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Strongly gamma-starlike functions of order alpha

ABSTRACT. In this work we consider the class of analytic functions $\mathcal{G}(\alpha, \gamma)$, which is a subset of gamma-starlike functions introduced by Lewandowski, Miller and Złotkiewicz in *Gamma starlike functions*, Ann. Univ. Mariae Curie-Skłodowska, Sect. A **28** (1974), 53–58. We discuss the order of strongly starlikeness and the order of strongly convexity in this subclass.

1. Introduction. Let $\mathcal{H}$ denote the class of analytic functions in the unit disc $\mathbb{D} = \{z : |z| < 1\}$ on the complex plane $\mathbb{C}$. For $a \in \mathbb{C}$ and $n \in \mathbb{N}$ we denote by

$$\mathcal{H}[a, n] = \{f \in \mathcal{H} : f(z) = a + a_n z^n + \dots\}$$

and

$$\mathcal{A}_n = \left\{f \in \mathcal{H} : f(z) = z + a_{n+1} z^{n+1} + \dots\right\},$$

so $\mathcal{A} = \mathcal{A}_1$. Let $\mathcal{S}$ be the subclass of $\mathcal{A}$ whose members are univalent in $\mathbb{D}$.

The class $\mathcal{S}_\alpha^*$ of starlike functions of order $\alpha < 1$ may be defined as

$$\mathcal{S}_\alpha^* = \left\{f \in \mathcal{A} : \; \Re\mathfrak{e}\frac{zf'(z)}{f(z)} > \alpha, \; z \in \mathbb{D}\right\}.$$

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The class $\mathcal{S}_\alpha^*$ and the class $\mathcal{K}_\alpha$ of convex functions of order $\alpha < 1$

$$\mathcal{K}_\alpha := \left\{ f \in \mathcal{A} : \ \mathfrak{Re}\left(1 + \frac{zf''(z)}{f'(z)} \right) > \alpha, \ z \in \mathbb{D} \right\}$$
$$= \left\{ f \in \mathcal{A} : \ zf' \in \mathcal{S}_\alpha^* \right\}$$

were introduced by Robertson in [6], see also [2]. If $\alpha \in [0; 1)$, then a function in either of these sets is univalent, if $\alpha < 0$ it may fail to be univalent. In particular we denote $\mathcal{S}_0^* = \mathcal{S}^*$, $\mathcal{K}_0 = \mathcal{K}$, the classes of starlike and convex functions, respectively. Furthermore, note that if $f \in \mathcal{K}_\alpha$, then $f \in \mathcal{S}_{\delta(\alpha)}^*$, see [9], where

$$(1.1) \qquad\qquad \delta(\alpha) = \begin{cases} \frac{1-2\alpha}{2^{2-2\alpha}-2} & \text{for } \alpha \neq \frac{1}{2}, \\ \frac{1}{2\log 2} & \text{for } \alpha = \frac{1}{2}. \end{cases}$$

Let $\mathcal{SS}^*(\beta)$ denote the class of strongly starlike functions of order β, $0 < \beta < 2$,

$$(1.2) \qquad \mathcal{SS}^*(\beta) := \left\{ f \in \mathcal{A} : \ \left| \mathrm{Arg}\,\frac{zf'(z)}{f(z)} \right| < \frac{\beta\pi}{2}, \ \ z \in \mathbb{D} \right\},$$

which was introduced in [8] and [1]. Furthermore,

$$\mathcal{SK}(\beta) = \left\{ f \in \mathcal{A} : \ zf' \in \mathcal{SS}^*(\beta) \right\}$$

denotes the class of strongly convex functions of order β. Analogously to (1.1), in the work [5] it was proved that if $\beta \in (0,1)$ and $f \in \mathcal{SK}(\alpha(\beta))$, then $f \in \mathcal{SS}(\beta)$, where

$$(1.3) \qquad \alpha(\beta) = \beta + \frac{2}{\pi}\tan^{-1}\left(\frac{\beta n(\beta)\sin(\pi(1-\beta)/2)}{m(\beta) + \beta n(\beta)\cos(\pi(1-\beta)/2)} \right),$$

and where

$$m(\beta) = (1+\beta)^{(1+\beta)/2}, \qquad n(\beta) = (1-\beta)^{(\beta-1)/2}.$$

The class $\mathcal{G}(\alpha, \gamma)$, $\gamma > 0$, $0 < \alpha \leq 1$ of γ-strongly starlike functions of order α consists of functions $f \in \mathcal{A}$ satisfying

$$(1.4) \qquad \left| \mathrm{Arg}\left\{ \left(\frac{zf'(z)}{f(z)} \right)^{1-\gamma} \left(1 + \frac{zf''(z)}{f'(z)} \right)^\gamma \right\} \right| < \frac{\alpha\pi}{2}, \ \ z \in \mathbb{D},$$

and such that

$$(1.5) \qquad f(z)f'(z)\left(1 + \frac{zf''(z)}{f'(z)} \right) \neq 0, \ \ z \in \mathbb{D} \setminus \{0\}.$$

Note that Lewandowski, Miller and Złotkiewicz, 1974 [3] have introduced the class of γ-starlike functions, denoted here by $\mathcal{G}(1, \gamma)$, which satisfy (1.5) and such that

$$(1.6) \qquad \mathfrak{Re}\left\{ \left(\frac{zf'(z)}{f(z)} \right)^{1-\gamma} \left(1 + \frac{zf''(z)}{f'(z)} \right)^\gamma \right\} > 0, \ \ z \in \mathbb{D}.$$

2. Preliminaries. To prove the main results, we need the following Nunokawa's Lemma.

Lemma 2.1 ([4], [5]). *Let p be an analytic function in $|z| < 1$ with $p(0) = 1$, $p(z) \neq 0$. If there exists a point z_0, $|z_0| < 1$, such that*

$$|\operatorname{Arg}\{p(z)\}| < \frac{\pi\alpha}{2} \quad \text{for} \quad |z| < |z_0|$$

and

$$|\operatorname{Arg}\{p(z_0)\}| = \frac{\pi\alpha}{2}$$

for some $\alpha > 0$, then we have

$$\frac{z_0 p'(z_0)}{p(z_0)} = ik\alpha,$$

where

$$k \geq \frac{1}{2}\left(a + \frac{1}{a}\right) \quad \text{when} \quad \operatorname{Arg}\{p(z_0)\} = \frac{\pi\alpha}{2}$$

and

$$k \leq -\frac{1}{2}\left(a + \frac{1}{a}\right) \quad \text{when} \quad \operatorname{Arg}\{p(z_0)\} = -\frac{\pi\alpha}{2},$$

where

$$\{p(z_0)\}^{1/\alpha} = \pm ia, \quad \text{and} \quad a > 0.$$

Moreover,

$$(2.1) \quad \operatorname{Arg}\left\{1 + \frac{z_0 p'(z_0)}{p^2(z_0)}\right\} \geq \tan^{-1}\left(\frac{\alpha n(\alpha)\sin(\pi(1-\alpha)/2)}{m(\alpha) + \alpha n(\alpha)\cos(\pi(1-\alpha)/2)}\right),$$

where

$$m(\alpha) = (1+\alpha)^{(1+\alpha)/2} \quad n(\alpha) = (1-\alpha)^{(\alpha-1)/2}.$$

3. Main result.

Theorem 3.1. *Let $f(z) = z + a_2 z^2 + a_3 z^3 + \dots$ be an analytic function in $\mathbb{D}$. Suppose also that $0 < \alpha \leq 1$ and γ is a positive real number such that f satisfies*

$$(3.1) \quad \left|\operatorname{Arg}\left\{\left(\frac{zf'(z)}{f(z)}\right)^{1-\gamma}\left(1 + \frac{zf''(z)}{f'(z)}\right)^{\gamma}\right\}\right| < \frac{\alpha\pi}{2} \quad \text{for} \quad |z| < 1.$$

If the equation, with respect to x,

$$(3.2) \quad x + \frac{2\gamma}{\pi}\tan^{-1}\left(\frac{xn(x)\sin(\pi(1-x)/2)}{m(x) + xn(x)\cos(\pi(1-x)/2)}\right) = \alpha,$$

where

$$m(x) = (1+x)^{(1+x)/2}, \qquad n(x) = (1-x)^{(x-1)/2},$$

has a solution $\beta \in (0,1]$, then f is strongly starlike of order β.

Proof. Let us put

(3.3)
$$p(z) = \frac{zf'(z)}{f(z)}, \ p(0) = 1, \ (z \in \mathbb{D}).$$

Then we have

$$f(z)f'(z)\left(1 + \frac{zf''(z)}{f'(z)}\right) \neq 0 \ \text{ for } \ 0 < |z| < 1$$

because of the assumption (3.1). Moreover,

(3.4)
$$\left(\frac{zf'(z)}{f(z)}\right)^{1-\gamma}\left(1 + \frac{zf''(z)}{f'(z)}\right)^{\gamma} = p(z)\left(1 + \frac{zp'(z)}{p^2(z)}\right)^{\gamma}.$$

If there exists a point z_0, $|z_0| < 1$, such that

$$|\text{Arg}\,\{p(z)\}| < \frac{\pi\beta}{2} \quad \text{for} \ |z| < |z_0|$$

and

$$|\text{Arg}\,\{p(z_0)\}| = \frac{\pi\beta}{2},$$

then by Nunokawa's Lemma 2.1, we have

$$\frac{z_0 p'(z_0)}{p(z_0)} = i\beta k,$$

where

$$k \geq 1 \quad \text{when} \quad \text{Arg}\,\{p(z_0)\} = \frac{\pi\beta}{2}$$

and

$$k \leq -1 \quad \text{when} \ \text{Arg}\,\{p(z_0)\} = -\frac{\pi\beta}{2}.$$

For the case $\text{Arg}\,\{p(z_0)\} = \pi\beta/2$, we have from (3.4) and (2.1)

$$\text{Arg}\left\{\left(\frac{z_0 f'(z_0)}{f(z_0)}\right)^{1-\gamma}\left(1 + \frac{z_0 f''(z_0)}{f'(z_0)}\right)^{\gamma}\right\}$$

$$= \text{Arg}\,\{p(z_0)\} + \gamma\text{Arg}\left\{1 + \frac{z_0 p'(z_0)}{p^2(z_0)}\right\}$$

$$\geq \frac{\pi\beta}{2} + \gamma\tan^{-1}\left(\frac{\beta n(\beta)\sin(\pi(1-\beta)/2)}{m(\beta) + \beta n(\beta)\cos(\pi(1-\beta)/2)}\right)$$

$$= \frac{\alpha\pi}{2}$$

because β is the solution of (3.2). For the case $\text{Arg}\,\{p(z_0)\} = -\pi\beta/2$, applying the same method as the above, we have

$$\text{Arg}\left\{\left(\frac{z_0 f'(z_0)}{f(z_0)}\right)^{1-\gamma}\left(1 + \frac{z_0 f''(z_0)}{f'(z_0)}\right)^{\gamma}\right\} < -\frac{\alpha\pi}{2}.$$

In both of the above cases we have

$$\left| \mathrm{Arg} \left\{ \left(\frac{z_0 f'(z_0)}{f(z_0)} \right)^{1-\gamma} \left(1 + \frac{z_0 f''(z_0)}{f'(z_0)} \right)^{\gamma} \right\} \right| \geq \frac{\alpha \pi}{2}$$

for $z_0 \in \mathbb{D}$, which contradicts hypothesis (3.1) of the theorem and therefore,

$$|\mathrm{Arg}\{p(z)\}| < \frac{\pi \beta}{2} \quad \text{for} \quad |z| < 1,$$

which completes the proof. $\hfill\square$

Theorem 3.1 says that a function in the class $\mathcal{G}(\alpha, \gamma)$, see (1.4), of γ-strongly starlike functions of order α is strongly starlike function, see (1.2), of order at least β, where β is the solution of (3.2). Note that if $f \in \mathcal{G}(\alpha, 0)$, then f is strongly starlike of order α. For a related result we refer to [7]. If $\alpha = 1$, then Theorem 3.1 becomes the following result on the class $\mathcal{G}(\gamma, 1)$ introduced by Lewandowski, Miller and Złotkiewicz [3].

Corollary 3.1. *Assume that $f \in \mathcal{G}(\gamma, 1)$ or that f satisfies (1.5) and (1.6). If the equation*

$$x + \frac{2\gamma}{\pi} \tan^{-1} \left(\frac{xn(x) \sin(\pi(1-x)/2)}{m(x) + xn(x) \cos(\pi(1-x)/2)} \right) = 1,$$

has a solution $\beta \in (0, 1]$, then f is strongly starlike of order β.

In the corollary below there are examples of the choice α, γ and β which satisfies Corollary 3.1 or Theorem 3.1.

Corollary 3.2. *If $f \in \mathcal{G}(\gamma_{(1;1/2)}, 1)$, then $f \in \mathcal{SS}^*(1/2)$, where*

$$\gamma_{(1;1/2)} = \frac{\pi}{4 \tan^{-1} \left(\frac{1}{3 \sqrt[4]{4/3} + 1} \right)} \approx 3.378.$$

If $f \in \mathcal{G}(\gamma_{(3/4;1/2)}, 3/4)$, then $f \in \mathcal{SS}^*(1/2)$, where

$$\gamma_{(3/4;1/2))} = \frac{\pi}{8 \tan^{-1} \left(\frac{1}{3 \sqrt[4]{4/3} + 1} \right)} \approx 1.689.$$

If $f \in \mathcal{G}(\gamma_{(3/5;1/2)}, 3/5)$, then $f \in \mathcal{SS}^*(1/2)$, where

$$\gamma_{(3/5;1/2)} = \frac{\pi}{20 \tan^{-1} \left(\frac{1}{3 \sqrt[4]{4/3} + 1} \right)} \approx 0.675.$$

If $f \in \mathcal{G}(\gamma_{(3/4;2/3)}, 3/4)$, then $f \in \mathcal{SS}^*(2/3)$, where

$$\gamma_{(3/4;2/3)} = \frac{\pi}{8 \tan^{-1} \left(\frac{\sqrt[6]{3}}{3(5/3)^{5/3} + \sqrt[6]{81}} \right)} \approx 1.481.$$

If $\alpha = \beta$, then from (3.2) we get $\gamma = 0$, and $\mathcal{G}(0, \alpha) \subset \mathcal{SS}^*(\alpha)$, but this case is trivial. In the next theorem we consider the order of strongly starlikeness for functions, in some sense, in the class $\mathcal{G}(\alpha, \gamma)$ of negative order γ.

Theorem 3.2. *Let* $f(z) = z + a_2 z^2 + a_3 z^3 + \ldots$ *be an analytic function in* $\mathbb{D}$. *Suppose also that* γ *is a negative real number such that* f *satisfies*

$$(3.5) \qquad \left| \mathrm{Arg} \left\{ \left(\frac{zf'(z)}{f(z)} \right)^{1-\gamma} \left(1 + \frac{zf''(z)}{f'(z)} \right)^{\gamma} \right\} \right| < \frac{\alpha\pi}{2} \quad for \quad |z| < 1,$$

where $0 < \alpha \leq 1$ *and suppose that* β *is the root of the equation*

$$(3.6) \qquad \beta + \gamma(1 - \beta) = \alpha$$

in the interval $(0, 1]$. *Then we have*

$$\left| \mathrm{Arg} \left\{ \frac{zf'(z)}{f(z)} \right\} \right| < \frac{\beta\pi}{2} \quad for \quad |z| < 1.$$

Proof. In the first part of the proof we apply the same method as in the proof of Theorem 3.1. Let us put

$$p(z) = \frac{zf'(z)}{f(z)}, \ p(0) = 1, \ (z \in \mathbb{D}).$$

If there exists a point $z_0 \in \mathbb{D}$ such that

$$(3.7) \qquad \left| \mathrm{Arg} \left\{ \frac{zf'(z)}{f(z)} \right\} \right| < \frac{\pi\beta}{2} \quad \text{for} \quad |z| < |z_0|$$

and

$$\left| \mathrm{Arg} \left\{ \frac{z_0 f'(z_0)}{f(z_0)} \right\} \right| = \frac{\pi\beta}{2},$$

then by Nunokawa's Lemma 2.1, we have

$$(3.8) \qquad \frac{z_0 p'(z_0)}{p(z_0)} = i\beta k,$$

where

$$k \geq \frac{1}{2} \left(a + \frac{1}{a} \right) \quad \text{when} \ \ \mathrm{Arg} \left\{ p(z_0) \right\} = \frac{\pi\alpha}{2}$$

and

$$k \leq -\frac{1}{2} \left(a + \frac{1}{a} \right) \quad \text{when} \ \ \mathrm{Arg} \left\{ p(z_0) \right\} = -\frac{\pi\alpha}{2},$$

where

$$(3.9) \qquad \{ p(z_0) \}^{1/\beta} = \pm ia, \ \ \text{and} \ \ a > 0.$$

For the case $\{p(z_0)\}^{1/\beta} = ia$, $a > 0$, we have from (3.8) and (3.9)

$$\text{Arg}\left\{\left(\frac{z_0 f'(z_0)}{f(z_0)}\right)^{1-\gamma}\left(1+\frac{z_0 f''(z_0)}{f'(z_0)}\right)^{\gamma}\right\}$$

$$= \text{Arg}\left\{p(z_0)\left(1+\frac{z_0 p'(z_0)}{p^2(z_0)}\right)^{\gamma}\right\}$$

$$= \text{Arg}\left\{p(z_0)\right\} + \gamma\text{Arg}\left\{1+\frac{z_0 p'(z_0)}{p^2(z_0)}\right\}$$

$$= \frac{\pi\beta}{2} + \gamma\text{Arg}\left\{1+\frac{i\beta k}{(ia)^{\beta}}\right\}$$

$$= \frac{\pi\beta}{2} + \gamma\text{Arg}\left\{1+\frac{\beta k}{a^{\beta}}e^{i\frac{\pi(1-\beta)}{2}}\right\}$$

$$\geq \frac{\pi\beta}{2} + \gamma\frac{\pi(1-\beta)}{2}$$

$$= \frac{\alpha\pi}{2},$$

because β is the solution of (3.6). For the case $\text{Arg}\{p(z_0)\} = -\pi\beta/2$, applying the same method as the above, we have

$$\text{Arg}\left\{\left(\frac{z_0 f'(z_0)}{f(z_0)}\right)^{1-\gamma}\left(1+\frac{z_0 f''(z_0)}{f'(z_0)}\right)^{\gamma}\right\} \leq -\frac{\alpha\pi}{2}.$$

The above cases show that

$$\left|\text{Arg}\left\{\left(\frac{z_0 f'(z_0)}{f(z_0)}\right)^{1-\gamma}\left(1+\frac{z_0 f''(z_0)}{f'(z_0)}\right)^{\gamma}\right\}\right| \geq \frac{\alpha\pi}{2}, \quad z_0 \in \mathbb{D},$$

which contradicts hypothesis (3.5) of the theorem and therefore,

$$\left|\text{Arg}\left\{p(z)\right\}\right| < \frac{\pi\beta}{2} \quad \text{for} \quad |z| < 1$$

which completes the proof. □

Theorem 3.3. *Let $f(z) = z + a_2 z^2 + a_3 z^3 + \ldots$ be an analytic function in $\mathbb{D}$. Suppose also that $0 < \alpha \leq 1$ and $0 < \gamma \leq 1$ are such that f satisfies (3.1). If the equation (3.2) has a solution $\alpha_0 \in (0,1]$, then f is strongly convex of order $\{(1-\gamma)\alpha_0 + \alpha\}/\gamma$.*

Proof.

$$\left|\text{Arg}\left\{\left(1+\frac{zf''(z)}{f'(z)}\right)^{\gamma}\right\}\right| - \left|\text{Arg}\left\{\left(\frac{zf'(z)}{f(z)}\right)^{1-\gamma}\right\}\right|$$

$$\leq \left|\text{Arg}\left\{\left(\frac{zf'(z)}{f(z)}\right)^{1-\gamma}\left(1+\frac{zf''(z)}{f'(z)}\right)^{\gamma}\right\}\right| < \frac{\alpha\pi}{2} \quad \text{for} \quad |z| < 1.$$

Then by Theorem 3.1 we have

$$\left| \mathrm{Arg} \left\{ \left(1 + \frac{zf''(z)}{f'(z)} \right)^{\gamma} \right\} \right|$$

$$\leq \left| \mathrm{Arg} \left\{ \left(\frac{zf'(z)}{f(z)} \right)^{1-\gamma} \right\} \right| + \frac{\alpha\pi}{2}$$

$$< \frac{\pi(1-\gamma)\alpha_0}{2} + \frac{\alpha\pi}{2},$$

and so

$$\left| \mathrm{Arg} \left\{ 1 + \frac{zf''(z)}{f'(z)} \right\} \right| < \frac{\pi\left\{ (1-\gamma)\alpha_0 + \alpha \right\}}{2\gamma} \quad \text{for} \quad |z| < 1. \qquad \square$$

Theorem 3.4. *Assume that the equation* (3.2) *has a solution* α_0, $0 < \alpha_0 < \alpha \leq 1$. *If* $0 < \delta < \gamma$, *then* $\mathcal{G}(\alpha, \gamma) \subset \mathcal{G}(\alpha, \delta)$.

Proof. Let us suppose that f is a member of $\mathcal{G}(\alpha, \gamma)$ and let us put

$$A = \left\{ B^{1-\delta} C^{\delta} \right\}^{\gamma/\delta},$$

where

$$B = zf'(z)/f(z) \quad \text{and} \quad C = 1 + zf''(z)/f'(z).$$

Then we have

$$A = B^{1-\gamma} C^{\gamma} B^{\gamma/\delta - 1}$$

and by Theorem 3.1 we obtain

$$|\mathrm{Arg}\{A\}| = \frac{\gamma}{\delta} \left| \mathrm{Arg} \left\{ B^{1-\delta} C^{\delta} \right\} \right|$$

$$= \left| \mathrm{Arg} \left\{ B^{1-\gamma} C^{\gamma} \right\} + \mathrm{Arg} \left\{ B^{\gamma/\delta - 1} \right\} \right|$$

$$< \frac{\alpha\pi}{2} + \left(\frac{\gamma}{\delta} - 1 \right) \frac{\alpha_0 \pi}{2}$$

$$< \frac{\alpha\pi}{2} + \left(\frac{\gamma}{\delta} - 1 \right) \frac{\alpha\pi}{2}$$

$$\leq \frac{\gamma}{\delta} \frac{\alpha\pi}{2}.$$

This shows that

$$\left| \mathrm{Arg} \left\{ B^{1-\delta} C^{\delta} \right\} \right| < \frac{\alpha\pi}{2} \quad z_0 \in \mathbb{D}.$$

Therefore, $f \in \mathcal{G}(\alpha, \delta)$. $\qquad \square$

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