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Importance of the Neutron-Proton Interactions for the Even Ra-Th Nuclei

INTRODUCTION

The concept of deformation has extensively been used since 1953 [1] to describe "collective" properties of heavy and light nuclei but up to now very little is known about the microscopic structure of deformation. This problem was discussed in a shell-model framework [2,3,4]. The common conclusion was that the neutron-proton interaction could be essential to the development of collectivity and deformation in nuclei. In light nuclei neutrons and protons occupy the same shell model levels and this idea is commonly accepted. In such nuclei the deuteron pairs ($J^\pi = 1^+$, $T = 0$) and charge independent pairing pairs (i.e. $n-n, p-p$ and $n-p$ with $J^\pi = 0^+$, $T = 1$) are important. The Hamiltonian, containing the pairing and isopairing terms, has been diagonalized using e.g. quasispin group SO(8) [5, 6, 7] or IBM-4 [8,9].

In heavy nuclei the valence neutrons and protons occupy different shells. So, in most of theoretical papers the neutron-proton interaction is neglected or included indirectly as the interaction of the $n-n$ and $p-p$ pairs (e.g. as in the quartet model [10,11] and IBM-2 [12,13]).

Many years ago [2] it has been shown that, for two nucleons moving in a harmonic oscillator potential and interacting via a delta force, the energy of the $J = 0$ state is minimal for $l_1 = l_2$. It can be seen (Fig. 1) that the difference of the overlaps of $(n_1 l_1 j_1)$ and $(n_2 l_2 j_2)$ states

$$\int_0^\infty R^2_{n_1 l_1} R^2_{n_2 l_2} \frac{dr}{r^2} - \int_0^\infty R^2_{n'_1 l'_1} R^2_{n'_2 l'_2} \frac{dr}{r^2}$$

decreases with increasing l if $l_1 = l_2 \equiv l$, $l'_1 = l$, $l'_2 = l - 1$. This means, we expect relatively large neutron-proton interaction in heavy nuclei, too. Fig. 2 shows some experimental effective interactions in light (a, b lines) and heavy (c, d lines)

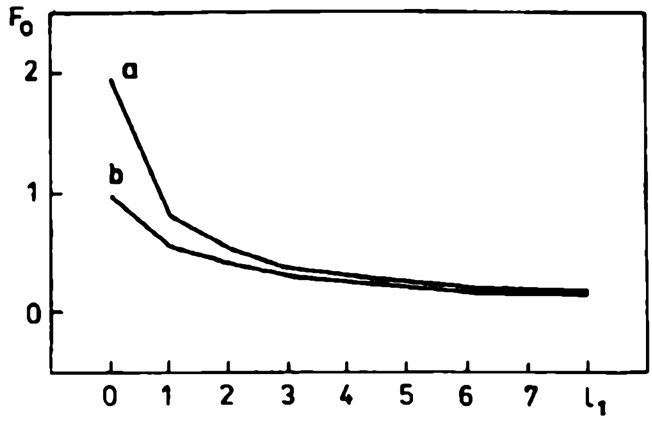

Fig. 1. The overlap $F_0 = \int_0^\infty R^2_{n_1 l_1} R^2_{n_2 l_2} dr/r^2$ for "a" $n_1 = n_2 = 1$, $l_1 = l_2$, "b" $n_1 = n_2 = 1$, $l_1 = l_2 - 1$

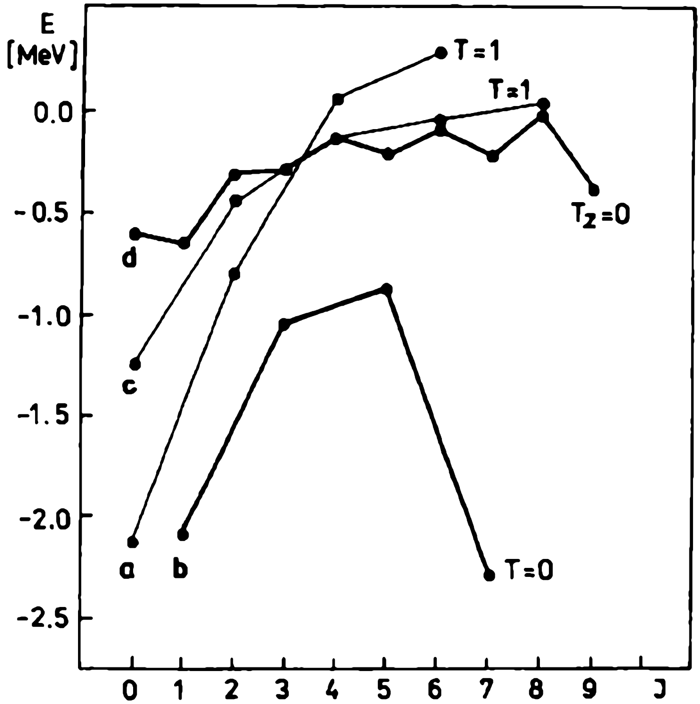

Fig. 2. Experimental [14] diagonal matrix elements of nucleon-nucleon interaction. The line "a" corresponds to $(1f_{7/2})^{2n}$ configuration in ^{42}Ca, "b" — to $(1f_{7/2})^{pn}$ in ^{48}S, "c" — to $(2g_{9/2})^{2n}$ in ^{210}Pb and "d" — to $(1h_{9/2})^{p}(2g_{9/2})^{n}$ in ^{210}Bi

nuclei. Such a picture is typical for other j-levels [14,15]. We can conclude that the interaction of two neutrons or two protons is the strongest in $J^\pi, T = 0^+, 1$ state and rapidly decreases when J increases. The result depends on lj values in neutron-proton pair:

$$\left|V^{eff}_{J=2j,T=0}\right| > \left|V^{eff}_{J=0,T=1}\right| \quad \text{for} \quad j = 3/2, 5/2, 7/2 \quad \text{and}$$
$$\left|V^{eff}_{J=0,T=1}\right| > \left|V^{eff}_{J=\text{odd},T=0}\right| \quad \text{for} \quad j = 9/2.$$

In the case $j_n \neq j_p$ one can find the strongest interaction for larger j and $|l_n - l_p| = 1$. As a consequence we see very low-lying 0^- states in the odd-odd nuclei (e.g. 0.047 MeV in $^{210}_{83}$Bi, 0.002 MeV in $^{236}_{91}$Pa, 0.049 MeV in $^{192}_{63}$Eu, 0.043 MeV in $^{166}_{71}$Lu and so on) and also low excited 1^- states in the odd-odd and even-even deformed nuclei.

Lately [16] the importance of the neutron-proton interaction in generating the nuclear deformation is supported by the self-consistent calculations within the Hartree-Fock method with the Skyrme interaction. The conclusion [17], that low--lying states ($E \leq 1.5$ MeV) in ^{218}Ra are mostly described by s and p bosons, is such a backing in the spdf — IBM model.

Encouraged by experimental data and the results of the papers [16–20], we take the strongest interactions in $J^\pi T = 0^+, 1$ and $1^-, 0$ states and we obtain the model with the same group structure as in ref. [21].

In § THE MODEL we give a sketch of the model, § RESULTS includes the results for even $^{222-228}$Ra and $^{226-232}$Th, § CONCLUSIONS presents summary and conclusions, the appendix contains some details of the calculations.

THE MODEL

In the actinide region the correlation energy of the nucleon pairs is of about a few hundred keV. Then, for low excitation energy ($E \leq 1$ MeV), we will treat the nucleon pairs as building blocks of a nucleus [22]. With the above assumption, the nucleon pair with quantum numbers of the total angular momentum J, the parity Π and the isospin T corresponds to the boson with the same quantum numbers. Taking into account the strongest interactions we have six bosons: s^+_μ with $J^\pi = 0^+$, $T = 1$, $\mu = 0, \pm 1$ and p^+_μ with $J^\pi = 1^-$, $\mu = 0, \pm 1$, $T = 0$. The boson s^+ corresponds to a pair of nucleons coupled by pairing forces, the boson p^+ substitutes for a neutron-proton pair based on single particle shell model levels with $|l_n - l_p| = 1$.

The most general Hamiltonian H for a system of interacting s and p bosons is:

$$\begin{aligned} H &= \epsilon_1 \hat{n}_s + \epsilon_2 \hat{n}_p + \epsilon_3 [p^+ p^+]^{J=0,T=0} [\tilde{p}\tilde{p}]^{00} + \epsilon_4 [s^+ s^+]^{00} [\tilde{s}\tilde{s}]^{00} + \\ &+ \epsilon_5 [p^+ p^+]^{20} [\tilde{p}\tilde{p}]^{20} + \epsilon_6 [s^+ s^+]^{02} [\tilde{s}\tilde{s}]^{02} + \epsilon_7 [p^+ s^+]^{11} [\tilde{s}\tilde{p}]^{11} + \\ &+ \epsilon_8 ([p^+ p^+]^{00} [\tilde{s}\tilde{s}]^{00} + [s^+ s^+]^{00} [\tilde{p}\tilde{p}]^{00}). \end{aligned} \tag{1}$$

Square brackets denote spin and/or isospin coupling and

$$T^k T^k = (-1)^k (2k+1)^{1/2} [T^k T^k]^0; \quad \tilde{b}_\mu = (-1)^{1-\mu} b_{-\mu}.$$

The first two terms in (1) represent the familiar pairing and iso-pairing interactions, the subsequent terms — different effective four-nucleon interactions. Hamiltonian (1) conserves the total number of bosons $N = n_p + n_s$, total angular momentum J and isospin T. It can be rewritten in the generators

$$[p^+\tilde{p}]_\mu^{J=0,1,2;T=0}, \ [s^+\tilde{s}]_\nu^{J=0;T=0,1,2}, \ [p^+\tilde{s}]_{\mu\nu}^{11}, \ [s^+\tilde{p}]_{\mu\nu}^{11} \tag{2}$$

of the unitary group U(6). One from the two possible complete chains of subgroups of this U(6)

$$\mathrm{U}(6) \supset \mathrm{U}_{n_p}(3) \otimes \mathrm{U}_{n_s}(3) \supset \mathrm{SO}_J(3) \otimes \mathrm{SO}_T(3) \supset \mathrm{SO}_{M_J}(2) \otimes \mathrm{SO}_{M_T} \tag{3}$$

provides the basis

$$|N n_p J M_J T M_T\rangle \tag{4}$$

in which H is diagonalized.

RESULTS

The actinide nuclei, containing a ^{208}Pb core with valence protons filling the $1h_{9/2}$, $2f_{7/2}$, $1i_{13/2}$ and $2f_{5/2}$ orbitals and valence neutrons in $2g_{9/2}$, $1i_{11/2}$, $1j_{15/2}$ and $2d_{5/2}$ orbitals, are suitable for verifying the importance of the effective neutron-proton interaction. Even nuclei of Ra with $E(4_1^+)/E(2_1^+) < 3.2$ and Th with $E(4_1^+)/E(2_1^+) > 3.2$ are chosen for the study.

The Hamiltonian (1) was diagonalized in the basis (4) for the boson numbers equal to 1/2 of the number of nucleons over the core ^{208}Pb and for the isospin numbers $T = T_z$ for the valence nucleons. From 8 one- and two-boson energies ϵ_i only 6 parameters $k_i = f_i(\epsilon_1 \ldots \epsilon_8)$ are independent [21]. They were fitted in order to obtain low-lying spectra of searching nuclei (Fig. 3 and Fig. 4).

The eigenstates of H

$$|NJM_JTM_T, E\rangle = \sum_{n_p=J \text{ step } 2}^{N \text{ or } N-1} a_{n_p}(JT, E)|n_p, JM_J\rangle|N - n_p, TM_T\rangle \tag{5}$$

make it possible to find the reduced $E1$ and $E2$ transitions defined as usual

$$B(E\lambda; J_1 \to J_2) = (2J_1 + 1)^{-1}|\langle J_2||\hat{B}(E\lambda)||J_1\rangle|^2$$

with:

$$\hat{B}_{\mu\nu}(E1) = C_1[s^+\tilde{p} + p^+\tilde{s}]_{\mu\nu}^{11}, \tag{6}$$

$$\hat{B}_\mu(E2) = C_2[p^+\tilde{p}]_\mu^2. \tag{7}$$

Table 1 shows the ratios for reduced probabilities of the electric dipole transitions from the 1^- state ("octupole") to the yrast band states. Tables 2 and 3

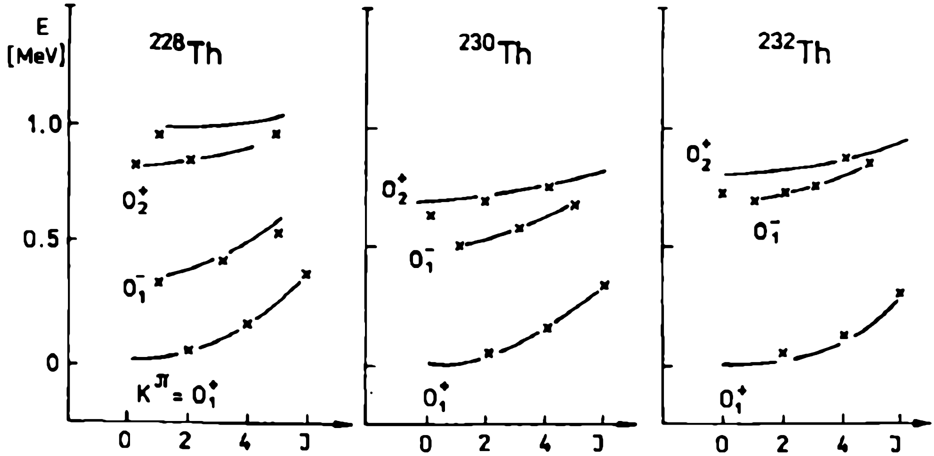

Fig. 3. The calculated (lines) with parameters (in MeV): $H_0 = 0$, $k_1 = 1.6252$, $k_2 = -0.0100$, $k_3 = -0.0098$, $k_4 = 0.1317$, $k_5 = 0.0129$, $k_6 = 0.0581$ and experimental (crosses) spectra of ^{228}Th [23], ^{230}Th [24] and ^{232}Th [25]

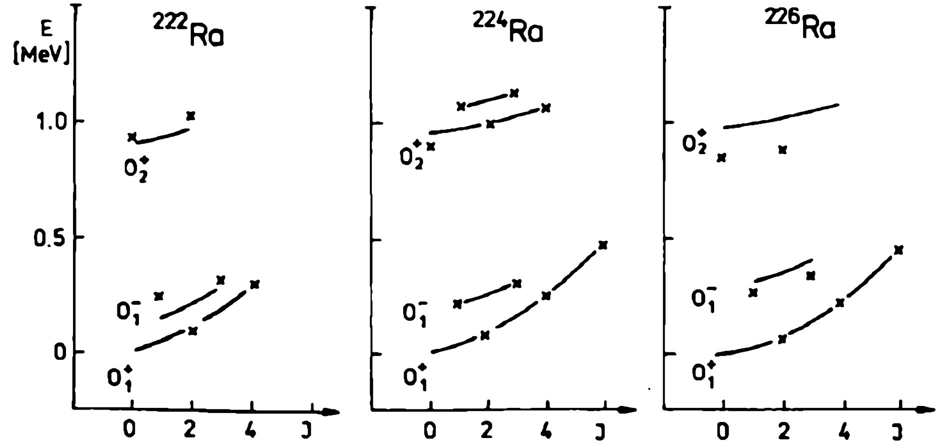

Fig. 4. The calculated (lines) with parameters (in MeV): $H_1 = 0$, $k_1 = 3.1722$, $k_2 = -0.0146$, $k_3 = -0.0218$, $k_4 = 0.2946$, $k_5 = 0.0212$, $k_6 = 0.0333$ and experimental (crosses) spectra of ^{222}Ra [26], ^{224}Ra [23] and ^{226}Ra [24]

Table 1. The ratio for reduced probabilities of the electric dipole transitions from the 1^- state to the yrast band states

$\frac{B(E1,1^- - 2^+)}{B(E1,1^- - 0^+)}$	^{222}Ra	^{224}Ra	^{226}Ra	^{226}Th	^{228}Th	^{230}Th
exp. [ref.]	2.03 [26]	2.18 [23]	1.85 [24]	1.76 [24]	2.06 [23]	2.33 [28] 1.72 [29]
calc.	2.37	2.17	2.60	2.13	2.24	2.30

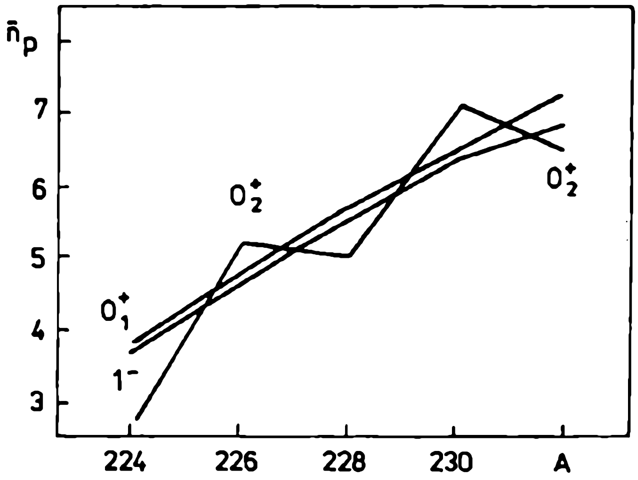

Fig. 5. The average number of $J = 1$, $T = 0$ pairs in the ground state, 1^- state and 0_2^+ state against the mass of ^{A}Th

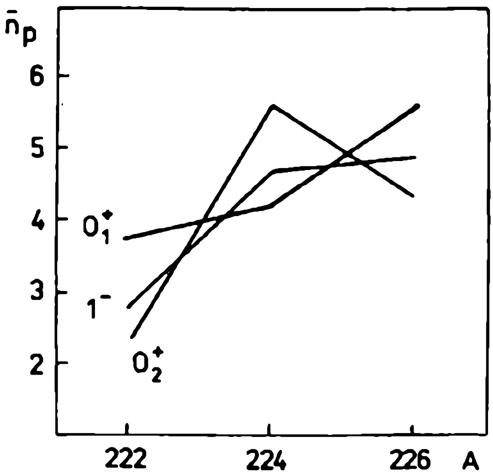

Fig. 6. The same as in Fig. 5 but for ^{A}Ra nuclei

Table 2. Reduced probabilities of $E2$ transitions $0^+_{g.s.} \to 2^+_1$ in ^{A}Ra nuclei

$B(E2, 0^+ \to 2^+)$ (e^2b^2)	^{222}Ra	^{224}Ra	^{226}Ra	^{228}Ra
exp [27]	4.52±38	3.99±16	5.13±28	6.01±49
calc.	3.34	4.09	6.43	7.01

Table 3. Reduced probabilities of $E2$ transitions $0^+_{g.s.} \to 2^+_1$ in ^{A}Th nuclei

$B(E2, 0^+ \to 2^+)$ (e^2b^2)	^{226}Th	^{228}Th	^{230}Th	^{232}Th
exp [27]	6.85±40	7.07±27	8.04±10	9.28±9
calc.	4.80	6.58	8.18	10.14

contain the reduced probabilities of $E2$ transitions $0^+_{g.s.} \to 2^+_1$ in ^{A}Ra and ^{A}Th nuclei.

The calculated values were obtained with the parameter $C_2^2 = 0.200e^2b^2$. Table 4 and Table 5 show the ratios for the reduced $E2$ and $E1$ probabilities but only for ^{230}Th compared with the latest experimental data [29].

Then, we calculate the average number of p and s bosons in any state

$$\bar{n} = \sum_{n_i} |a_{n_i}(J,T,E)|^2 n_i. \tag{8}$$

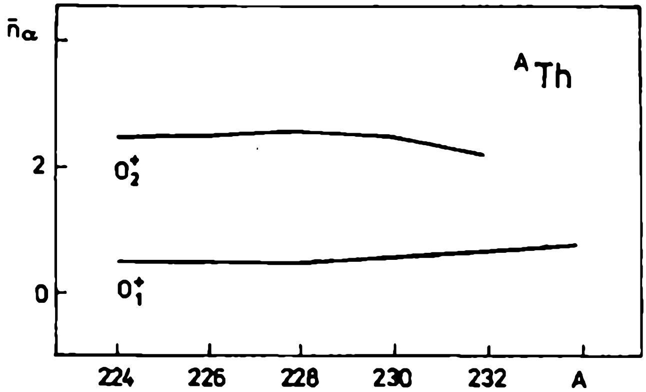

Fig. 7. The average number of α-cluster in the ground state and 0^+_2 state. We obtain in 1^- state the same values of $\bar{n}_\alpha$ as in 0^+_1 state

Table 4. The ratios for reduced probabilities of the quadrupole transitions from some β-band states to the yrast-band states in ^{230}Th

J_β	J_g	J'_g	$B(E2, J_\beta \to J_g)/B(E2, J_\beta \to J'_g)$ exp. [29]	calc.	Alaga
2	2	0	2.31±0.31	1.58	1.42
2	2	4	0.58±0.10	0.35	0.55
2	4	0	3.44±0.51	4.33	2.57
4	4	2	2.06±1.41	1.01	0.91

One can estimate also the average number of α-like clusters

$$\bar{n}_\alpha = 1/2(N - \bar{\omega}), \tag{9}$$

Table 5. The ratios for reduced probabilities of the electric dipole transitions from some octupole states to the yrast-band states in ^{230}Th

J^{oct}	J_g	J'_g	$B(E1, J^{oct} \to J_g)/B(E2, J^{oct} \to J'_g)$		
			exp. [ref.]	calc.	ref. [18]
1	2	0	2.44 [28] 1.72 [29]	2.30	2.30
3	4	2	2.15 [30] 1.95 [28] 1.61 [29]	1.86	1.78
5	6	4	3.17 [28] 2.08 [29]	2.08	2.04

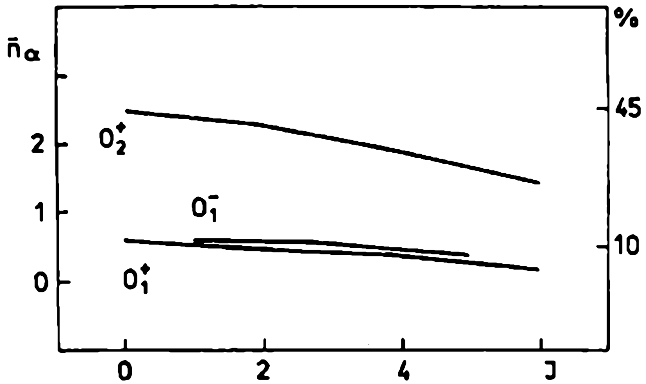

Fig. 8. α-clustering in some states of the $K^\pi = O_1^+$, O_1^- and O_2^+ bands in ^{230}Th

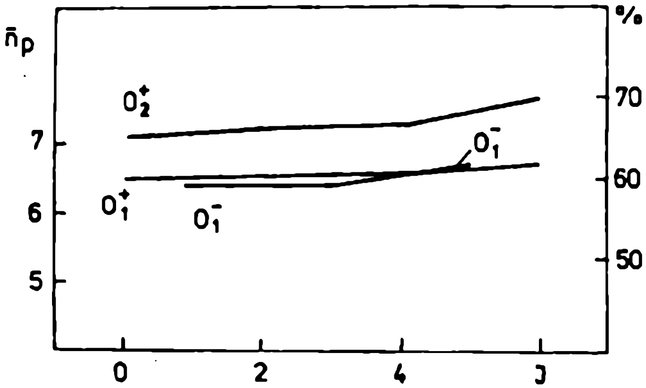

Fig. 9. Percentage of neutron-proton pairs in some states of ^{230}Th

where $\bar{\omega}$ means the average number of bosons not coupled in $J = 0, T = 0$ pairs and it can be extracted from a given eigen-energy of (1) [21]. Figs. 5 and 7 show that the ground state and octupole 1^- state in the studied ^{A}Th nuclei have very similar neutron-proton and α-like structure. This is consistent with the remark [31] that the ground states of heavier ^{A}Th can be octupole-deformed. The fact that reduced width for α-decay is nearly constant in the well-deformed actinides implies that the number of "α-clusters" in the ground states is nearly constant. The calculations confirm this mention, too (Fig. 7). It is noteworthy that our results (Table 5 and others) are similar to the ones obtained in the paper [18] in which s, s^*, p and d bosons are included in the model with $U_{sd}(6) \otimes U_{sp}(4)$ group structure. We can see in Figs. 8 and 9 that within a given band ($K^\pi = 0_1^+, 0_1^-$ and 0_2^+) of ^{230}Th the average number of neutron-proton pairs $\bar{n}_p$ slowly increases with increasing J: simultaneously, $\bar{n}_\alpha$ slowly decreases. In every case we obtain more neutron-proton pairs than neutron-neutron and proton-proton pairs. a similar result was obtained in the paper [17] and in our earlier calculations for the light [33] and rare earth [20] nuclei. It can follow from larger range of the neutron-proton interaction in the $S = 1$ states. Further calculations in the actinide region are in progress.

CONCLUSIONS

Experimental data and many successful calculations in different versions of the interacting model confirm that for low excitation energy the nucleon pairs can be treated as building blocks of a nucleus. By the above assumption and taking into account the most interacting pairs with $J^\pi, T = 0^+, 1$ and $1^-, 0$ we are able to reproduce the experimental energies and the reduced probabilities of $E2$ and $E1$ transitions in even Ra-Th nuclei. It is interesting and unexpected that in every case of deformed nucleus (light [33]) rare earth [20] and actinide nuclei), we obtain over 50% of p-bosons even in the ground states! This suggests that the neutron—proton interactions can be of an origin of collectivity and deformation in any nucleus. These aspects (collectivity and deformation) are the most pronounced in the actinide region where valence neutrons and protons can fill many single particle levels with large j and $j_n = j_p \pm 1$.

APPENDIX

We start, for N-boson system, from the total symmetric $[N\ 00000]$ irreducible representation (IR) of U(6) group generated by operators in (2). This IR contains only total symmetric IR's of $U_{n_p}(3) \otimes U_{n_s}(3)$ (or its subgroup $SU_{n_p}(3) \otimes SU_{n_s}(3)$ and

$$[N00000] = \sum_{\substack{n_p, n_s \\ n_p + n_s = N}} (n_p, 0) \otimes (n_s, 0), \tag{10}$$

Correctness of the reduction in (10) can be checked by comparison of dimensions of IR's:

$$\dim_{U(s)}[N\underbrace{00\ldots0}_{s-1}] = \frac{(N+s-1)!}{N!(s-1)!}. \tag{11}$$

Three quantum numbers distinguish states within IR $(\lambda\mu)$ of SU(3). They are:

$$\begin{aligned} K &= \min\{\lambda,\mu\},\ \min\{\lambda,\mu\}-2,\ \ldots 1 \text{ or } 0, \\ J &= \begin{cases} K, K+1, \ldots K+\max\{\lambda\mu\} & \text{for } K \neq \\ \max\{\lambda\mu\}, \max\{\lambda\mu\}-2, \ldots 1 \text{ or } 0 & \text{for } K=0, \end{cases} \\ M_J &= J, J-1, \ldots, -J+1, -J. \end{aligned} \tag{12}$$

According to (10) all states of the basis (4), and as well calculated eigenstates of Hamiltonian (1), have $K = 0$. K has not any physical meaning and for IR $(n_p, 0)$ is not necessary but in Fig. 3 (and later) $K_i = 0_i$ with $i = 1, 2, \ldots$ is conserved as a sign of a band: 0_1 signifies yrast line, 0_2 — next higher energy and so on.

Table 6. Classification scheme for the group chain (3) and $N = 8$

n_p	J	n_s	T
8	8, 6, 4, 2, 0	0	0
7	7, 5, 3, 1	1	1
6	6, 4, 2, 0	2	2, 0
5	5, 3, 1	3	3, 1
4	4, 2, 0	4	4, 2, 0
3	3, 1	5	5, 3, 1
2	2, 0	6	6, 4, 2, 0
1	1	7	7, 5, 3, 1
0	0	8	8, 6, 4, 2, 0

Table 7. Experimental [23] and calculated relative ground state part of the spectrum ^{224}Ra together with $a_n(02, E)$

E exp MeV calc.	0.0 0.0	0.92 0.94	 2.20	 6.59
$n_p = 0$	0.007	-0.006	0.063	0.998
$n_p = 2$	0.162	-0.120	0.977	-0.064
$n_p = 4$	0.906	-0.375	-0.196	0.004
$n_p = 6$	0.391	0.919	0.048	0.000

The calculations are very simple in spite of the large basis (4) because the matrix of H is quasidiagonal in J and T, an energy is independent with M_J and M_T and the eigenstates of H with even (odd) J are built from the states with $n_p = J, J+2, \ldots N$ or $N-1$. For example in the case of ^{224}Ra with $N = 8$ and $T = 2$, we diagonalise the following matrix: 4×4 for $J = 0^+$, 3×3 for 2^+, 2×2 for 4^+ and 1×1 for 6^+. Next dimensions of H matrix for $N = 8$ can be seen in Table 6. Six parameters are fitted to yrast states with $J^\pi = 2^+, 4^+$, 1^- and 3^- of a few nuclei with $T/N \leq 1/3$. For higher T/N we obtain too small moment of inertia because we limit $\bar{n}_p/N$. Table 7 presents $a_n(J=0, T=2, E)$ for the 0^+ states of ^{224}Ra. We see, the 0^+ states with higher energy have more neutron-neutron and proton-proton pairs than low-lying ones.

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