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Institute of Physics, Maria Curie-Skłodowska University

Jerzy KRAŚKIEWICZ

The Analysis of the Scattering of the Polarized Electron on the Polarized Proton

Analiza rozpraszania spolaryzowanego elektronu na spolaryzowanym protonie

1. INTRODUCTION

The lepton-hadron scattering provide a test of models of elementary particles. The predictions of QCD for polarized protons disagree with experimental results [1]. The proton in that model consists with three quarks and the process $e^- + p \to e^- + p$ is the four to four-body process for which we have no reliable method of analysis.

The aim of that work is to analyze the elastic scattering of the polarized electron on the polarized proton. We use the Hadronic Standard Model [2] in which the proton is represented as an elementary particle. The inner structure of a proton is described by form factors.

Our primary object of study is a differential cross section of the process $e^{-\uparrow} + p^{\uparrow} \to e^{-\uparrow} + p^{\uparrow}$ whose formalizm is presented in section 2. Matrix elements of that process we calculate in the lowest order of perturbation series taking into account the electromagnetic and weak interaction.

In section 3 we briefly describe electromagnetic and weak form factors of a proton. These form factors were calculated analytically in [3–5] from the vector dominance model with analyticity and unitarity conditions. These representation of form factors is very natural in Hadronic Standard Model since vector mesons are elementary particles in this model.

The numerical results of the differential cross section versus polar and azimuthal angles as well as energy are presented in section 4.

2. DIFFERENTIAL CROSS SECTION FOR $e^- + p \to e^- + p$ PROCESS

We consider the elastic scattering $e^- + p \to e^- + p$ in the lowest order of perturbative series with photon and Z exchange:

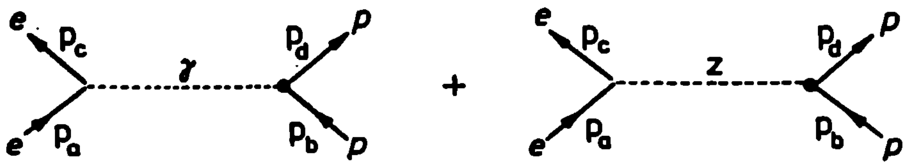


Fig. 1. The lowest order diagrams for $e^- + p \to e^- + p$ process with photon and Z boson exchange

Ryc. 1. Diagramy najniższego rzędu dla reakcji $e^- + p \to e^- + p$ z wymianą fotonu i bozonu Z

where we put electromagnetic form factors at $p - p - \gamma$ vertex Γ^{ph} and weak form factors of the proton at $p - p - Z$ vertex Γ^Z.

The forms of these two vertices are the following:

$$\Gamma^\gamma_\mu = F_1^p(t)\gamma_\mu - \frac{1}{2M}F_2^p(t)\sigma_{\mu\nu}q^\nu. \tag{1}$$

$$\Gamma^Z_\mu = F_V^0(t)\gamma_\mu - \frac{1}{2M}F_M^0(t)\sigma_{\mu\nu}q^\nu + \gamma_\mu\gamma_5 F_A^0(t), \tag{2}$$

where $\sigma^{\mu\nu} = \frac{1}{2}[\gamma^\mu\gamma^\nu - \gamma^\nu\gamma^\mu]$, $\mu = 0,1,2,3$, $\gamma^5 = i\gamma^0\gamma^1\gamma^2\gamma^3$, $q = p_c - p_a$, $t = q^2$.

The partial polarization of a particle beam with momentum $\mathbf{p}$ is given by the 4-vector s^μ which is defined as

$$s^\mu \equiv \left(P^3\frac{|\,\mathbf{p}\,|}{m}, P^1, P^2, P^3\frac{p^0}{m}\right)$$

where $p^0 = \sqrt{\mathbf{p}^2 + m^2}$,
$P^3 \equiv P_{||}$ describe longitudinal polarization (in the $\mathbf{p}$ direction),
$(P^1, P^2) \equiv P_\perp$ describe transverse polarization (perpendicular to $\mathbf{p}$)
with

$$(P^1)^2 + (P^2)^2 + (P^3)^2 \leqslant 1.$$

The 4-vector s^μ has the following properties:

$$s^\mu p_\mu = 0, \qquad s^\mu s_\mu = -\sqrt{(P^1)^2 + (P^2)^2 + (P^3)^2}.$$

According to Dirac theory a free fermion quantum state is described by a 4-component spinor u(p,s) which satisfies the equation

$$(p - m)u(p, s) = 0$$

with normalization

$$\bar{u}(p, s)u(p, s) = 1.$$

The covariant density matrix for the polarized particle beam is the following

$$\rho(p, s) = u(p, s)\bar{u}(p, s) = \frac{(p^{\mu}\gamma_{\mu} + m)(1 + \gamma^5 s^{\mu}\gamma_{\mu})}{4m}.$$

The scattering amplitude for polarized particles is the following:

$$\begin{aligned} R(s_a, s_b, s_c, s_d) &= \\ & - e^2 \left\{ \bar{u}(p_d, s_d)\Gamma^{ph}_{\mu} u(p_b, s_b) u(p_c, s_c)\gamma^{\mu} \frac{-1}{t} u(p_a, s_a) + \right. \\ & + \bar{u}(p_d, s_d)\Gamma^{Z}_{\mu} u(p_b, s_b)\bar{u}(p_c, s_c)\gamma^{\mu}(a - b\gamma^5) \times \\ & \left. \times \frac{1}{t - M_Z - iM_Z\Gamma_Z} u(p_a, s_a) \right\} \end{aligned}$$

where $\Gamma^{ph}_{\mu}, \Gamma^{Z}_{\mu}$ are vertices depending on form factors; $t = (p_c - p_a)$; M_Z, Γ_Z — mass and width of Z boson; e, a, b — coupling constants of electromagnetic and weak interections.

We neglected the term with $q^{\nu}q^{\mu}$ in the Z propagator which is proportional to $\frac{m_e}{M_Z}$ when coupled to light fermion like electron.

The differential cross section for elastic scattering of polarized initial particles and without detecting polarization of final particles is given by:

$$\frac{d\sigma}{d\Omega}(s, \theta, \varphi) = \frac{m_e^2 M^2}{\sqrt{p_a P_c - M^2 m_e^2}(2\pi)^2} \frac{\mid \mathbf{p}_a \mid}{\sqrt{s}} \mid R_{ab\to cd}(s_a, s_b, s_c = 0, s_d = 0) \mid^2 .$$

The expression for differential cross section after calculations of traces in $\mid R \mid^2$ is too long to present here and is evaluated numerically. The covariant form of differential cross section for electron-proton scattering but only with one photon exchange one can find in [6].

3. NUCLEON FORM FACTORS

The proton current coupled to a photon can be written in terms of Dirac $F_1^p(t)$ and Pauli $F_2^p(t)$ form factors as follows:

$$J_\mu^\gamma = \overline{u}(p')\left[F_1^p(t)\gamma_\mu - \frac{1}{2M}F_2^p(t)\sigma_{\mu\nu}q^\nu\right]u(p). \tag{3}$$

The proton current coupled to Z-boson has the similar form:

$$J_\mu^\gamma = \overline{u}(p')\left[F_V^0(t)\gamma_\mu - \frac{1}{2M}F_M^0(t)\sigma_{\mu\nu}q^\nu + \gamma_\mu\gamma_5 F_A^0(t)\right]u(p). \tag{4}$$

The proton and neutron electromagnetic form factors can be decomposed into isoscalar and isovector part:

$$F_{1,2}^p = F_{1,2}^S + F_{1,2}^V$$

$$F_{1,2}^n = F_{1,2}^S - F_{1,2}^V.$$

The weak form factors of a proton are determined with the help of current algebra

$$F_V^0 = F_1^V - 2\sin^2\theta_W\ F_1^p$$

$$F_M^0 = F_2^V - 2\sin^2\theta_W\ F_2^p,$$

where θ_W is the Weinberg angle and F_A^0 is represented by the dipol formula [7]

$$F_A^0(t) = \frac{-0.6}{\left(1 - \frac{t}{0.86^2 GeV^2}\right)^2}.$$

One obtains the analytic form of the functions $F_{1,2}^s(t)$, $F_{1,2}^v(t)$ using the vector-meson dominance (VMD) hypothesis which give the following parameterization

$$\begin{aligned} F_1^s(t) &= \sum_{s=\omega,\omega',\omega''} \frac{m_s^2}{m_s^2 - t}(f_{sp\bar{p}}^{(1)}/f_s) \\ F_2^s(t) &= \sum_{s=\omega,\omega',\omega''} \frac{m_s^2}{m_s^2 - t}(f_{sp\bar{p}}^{(2)}/f_s) \end{aligned} \tag{5}$$

$$F_1^v(t) = \sum_{v=\rho,\rho',\rho'',\rho'''} \frac{m_v^2}{m_v^2 - t}(f^{(1)}_{vp\bar{p}}/f_v)$$

$$F_2^v(t) = \sum_{v=\rho,\rho',\rho'',\rho'''} \frac{m_v^2}{m_v^2 - t}(f^{(2)}_{vp\bar{p}}/f_v)$$

The sum over vector mesons in 5 is natural in our model since vector mesons are elementary in Hadronic Standard Model and resonances $\omega, \omega', \omega'', \rho, \rho', \rho'', \rho'''$ etc. appear in daughter Regge trajectories (see [8] Ch. III).

We use in the present work the VMD representation of form factors derived by Dubnička, who took into account some analyticity and unitarity conditions [3–5].

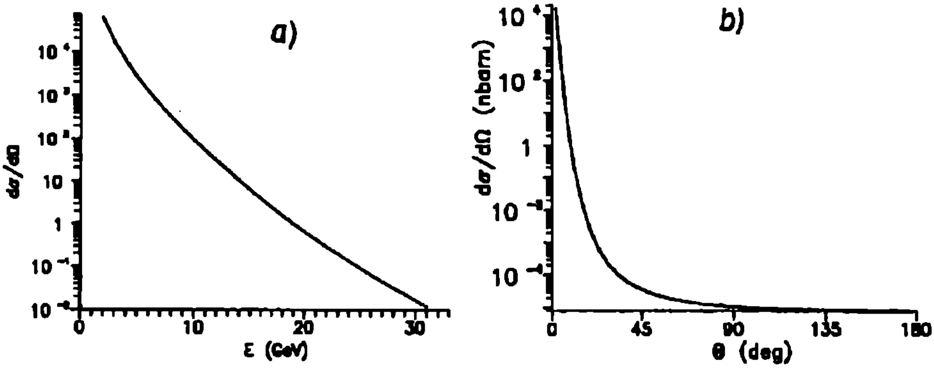


Fig. 2. The differential cross section versus: a) energy for $\varphi = 0°$, $\theta = 5°$, $P_{\parallel}^{el} = 1$, $P_{\perp}^{p} = 1$; b) longitudinal polarization of an electron for $E_{LAB} = 30$ GeV, $\theta = 5°$, $\varphi = 0$, $P_{\parallel}^{p} = 1$;
Ryc. 2. Różniczkowy przekrój czynny w zależności od: a) energii dla $\varphi = 0°$, $\theta = 5°$, $P_{\parallel}^{el} = 1$, $P_{\perp}^{p} = 1$; b) polaryzacji podłużnej elektronu dla $E_{LAB} = 30$ GeV, $\theta = 5°$, $\varphi = 0$, $P_{\parallel}^{p} = 1$;

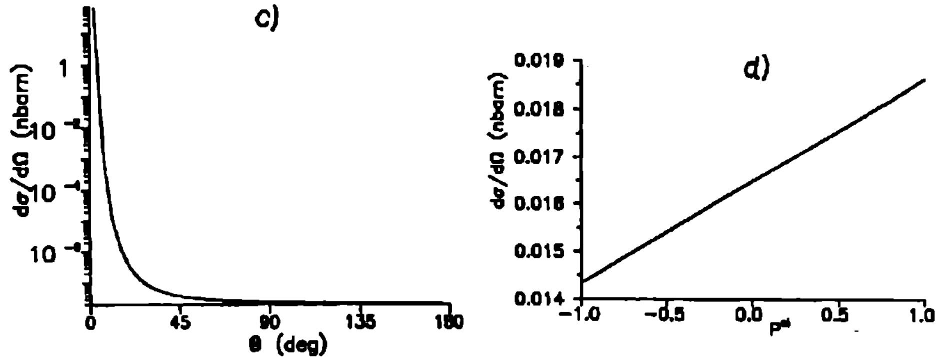
4. RESULTS

We have noticed that contribution of the diagram with Z exchange is about 0.01% that of photon exchange.

All input parameters as energy, azimuthal and polar angles are given in the LAB system but the differential cross section we express in the CM system.

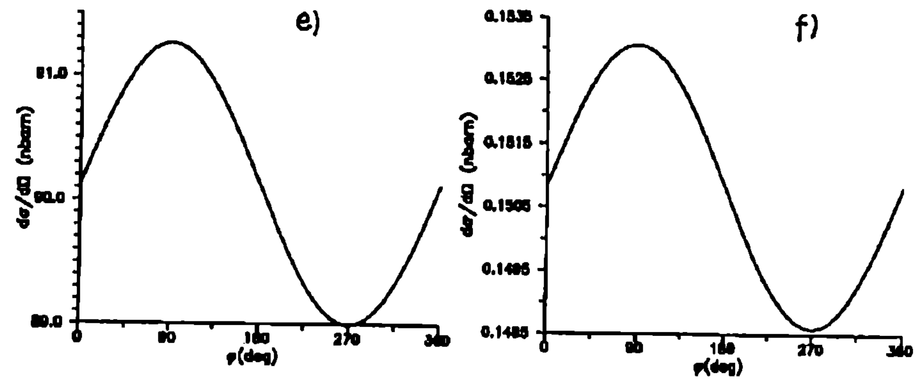
If we fixed z axis along momentum of an electron then $P_{\parallel}^{el} > 0, P_{\parallel}^{p} < 0$ means parallel polarization and $P_{\parallel}^{el} < 0, P_{\parallel}^{p} > 0$ antiparallel polarization of an electron and a proton.

Figure 2a shows energy dependence of the differential cross section (d.c.s.) of the elastic scattering an electron on a proton. The d.c.s. decreases very fast due to the form factors of a proton.



c) azimuthal angle for $E_{LAB} = 10$ GeV, $\theta = 5°$; d) azimuthal angle for $E_{LAB} = 30$ GeV, $\theta = 4°$;

c) kąta azymutalnego dla $E_{LAB} = 10$ GeV, $\theta = 5°$; d) kąta azymutalnego dla $E_{LAB} = 30$ GeV, $\theta = 4°$;



e) polar angle for $E_{LAB} = 10$ GeV, $\varphi = 0$, $P_{\parallel}^{el} = 0.4$, $P_{\parallel}^{p} = 1$; f) polar angle for $E_{LAB} = 30$ GeV, $\varphi = 0$, $P_{\parallel}^{el} = 0.4$, $P_{\parallel}^{p} = 1$

e) kąta polarnego dla $E_{LAB} = 10$ GeV, $\varphi = 0$, $P_{\parallel}^{el} = 0.4$, $P_{\parallel}^{p} = 1$; f) kąta polarnego dla $E_{LAB} = 30$ GeV, $\varphi = 0$, $P_{\parallel}^{el} = 0.4$, $P_{\parallel}^{p} = 1$

Figures $2bc$ show the dependence of the d.c.s. on longitudinal polarization of an electron for fixed polarization of a proton.

The dependence of d.c.s. on azimuthal angle for two different energies is shown in Figure $2d$.

The dependence of d.c.s. on a polar angle is shown in Figures $2ef$.

ACKNOWLEDGMENTS

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STRESZCZENIE

Analizuję proces rozpraszania spolaryzowanych elektronów na spolaryzowanych protonach. Używając analitycznej formuły dla formfaktorów protonu, obliczam różniczkowy przekrój czynny jako funkcję kątów i energii.