac{\|T^n x - T^n y\|}{\|x - y\|} : x \neq y, x, y \in C\right\} \leq k,$$

for $n = 1, 2, \ldots$.

It is clear that nonexpansive mappings are uniformly 1-Lipschitzian mappings. Again, when E has some "nice" geometric properties and $k > 1$ is not too large, we can assure that T has a fixed point [3], [1]. For example, Lifshitz [9] proved that in a Hilbert space a uniformly k-Lipschitzian mapping with $k < \sqrt{2}$ has a fixed point and found an example of a fixed point free uniformly $\frac{\pi}{2}$-Lipschitzian mapping which leaves invariant a bounded closed convex subset of l^2 (cf. Baillon [2]). The validity of Lifshitz's Theorem for $\sqrt{2} \leq k < \frac{\pi}{2}$ remains open.

In 1988, Górnicki and Krüppel [5], [6] indicated some applications of an asymptotic density in fixed point theorems for Lipschitzian mappings and proved, among others, that in a Hilbert space a Lipschitzian mapping $T : C \to C$ with

$$\varliminf_{n\to\infty} \inf_{m=0,1,2,\ldots} \frac{1}{n}\sum_{i=1}^{n} \|T^{i+m}\|^2 < 2$$

has a fixed point [3]. This result generalizes Lifshitz's Theorem and shows that the theorem admits certain perturbations in the behavior of the norm of successive iterations in infinite but "small" sets. In this paper we continue this study and we prove a fixed point theorem for mappings with generalized Lipschitzian iterates.

2. Preliminaries and lemmas. In the present paper we consider in a Hilbert space H a class of mappings (not necessarily continuous) $T : C \to C$ (C is nonempty subset of H) whose n-th iterate satisfying the following condition:

$$(*)\qquad \begin{aligned} \|T^n x - T^n y\|^2 \leq a_n\|x - y\|^2 &+ b_n\{\|x - T^n x\|^2 + \|y - T^n y\|^2\} \\ &+ c_n\{\|x - T^n y\|^2 + \|y - T^n x\|^2\} \end{aligned}$$

for all x, y in C, $n = 1, 2, \ldots$, where a_n, b_n, c_n are nonnegative constants satisfying the following inequality,

$$(**)\qquad b_n + 2c_n < 1/3, \quad n = 1, 2, \ldots .$$

Note, in case $b_n = c_n = 0$, $n = 1, 2, \ldots$, T is a mapping with Lipschitzian iterates.

In the present paper we discuss the worst-case scenario, when $a_n b_n c_n > 0$ for $n = 1, 2, \ldots$.

Lemma 1. *If $T : C \to C$ is a mapping satisfying the condition $(*)$ with $2b_n + 3c_n < 1$ for $n = 1, 2, \ldots$, then for $i > k$, we have:*

$$\|T^i x - T^k y\|^2 \le \frac{a_k + 2b_k + 3c_k}{1 - (2b_k + 3c_k)}\|y - T^{i-k}x\|^2 + \frac{4(b_k + c_k)}{1 - (2b_k + 3c_k)}\|y - T^i x\|^2.$$

Proof. For $i > k$, we have

$$\begin{aligned}\|T^i x - T^k y\|^2 =& \|T^k(T^{i-k}x) - T^k y\|^2\\ \le& a_k\|T^{i-k}x - y\|^2 + b_k\{\|T^{i-k}x - T^i x\|^2 + \|y - T^k y\|^2\}\\ &+ c_k\{\|T^{i-k}x - T^k y\|^2 + \|y - T^i x\|^2\} \text{ (by (*))}\\ \le& a_k\|T^{i-k}x - y\|^2 + b_k\{\|T^{i-k}x - y\| + \|y - T^i x\|\}^2\\ &+ b_k\{\|y - T^i x\| + \|T^i x - T^k y\|\}^2 + c_k\{\|T^{i-k}x - y\|\\ &+ \|y - T^i x\| + \|T^i x - T^k y\|\}^2 + c_k\|y - T^i x\|^2\\ \le& a_k\|T^{i-k}x - y\|^2 + 2b_k\|T^{i-k}x - y\|^2 + 4b_k\|y - T^i x\|^2\\ &+ 2b_k\|T^i x - T^k y\|^2 + 3c_k\|T^{i-k}x - y\|^2\\ &+ 4c_k\|y - T^i x\|^2 + 3c_k\|T^i x - T^k y\|^2 \text{ (by Lemma 1)},\end{aligned}$$

and hence $[1 - (2b_k + 3c_k)]\|T^i x - T^k y\|^2 \le (a_k + 2b_k + 3c_k)\|y - T^{i-k}x\|^2 + 4(b_k + c_k)\|y - T^i x\|^2$.

□

Now, we use techniques of asymptotic center in Banach spaces.

Lemma 2 [5]. *Let C be a nonempty closed convex and bounded subset of a Hilbert space H and let $\{x_i\} \subset H$ be a bounded sequence. We define the functional $r : C \to [0, +\infty)$ by the formula*

$$r(x) = \varlimsup_{n\to\infty} \frac{1}{n}\sum_{i=1}^{n} \|x - x_i\|^2, \quad x \in C.$$

Then $r(\cdot)$ is continuous and strictly convex.

Lemma 3 [7]. *Let C be a nonempty closed convex and bounded subset of a uniformly convex Banach space E and $f : C \to [0, +\infty)$ be continuous. If f is strictly convex, then f attains its minimum at exactly one point.*

And so, by Lemma 3, for the bounded sequence $\{x_i\}$ in a Hilbert space there exists a unique point $z \in C$ such that $r(z) = \inf\{r(x) : x \in C\}$.

The element z in C is said to be an *asymptotic center* of $\{x_i\}$ with respect to $r(\cdot)$ and C.

Now we establish some lemmas. Let C be a nonempty closed convex and bounded subset of a Hilbert space H and $T : C \to C$ satisfy conditions $(*)$ and $(**)$. We define the functional

$$d(u) = \overline{\lim_{n\to\infty}} \frac{1}{n} \sum_{i=1}^{n} \|u - T^i u\|^2, \quad x \in C,$$

and let z be the asymptotic center in C which minimalizes the functional

$$r(x) = \overline{\lim_{n\to\infty}} \frac{1}{n} \sum_{i=1}^{n} \|x - T^i u\|^2,$$

over x in C (for fixed $u \in C$). Then we have the following lemmas.

Lemma 4 [5]. $r(z) \leq d(u)$.

Lemma 5 [5]. $\|z - u\| \leq 2\sqrt{d(u)}$.

Lemma 6. *For each $k \in \mathbb{N}$ holds*

$$r(T^k z) \leq \frac{a_k + 6b_k + 7c_k}{1 - (2b_k + 3c_k)} r(z).$$

Proof. Fix $k \in \mathbb{N}$. Then by Lemma 1, we have:

$$\frac{1}{n}\sum_{i=1}^{n} \|T^k z - T^i u\|^2 = \frac{1}{n}\sum_{i=1}^{k} \|T^k z - T^i u\|^2 + \frac{1}{n}\sum_{i=k+1}^{n} \|T^k z - T^i u\|^2$$

$$\leq \frac{1}{n}\sum_{i=1}^{k} \|T^k z - T^i u\|^2$$

$$+ \frac{1}{n}\sum_{i=k+1}^{n} \left(\frac{a_k + 2b_k + 3c_k}{1 - (2b_k + 3c_k)} \|z - T^{i-k} u\|^2 + \frac{4(b_k + c_k)}{1 - (2b_k + 3c_k)} \|z - T^i u\|^2 \right)$$

$$= \frac{1}{n}\sum_{i=1}^{k} \|T^k z - T^i u\|^2$$

$$+ \frac{a_k + 2b_k + 3c_k}{1 - (2b_k + 3c_k)} \frac{1}{n} \sum_{i=1}^{n-k} \|z - T^i u\|^2 + \frac{4(b_k + c_k)}{1 - (2b_k + 3c_k)} \frac{1}{n} \sum_{i=k+1}^{n} \|z - T^i u\|^2$$

$$= \frac{1}{n}\sum_{i=1}^{n} \|T^k z - T^i u\|^2$$

$$+\frac{a_k+2b_k+3c_k}{1-(2b_k+3c_k)}\left(\frac{1}{n}\sum_{i=1}^{n}\|z-T^iu\|^2-\frac{1}{n}\sum_{i=n-k+1}^{n}\|z-T^iu\|^2\right)$$

$$+\frac{4(b_k+c_k)}{1-(2b_k+3c_k)}\left(\frac{1}{n}\sum_{i=1}^{n}\|z-T^iu\|^2-\frac{1}{n}\sum_{i=1}^{k}\|z-T^iu\|^2\right).$$

Taking the limit superior on each side as $n \to +\infty$, we get

$$\overline{\lim_{n\to\infty}}\,\frac{1}{n}\sum_{i=1}^{n}\|T^kz-T^iu\|^2 \le \frac{a_k+6b_k+7c_k}{1-(2b_k+3c_k)}\,\overline{\lim_{n\to\infty}}\,\frac{1}{n}\sum_{i=1}^{n}\|z-T^iu\|^2,$$

since

(a) $$\frac{1}{n}\sum_{i=1}^{k}\|T^kz-T^iu\|^2 \to 0, \quad \text{as } n \to +\infty,$$

(b) $$\frac{1}{n}\sum_{i=n-k+1}^{n}\|z-T^iu\|^2 \to 0, \quad \text{as } n \to +\infty,$$

(c) $$\frac{1}{n}\sum_{i=1}^{k}\|z-T^iu\|^2 \to 0, \quad \text{as } n \to +\infty.$$

Therefore, for any fixed $k \in \mathrm{N}$, we obtain the estimate

$$r(T^kz) \le \frac{a_k+6b_k+7c_k}{1-(2b_k+3c_k)}\,r(z).$$

□

Lemma 7 [5]. *$r(z)+\|z-x\|^2 \le r(x)$ for every $x \in C$.*

3. Fixed point Theorem. In this section we show an exsistence result for fixed point of mapping satisfying $(*)$ and $(**)$.

Theorem 1. *Let C be a nonempty closed convex and bounded subset of a Hilbert space H. If $T : C \to C$ satisfy conditions $(*)$ and $(**)$ with*

(1) $$\underline{\lim}_{n\to\infty}\ \inf_{m=0,1,2,\dots}\ \frac{1}{n}\sum_{i=1}^{n}A_{i+m} < 2,$$

where $A_s = \dfrac{a_s+6b_s+7c_s}{1-(2b_s+3c_s)}$, $s = 1,2,\dots,$

(2) $$\sup\{\frac{1+a_i+4c_i}{1-3(b_i+2c_i)} : i \ge 1\} < +\infty,$$

(3) $$\sup\{\frac{1+b_i+2c_i}{1-3(b_i+2c_i)} : i \geq 1\} < +\infty,$$

then T has a fixed point in C.

Proof. Let $\{n_j\}$ and $\{m_j\}$ be sequences of natural numbers such that

$$\varliminf_{n\to\infty} \inf_{m=0,1,2,\ldots} \frac{1}{n}\sum_{i=1}^{n} A_{i+m} = \lim_{j\to\infty} \frac{1}{n_j}\sum_{i=1}^{n_j} A_{i+m_j} = a < 2.$$

By induction, we define a sequence $\{x_n\}$ in C in the following manner:

$$\begin{cases} x_1 \in C, \text{ arbitrary}, \\ x_{m+1} = z(x_m),\ m = 1,2,\ldots, \end{cases}$$

where $z(x_m)$ is the unique point in C (asymptotic center) which minimalizes the functional

$$r(x) = \varlimsup_{j\to\infty} \frac{1}{n_j}\sum_{i=1}^{n_j} \|x - T^{i+m_j}x_m\|^2$$

over x in C.

First we prove the following inequality:

(α) $$d(z) \leq (a-1)d(u),$$

where

$$d(u) = \varlimsup_{j\to\infty} \frac{1}{n_j}\sum_{i=1}^{n_j} \|u - T^{i+m_j}u\|^2$$

and z is the asymptotic center in C which minimalizes the functional

$$r(x) = \varlimsup_{j\to\infty} \frac{1}{n_j}\sum_{i=1}^{n_j} \|x - T^{i+m_j}u\|^2$$

over x in C (for fixed $u \in C$).

We put in Lemma 7, $x = T^{i+m_j}z$. Then, by Lemma 6, we get

$$r(x) + \|z - T^{i+m_j}z\|^2 \leq r(T^{i+m_j}z) \leq A_{i+m_j}r(z),$$

and $\|z - T^{i+m_j}z\|^2 \leq (A_{i+m_j} - 1)r(z)$. Summing up these inequalities from $i = 1$ to n_j, dividing through by n_j and taking the limit superior as $j \to +\infty$ on each side, we get

$$d(z) \leq \left(\lim_{j\to\infty} \frac{1}{n_j}\sum_{i=1}^{n_j} A_{i+m_j} - 1\right) r(z) = (a-1)r(z) \leq (a-1)d(u),$$

by Lemma 4. Hence, by Lemma 5, we have

$$(\beta) \qquad \|x_{m+1} - x_m\| \le 2\sqrt{d(x_m)} \le 2\sqrt{(a-1)^{m-1} d(x_1)},$$

and $\|x_{m+1} - x_m\| \to 0$ as $m \to +\infty$. We see from (β) that $\{x_m\}$ is norm Cauchy and hence strongly convergent. Let $x_0 = \lim_{m\to\infty} x_m$. Then, we have

$$\begin{aligned}
&\|x_0 - T^{i+m_j} x_0\|^2 \\
&\le \left(\|x_0 - x_m\| + \|x_m - T^{i+m_j} x_m\| + \|T^{i+m_j} x_m - T^{i+m_j} x_0\|\right)^2 \\
&\le 3\|x_0 - x_m\|^2 + 3\|x_m - T^{i+m_j} x_m\|^2 + 3\|T^{i+m_j} x_m - T^{i+m_j} x_0\|^2 \\
&\le 3\|x_0 - x_m\|^2 + 3\|x_m - T^{i+m_j} x_m\|^2 + 3\left(a_{i+m_j}\|x_0 - x_m\|^2\right. \\
&+ b_{i+m_j}\left\{\|x_m - T^{i+m_j} x_m\|^2 + \|x_0 - T^{i+m_j} x_0\|^2\right\} \\
&\left.+ c_{i+m_j}\left\{\|x_m - T^{i+m_j} x_0\|^2 + \|x_0 - T^{i+m_j} x_m\|^2\right\}\right) \\
&\le 3(1 + a_{i+m_j})\|x_0 - x_m\|^2 + 3(1 + b_{i+m_j})\|x_m - T^{i+m_j} x_m\|^2 \\
&+ 3b_{i+m_j}\|x_0 - T^{i+m_j} x_0\|^2 + 3c_{i+m_j}\left\{\|x_m - x_0\| + \|x_0 - T^{i+m_j} x_0\|\right\}^2 \\
&+ 3c_{i+m_j}\left\{\|x_0 - x_m\| + \|x_m - T^{i+m_j} x_m\|\right\}^2 \\
&\le 3(1 + a_{i+m_j} + 4c_{i+m_j})\|x_0 - x_m\|^2 \\
&+ 3(1 + b_{i+m_j} + 2c_{i+m_j})\|x_m - T^{i+m_j} x_m\|^2 \\
&+ 3(b_{i+m_j} + 2c_{i+m_j})\|x_0 - T^{i+m_j} x_0\|^2,
\end{aligned}$$

and hence

$$\begin{aligned}
\|x_0 - T^{i+m_j} x_0\|^2 \le& \frac{3(1 + a_{i+m_j} + 4c_{i+m_j})}{1 - 3(b_{i+m_j} + 2c_{i+m_j})}\|x_0 - x_m\|^2 \\
&+ \frac{3(1 + b_{i+m_j} + 2c_{i+m_j})}{1 - 3(b_{i+m_j} + 2c_{i+m_j})}\|x_m - T^{i+m_j} x_m\|^2.
\end{aligned}$$

Summing up these inequalities from $i = 1$ to n_j, dividing through by n_j, we get

$$\begin{aligned}
\frac{1}{n_j}\sum_{i=1}^{n_j}\|x_0 - T^{i+m_j}\|^2 \le& \frac{1}{n_j}\sum_{i=1}^{n_j}\frac{3(1 + a_{i+m_j} + 4c_{i+m_j})}{1 - 3(b_{i+m_j} + 2c_{i+m_j})}\|x_0 - x_m\|^2 \\
&+ \frac{1}{n_j}\sum_{i=1}^{n_j}\frac{3(1 + b_{i+m_j} + 2c_{i+m_j})}{1 - 3(b_{i+m_j} + 2c_{i+m_j})}\|x_m - T^{i+m_j} x_m\|^2 \\
&\le 3\|x_0 - x_m\|^2 \max\left\{\frac{1 + a_i + 4c_i}{1 - 3(b_i + 2c_i)} : 1 \le i \le n_j\right\}
\end{aligned}$$

$$+3\left[\frac{1}{n_j}\sum_{i=1}^{n_j}\left(\frac{3(1+b_{i+m_j}+2c_{i+m_j})}{1-3(b_{i+m_j}+2c_{i+m_j})}\right)^2\right]^{1/2}$$

$$\times\left[\frac{1}{n_j}\sum_{i=1}^{n_j}\|x_m-T^{i+m_j}x_m\|^4\right]^{1/2} \quad \text{(by Lemma 10 and 9)}$$

$$\leq 3\|x_0-x_m\|^2\max\left\{\frac{1+a_i+4c_i}{1-3(b_i+2c_i)}:1\leq i\leq n_j\right\}$$

$$+3\max\left\{\frac{1+a_i+4c_i}{1-3(b_i+2c_i)}:1\leq i\leq n_j\right\}$$

$$\times\operatorname{diam}(C)\left[\frac{1}{n_j}\sum_{i=1}^{n_j}\|x_m-T^{i+m_j}x_m\|^4\right]^{1/2}.$$

Taking the limit superior as $j\to+\infty$ on each side, we get

$$d(x_0)=\overline{\lim_{j\to\infty}}\frac{1}{n_j}\sum_{i=1}^{n_j}\|x_0-T^{i+m_j}\|^2$$

$$\leq 3\|x_0-x_m\|^2\sup\left\{\frac{1+a_i+4c_i}{1-3(b_i+2c_i)}:i\geq 1\right\}$$

$$+3\sup\left\{\frac{1+a_i+4c_i}{1-3(b_i+2c_i)}:i\geq 1\right\}\operatorname{diam}(C)$$

$$\times\left[\overline{\lim_{j\to\infty}}\frac{1}{n_j}\sum_{i=1}^{n_j}\|x_m-T^{i+m_j}x_m\|^4\right]^{1/2}$$

$$\leq 3\|x_0-x_m\|^2\sup\left\{\frac{1+a_i+4c_i}{1-3(b_i+2c_i)}:i\geq 1\right\}$$

$$+3\sup\left\{\frac{1+a_i+4c_i}{1-3(b_i+2c_i)}:i\geq 1\right\}\operatorname{diam}(C)\sqrt{(a-1)^{m-1}d(x_1)}\to 0$$

as $m\to+\infty$.

This implies that $Tx_0=x_0$. Indeed, for any $\epsilon>0$ there exist natural numbers n_ϵ, $n_\epsilon+1$ such that $\|x_0-T^{n_\epsilon}x_0\|<\epsilon$ and $\|x_0-T^{n_\epsilon+1}x_0\|<\epsilon$. Otherwise, we have for any n_j and m_j,

$$\frac{1}{n_j}\sum_{i=1}^{n_j}\|x_0-T^{i+m_j}\|^2\geq\frac{1}{2}\epsilon^2,$$

and therefore

$$d(x_0)=\overline{\lim_{j\to\infty}}\frac{1}{n_j}\sum_{i=1}^{n_j}\|x_0-T^{i+m_j}\|^2\geq\underline{\lim_{j\to\infty}}\frac{1}{n_j}\sum_{i=1}^{n_j}\|x_0-T^{i+m_j}\|^2\geq\frac{1}{2}\epsilon^2>0.$$

Thus, for any $\epsilon > 0$, we have

$$
\begin{aligned}
&\|Tx_0 - x_0\|^2 \\
&\quad \le (\|Tx_0 - T^{n_\epsilon+1}x_0\| + \|T^{n_\epsilon+1}x_0 - x_0\|)^2 \\
&\quad \le 2\|Tx_0 - T^{n_\epsilon+1}x_0\|^2 + 2\|T^{n_\epsilon+1} - x_0\|^2 \text{ (by Lemma 1)} \\
&\quad \le 2\left(a_1\|x_0 - T^{n_\epsilon}x_0\|^2 + b_1\left\{\|x_0 - Tx_0\|^2 + \|T^{n_\epsilon}x_0 - T^{n_\epsilon+1}x_0\|^2\right\}\right. \\
&\qquad \left. + c_1\left\{\|x_0 - T^{n_\epsilon+1}x_0\|^2 + \|T^{n_\epsilon}x_0 - T_0^x\|^2\right\}\right) + 2\|T^{n_\epsilon+1}x_0 - x_0\|^2 \\
&\quad \le 2a_1\|x_0 - T^{n_\epsilon}x_0\|^2 + 2b_1\|x_0 - Tx_0\|^2 \\
&\quad + 2b_1\left\{2\|T^{n_\epsilon}x_0 - x_0\|^2 + \|x_0 - T^{n_\epsilon+1}x_0\|^2\right\} + 2c_1\|x_0 - T^{n_\epsilon+1}x_0\|^2 \\
&\quad + 2c_1\left\{2\|T^{n_\epsilon}x_0 - x_0\|^2 + 2\|x_0 - Tx_0\|^2\right\} + 2\|T^{n_\epsilon+1}x_0 - x_0\|^2,
\end{aligned}
$$

and hence

$$
\begin{aligned}
&\|Tx_0 - x_0\|^2 \\
&\le \frac{2(a_1 + 2b_1 + 2c_1)}{1 - 2(b_1 + 2c_1)}\|x_0 - T^{n_\epsilon}x_0\|^2 + \frac{2(1 + 2b_1 + c_1)}{1 - 2(b_1 + 2c_1)}\|x_0 - T^{n_\epsilon+1}x_0\|^2 \\
&\quad < \frac{2(1 + a_1 + 4b_1 + 3c_1)}{1 - 2(b_1 + 2c_1)}\epsilon^2 \to 0
\end{aligned}
$$

as $\epsilon \downarrow 0+$, and consequently, $Tx_0 = x_0$. This completes the proof. □

As a simple consequence of Theorem 1, we have the following:

Corollary 1 [4]. *Let C be a nonempty closed convex and bounded subset of a Hilbert space H. If $T : C \to C$ is a Lipschitzian mapping such that*

$$
\varliminf_{n\to\infty} \inf_{m=0,1,2,\dots} \frac{1}{n}\sum_{i=1}^{n} \|T^{i+m}\|^2 < 2,
$$

then T has a fixed point in C.

Corollary 2. *Let C be a nonempty closed convex and bounded subset of a Hilbert space H. If $T : C \to C$ (necessarily continuous) satisfies the following condition*

$$
\begin{aligned}
\|T^n x - T^n y\| \le a\|x - y\|^2 &+ b\{\|x - T^n x\|^2 + \|y - T^n y\|^2\} \\
&+ c\{\|x - T^n y\|^2 + \|y - T^n x\|^2\}
\end{aligned}
$$

for all x, y in C and $n = 1, 2, \dots$ with $b + 2c < 1/3$, and $a + 10b + 13c < 2$, then T has a fixed point in C.

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Department of Mathematics
Rzeszów Institute of Technology
P.O.Box 85, 35-959 Rzeszów, Poland
e-mail: gornicki@prz.rzeszow.pl

received November 10, 1997